Dynamics of a Disordered Flux Line Lattice

S. Bhattacharya and M. J. Higgins
NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540
(Received 12 October 1992)

Flow behavior of a flux line lattice in the layered superconductor $2H$-$NbSe_2$ is studied with a magnetic field parallel to the layers in the vicinity of a pronounced peak effect. A striking crossover in the current-voltage characteristics is observed as the system enters the peak regime. The results yield a nonequilibrium phase diagram where conventional depinning of an elastic medium occurs for a rigid lattice. A defective (plastic) flow instability occurs as the lattice softens, but heals at large drives. This defective flow dominates the dynamics for a very soft lattice.

PACS numbers: 74.60.Ge, 71.45.Lr, 62.20.Fe, 74.60.Jg

Dynamics of an Abrikosov flux line lattice (FLL), pinned by quenched random disorder, is representative of a generic problem of collective dynamics of a disordered elastic medium with many degrees of freedom. The competition between interaction, i.e., the elasticity of the medium and disorder in the form of pinning centers leads to a threshold behavior. Above the threshold for depinning, the system moves collectively. In a pioneering theoretical work, Fisher [1] has suggested that the depinning transition represents a "dynamical critical phenomenon." The subject is of wide interest since similar behavior is observed in a broad class of systems, such as the incommensurate charge density wave (CDW) [2], two-fluid interface in a random medium [3] or a rough substrate [4], and Wigner solids in semiconductor heterostructures [5]. Despite the easy tunability of the interaction through the magnetic field, little is known about the dynamics of the moving FLL in the two different regimes of behavior, where interaction and disorder, respectively, dominate the dynamics.

In this paper we present an experimental determination of a "nonequilibrium phase diagram" describing the dynamics of the FLL in both regimes. We find a dramatic change in the I-V curve as the interaction among the flux lines is varied by varying the magnetic field in a low temperature superconductor. We attribute this to *a crossover between a coherent motion of an elastic medium for a relatively stiff FLL, to a primarily defective ("plastic") flow of a soft FLL.* Importantly, the plastic deformations heal at large drives and the elastic regime recovers. This recovery occurs at progressively larger drives as the FLL softens, such that the plastic regime dominates the dynamics for a very soft lattice.

Measurements were performed on single crystal samples of the layered superconductor $2H$-$NbSe_2$. The sample (of dimension 1 mm×1 mm×30 μm) has a T_c of 7.2 K (width ∼20 mK) and a residual resistance ratio $R \sim 20$. Both the magnetic field and the current are in the a-b plane and orthogonal to each other so as to maximize the driving Lorentz force $F_T = \mathbf{J}\times\mathbf{B}$. Pinning is weak in as-grown single crystals, i.e., the FLL is well formed; critical current density is in the 10^1–10^2 A/cm^2 range. The system is well described by the anisotropic Ginzburg-Landau (GL) model [6]. The GL parameter for $\mathbf{H}\|\mathbf{a},\mathbf{b}$ is $\kappa_\| \sim 30$; the anisotropy factor in H_{c2} between $\mathbf{H}\|\mathbf{a},\mathbf{b}$ and $\mathbf{H}\|\mathbf{c}$ is [6] ∼3.3.

Standard current versus voltage curves are used to measure the critical current I_c needed to depin the FLL using a condition of 100 nV as the onset voltage. Low resistance (< 100 μΩ) contacts were obtained by Ag-In solder. The absence of Joule heating was confirmed by (1) reversibility of the I-V data obtained for I increasing and decreasing, (2) ensuring an exact agreement between the differential resistance obtained from the dc I-V curves and from an ac technique using a 1 kHz modulation current, and (3) making the measurements with the sample submerged in liquid He.

The variations of the critical current density J_c and the pinning force $F_p = |\mathbf{J}_c\times\mathbf{B}|$ are shown in Fig. 1(a). The variation of F_p with H, and most specifically the peak in F_p near H_{c2} (∼7 T), i.e., the "peak effect," has been studied extensively [7]. Pippard [7] attributed it to the softening of the shear modulus of the FLL and the easy compliance of the FLL to the pinning configuration, which increases F_p. Subsequently, Larkin and Ovchinnikov (LO) [8] proposed the collective pinning theory. Disorder destroys the long range order of the FLL. Short range order persists with correlation lengths L_c and R_c, respectively parallel and perpendicular to $\mathbf{H}$. The pinning force F_p is given by

$$F_p = |\mathbf{J}_c \times \mathbf{B}| = (W/V_c)^{1/2}, \tag{1}$$

where W is a measure of the pinning: $W = N_p\langle f^2\rangle$; N_p is the volume density of pins and f is the elementary pinning interaction with an interaction range r_f. The correlation volume is $V_c = R_c^2 L_c$ where $R_c = (C_{44}^{1/2}C_{66}^{3/2}r_f^2)/W$ and $L_c = (C_{44}/C_{66})^{1/2}R_c$. As H approaches H_{c2}, the shear modulus C_{66} and the nonlocal tilt modulus C_{44} soften rapidly:

$$\begin{aligned} C_{66} &= (H_{c2}^2/4\pi)(1-1/2\kappa^2)b(1-b)^2(1-0.29b)/8\kappa^2, \\ C_{44} &= (H^2/4\pi\kappa)(1-b), \end{aligned} \tag{2}$$

where $b = H/H_{c2}$, the reduced field. This yields smaller correlation lengths of the FLL and a rapid increase in the pinning force. In the LO scenario, the peak occurs when $R_c \sim a_0$, the lattice constant. In thin films a peak effect

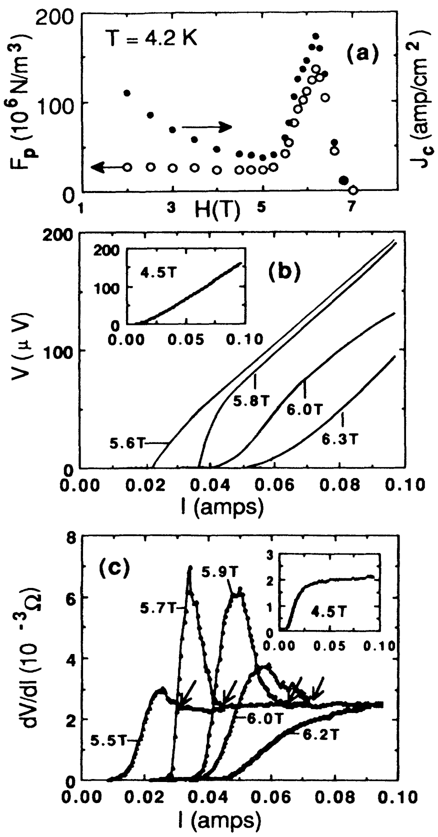

FIG. 1. (a) Field dependence of the critical current density and the pinning force density at $T=4.2$ K for $H\perp c$. (b) Evolution of the I-V curves near the peak regime. Data for $H=4.5$ T, shown in the inset, is the typical behavior below the peak regime. The opposite curvature is seen at $H=5.8$ T. Note the curvature returns to concave upwards at 6.3 T. (c) Evolution of the differential resistance with increasing H. For $H=4.5$ T, it grows monotonically to the flux flow resistance as shown in the inset. At higher fields R_d develops a peak, crossing over to the asymptotic behavior at larger currents. The arrows mark the crossover current I_{cr}. Note that R_d is monotonic for $H=6.2$ T, but it does not reach an asymptotic value at the largest available currents. See the text for discussions.

may also occur because of a dimensional crossover [9] as L_c becomes smaller than the sample thickness. For our samples we estimate R_c and L_c to be ~ 2 and 80 μm, respectively, at $H=4$ T and the sample is in the 3D regime.

While the critical current has been studied extensively, relatively little is understood about the I-V curves themselves. In a theoretical work in the specific context of the CDW systems, Fisher [1] has focused on the nonlinear dynamics above the onset of motion. He proposes, in analogy with static critical phenomena, that a scaling behavior between force and velocity is expected for the onset of motion:

$$V \sim (I-I_c)^{\zeta}, \tag{3}$$

where ζ is the critical exponent. The standard single particle model [10] yields $\zeta=\frac{1}{2}$; it is expected to be different for systems with many degrees of freedom. Little is known about the validity of such a scaling for the FLL and the value of the exponent.

We find remarkable changes in the gross features of the I-V curves as H is varied. In the regime below the peak, the I-V curves are qualitatively identical, the voltage rising concave upwards from I_c. A typical example is shown in Fig. 1(b) for $H=4.5$ T. Note that this is the generic form [10] of the I-V curves in FLL, interpreted as a rounding due to a distribution of I_c values. As is obvious in Fig. 1(b), a dramatic change in the I-V curve occurs as H enters the peak regime, becoming convex upwards with a pronounced inflection point, i.e., change of curvature [11]. As H is further increased, the inflection point moves to large values of I. The systematics of the evolution of these curves with increasing H is better illustrated by the differential resistance, R_d $(=dV/dI)$, whose I dependence is shown in Fig. 1(c). R_d shows a peak, i.e., a sign change of the curvature. The peak value exceeds the normal resistance and thus does not represent R_f, the asymptotic flux flow resistance. Indeed, R_d decreases rapidly at larger I and crosses over to the asymptotic flux flow resistance around a crossover current I_{cr}, marked by the arrows. The peak is smaller at larger H but the crossover at progressively larger values of I; above $H=6.2$ T, it is beyond the range of I values used. In this range of H, scaling fits as in Eq. (3) are poor. The I-V curves are also very noisy in this regime until I exceeds I_{cr}.

These results occur reproducibly in the same sample with a fixed quenched disorder where only the value of H is changed and thus imply an important conclusion: These remarkable differences are intrinsic properties of the FLL. In a recent theoretical work on a 2D FLL, Shi and Berlinsky (SB) [12] proposed that even in the case of arbitrarily weak random potential, there are regions of the FLL where the strain on the FLL is large enough that "phase slips" occur in the FLL and the motion is unstable to plastic flow in the form of channels. This issue has also been addressed by Coppersmith [13] in the context of the CDW systems. Remarkably, our data in this field regime, e.g., $H=5.6$ or 5.8 T in Fig. 1(b), are nearly identical to the results of SB.

SB find that the presence of these defects with density n_d, which is explicitly force/velocity dependent, yields an additional friction that reduces the velocity by an amount proportional to $(Dn_d a_0^2)\pi R_c^2/C_{66}$, where D is a diffusion constant for mobile dislocations. Since this effect is the result of disorder due to the pins, n_d is largest at small forces and vanishes at large forces where pinning becomes insignificant. Indeed, SB find through numerical simulations that n_d depletes rapidly at larger drives. This rapid increase in n_d, as the force (or current I) is decreased, leads to the anomalous convex curvature of the velocity (or voltage V) at threshold. They also note a sign change for the curvature of the I-V curve, which implies the peak in R_d. All these features are seen in our data: (1) a peak in R_d just above threshold, (2) its crossover to the asymptotic flux flow resistance above the crossover current, marked I_{cr} in Fig. 1(c), (3) the simultaneous occurrence of the anomalous I-V curves and a marked increase in F_p, and (4) a recovery of defect-free flow typically at $\sim 2I_c$. Moreover, the noisy dynamics noted in this regime is clearly indicative of a defective flow. A detailed theoretical description of the functional dependence of n_d on the applied force is needed for a quantitative test.

We now return to the region below $H=5$ T in the flat region for F_p and above $H=6.2$ T, i.e., above the peak. As can be seen in Fig. 1(b), the I-V curves for $H=4.5$ T, i.e., below the peak region and for $H=6.8$ T, i.e., above the peak, are quite similar in shape. In both cases V grows concave upwards from I_c, although it is steeper for the latter; indeed, this is the generic I-V curve for the FLL [10]. However, as can be easily seen in Figs. 1(b) and 1(c) the evolution of the curve for the latter field occurs as the plastic regime expands to fill the range of forces used.

In both cases an apparent scaling behavior is obtained with significant differences. Using the methods used for CDW systems [14], we have attempted a scaling fit [Eq. (3)]. For $H=4.0$ T, the apparent exponent $\zeta=1.24 \pm 0.07$, as shown in Fig. 2(a). Interestingly, this is the same as has been measured for the CDW systems [14]. The scaling fit extends over two decades but deviates downwards at current values $\sim$3–4 times I_c where the crossover to the asymptotic regime begins. The same result obtains for all fields studied below the peak regime. Since this is in agreement with the general features for an elastic medium [1], we attribute this regime to the coherent motion of an elastic FLL.

For H above the peak, R_d also rises concave upwards, as shown in Fig. 1(c), but fails to show a similar approach to the asymptotic value, i.e., a current-independent dV/dI that yields the flux flow resistance. The scaling plots illustrating these major differences are shown in Fig. 2(b). The exponent in this case is considerably larger, $\zeta \sim 1.75$. Moreover, as F_p decreases with increasing field above the peak, the scaling regime extends to rather large reduced forces; for $H=6.6$ T, the scaling form fits for the reduced value of current up to $\sim 8I_c$. This implies that the asymptotic flux flow behavior is not obtained even at these relatively large forces.

Note that the exponents in these two regimes signify different physical phenomena [15]. In the former, which is interaction dominated, the exponent measures the collective motion of the FLL which is uniform on average [1]. In the latter, which is disorder dominated, it is a measure of a nonuniform filamentary motion of a defective FLL and the growth of a tenuous structure of connected paths, similar to percolation. Recent theoretical work by Narayan and Fisher [15] speculates that in the disorder-dominated regime, too, a scaling form for the I-V curve may obtain. Our results lend strong support to this proposal. Detailed theoretical calculations or simulations are needed to compare with the results.

Combining these results we construct a nonequilibrium phase diagram for the FLL dynamics shown in Fig. 3, where the abscissa is the reduced field $b=H/H_{c2}$, i.e., the density of the FLL and the ordinate is the reduced force $F\xi/H_c^2$; here the T-dependent values [6] of the anisotropic coherence length ξ and the thermodynamic critical field H_c are used. We attribute the regime below the peak to be the conventional pinned FLL. F_p represents the depinning of an elastic medium above which a coherent motion of the FLL occurs. In this regime we obtain a scaling behavior with an apparent exponent close to the measured value for the CDW, but whether this is the true critical exponent must await further work [16]. In the peak regime, the onset of voltage is due to a plastic flow instabili-

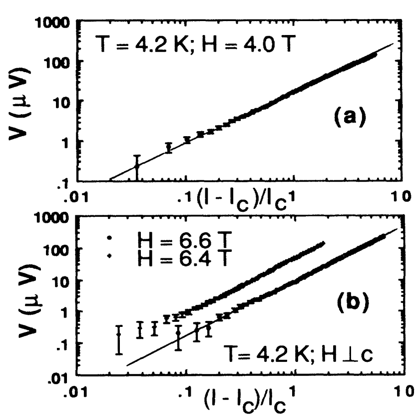

FIG. 2. (a) Scaling behavior in the field regime below the peak. The line marks the exponent 1.24. (b) Scaling behavior above the peak; the line marks an exponent of 1.75. Note the large scaling range at $H=6.6$ T.

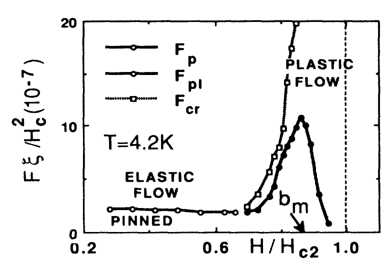

FIG. 3. A nonequilibrium phase diagram of the FLL dynamics. F_p is the conventional depinning threshold separating a pinned FLL from a moving elastic FLL. F_{pl} represents the onset of the plastic flow instability in a defective flux lattice. F_{cr} marks a crossover between the plastic flow and a defect-free elastic flow regime as the defects heal at large drives. b_m marks the Lindemann melting field for a disorder-free FLL. For fields above the peak the pinned FLL may be amorphous. See the text for discussions.

ty of a defective FLL and is marked by F_{pl}. The I-V curve is qualitatively different and in agreement with the simulations of SB [12] and, as they noted, outside the applicability of the LO scenario. In this regime the motion is noisy, as would be expected from such a filamentary motion. There exists another line, denoted by a crossover force F_{cr} $(=|\mathbf{J}_{\mathrm{cr}}\times\mathbf{B}|)$, above which the defects heal and the coherent motion of a defect-free FLL is recovered. As the name implies, this is a crossover and not a transition. As H increases even further, the crossover field increases very rapidly, and much of the available phase space is dominated by plastic flow. In this regime too, a scaling form is obtained, but with a larger exponent.

Finally we return to the case of thermal disorder, which cannot be ignored when H is very close to H_{c2}. The Lindemann criterion [17] has been used extensively for the high temperature superconductors to estimate the possibility of melting of the disorder-free FLL. The condition $u/a_0 \sim 0.1$ is satisfied on the melting line (H_m, T_m) in the H,T, space; here u is the rms fluctuation of the lattice position and a_0 is the lattice constant. Using the anisotropy and nonlocal form of the elastic moduli [17], we find the condition is satisfied for the reduced field ~ 0.87, marked by b_m in Fig. 3, close to where the peak in F_p occurs at $T=4.2$ K. Since the correlation lengths of the disordered FLL can be only smaller than that of a clean FLL, the pinned FLL is most probably amorphous above the peak. Note that in the likely case where the density of pins exceeds that of the FLL, an amorphous FLL is pinned [17] and the flux flow above the threshold resembles a fluid flow [15]. Note, however, that this description of a flux liquid is quite different from the case in the cuprates.

To conclude, we have presented three central results. (1) A dramatic change in the I-V curves occurs in a 3D FLL where a pronounced peak effect occurs which signals a crossover between the coherent motion of an elastic medium and a defective (plastic) flow [12,13] of a "liquid." (2) Scaling behavior indeed obtains in both regimes with different exponents which suggests dynamic critical phenomena belonging to different universality classes [1,15]. (3) These results allow the construction of a nonequilibrium phase diagram of the FLL covering both the interaction-dominated and disorder-dominated regimes. Detailed results from either theory or simulations of the dynamics of realistic FLL's are needed for further tests of the hypotheses presented here. It will be useful to know to what extent the proposed elastic-plastic crossover is relevant to other analogous systems [13] and whether the nonlocal elasticity enhances the plastic flow instability in a 3D FLL. These studies may lend insight into the problem of friction and wear associated with solid-on-solid motion [18].

We thank N. P. Ong for bringing this system to our attention and acknowledge discussions with A. J. Berlinsky, P. M. Chaikin, S. N. Coppersmith, D. S. Fisher, H. R. Kerchner, P. B. Littlewood, A. A. Middleton, O. Narayan, and P. Sibani.

[1] D. S. Fisher, in *Nonlinearity in Condensed Matter*, edited by A. R. Bishop, D. K. Campbell, P. Kumar, and S. E. Trullinger, Springer Series in Solid State Sciences Vol. 69 (Springer-Verlag, Berlin, 1987); also see D. S. Fisher, Phys. Rev. B **31**, 1396 (1985).

[2] G. Gruner, Rev. Mod. Phys. **60**, 1129 (1988).

[3] J. P. Stokes, A. P. Kushnick, and M. O. Robbins, Phys. Rev. Lett. **60**, 1386 (1988).

[4] J. P. Stokes, M. J. Higgins, A. P. Kushnick, S. Bhattacharya, and M. O. Robbins, Phys. Rev. Lett. **65**, 1885 (1990).

[5] Y. P. Li, T. Sajoto, L. W. Engel, D. C. Tsui, and M. Shayegan, Phys. Rev. Lett. **67**, 1630 (1991).

[6] P. deTrey, S. Gygax, and J.-P. Jan, J. Low Temp. Phys. **11**, 421 (1973).

[7] A. B. Pippard, Philos. Mag. **19**, 217 (1969). For an early study in $2H$-$NbSe_2$, see G. A. Spiering, E. Revolinsky, and D. J. Beernsten, J. Phys. Chem. Solids **27**, 535 (1966), and, for a recent study, see P. Koorevaar, J. Aarts, P. Berghuis, and P. H. Kes, Phys. Rev. B **42**, 1004 (1990); also see T. W. Jing and N. P. Ong, Phys. Rev. B **42**, 10781 (1990).

[8] A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. **34**, 409 (1979). For a quantitative analysis, see H. R. Kerchner, *ibid.* **50**, 337 (1983).

[9] R. Wordenweber and P. H. Kes, Cryogenics **29**, 321 (1989), and references therein.

[10] R. P. Huebener, *Magnetic Flux Structure in Superconductors* (Springer-Verlag, Berlin, 1979).

[11] I-V curves with a convex upward curvature have been reported before, specifically in the context of their history dependence. See, for example, M. Steingart, A. G. Putz, and E. J. Kramer, J. Appl. Phys. **44**, 5580 (1973); our results are, however, in the reversible regime. Also see P. Berghuis, A. L. F. van der Slot, and P. H. Kes, Phys. Rev. Lett. **65**, 2583 (1990).

[12] An-Chang Shi and A. John Berlinsky, Phys. Rev. Lett. **67**, 1926 (1991). Our results in Fig. 1(b) for $H=5.6$ and 5.8 T, for example, are nearly identical to that in Fig. 2 of this paper obtained from simulations of a defective FLL.

[13] S. N. Coppersmith, Phys. Rev. Lett. **65**, 1044 (1990).

[14] S. Bhattacharya, M. J. Higgins, and J. P. Stokes, Phys. Rev. Lett. **63**, 1503 (1989). We used the generation of the harmonics of an ac field to ascertain the presence of a single threshold in the elastic depinning regime, i.e., a single peak in the bias dependence of the second harmonic.

[15] O. Narayan and D. S. Fisher (to be published); O. Narayan, Ph.D. thesis, Princeton University, 1992.

[16] The true critical exponent for the CDW remains unclear. See O. Narayan and D. S. Fisher, Phys. Rev. Lett. **68**, 3615 (1992), and references therein.

[17] S. Seung and D. R. Nelson, Phys. Rev. B **39**, 9174 (1989); E. H. Brandt, Int. J. Mod. Phys. B **5**, 751 (1991); Eq. (30) in this reference is used to evaluate the mean square positional fluctuation: $\langle u^2 \rangle \sim k_B T (B\phi_0 C_{66})^{-1/2} \times [b\kappa^2/2(1-b)]^{1/2}(\lambda_\perp/\lambda_\parallel)$.

[18] P. W. Anderson, in *Basic Notions in Condensed Matter Physics* (Addison-Wesley, Reading, MA, 1984), pp. 162–163.