Polarized $q \to \Lambda$ Fragmentation Functions from $e^+e^- \to \Lambda + X$

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Measurement of the helicity asymmetric cross section for semi-inclusive production of Λ hyperons in e^+e^- annihilation near the Z^0 resonance allows a complete determination of the spin-dependent fragmentation functions for the different quark flavors into the Λ. The parity-violating, self-analyzing, decay of the final state Λ makes the experimental analysis of the helicity asymmetry possible. This experiment should be practical with present day technology at the LEP collider at CERN or at the SLAC Linear Collider.

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In the symmetric quark model, the Λ baryon has a rather simple spin-flavor wave function. All its spin is carried by the s quark, while the ud pair is coupled to $S=0$, $I=0$. The Λ thus seems to provide a particularly "clean" example to reexamine the spin crisis [1-3]. In general one has

$$\int_0^1 dx\, g_1^{e\Lambda}(x) = \tfrac{1}{2}\left(\tfrac{4}{9}\Delta u^\Lambda + \tfrac{1}{9}\Delta d^\Lambda + \tfrac{1}{9}\Delta s^\Lambda\right), \tag{1}$$

where Δq^Λ is the Λ matrix element of the q-quark axial charge or, equivalently, the fraction of the spin of the Λ carried by the spin of quarks and antiquarks of flavor q. The weak Q^2 dependence generated by QCD radiative corrections has been ignored for simplicity. In the nonrelativistic quark model $\Delta u^\Lambda = \Delta d^\Lambda = 0$ and $\Delta s^\Lambda = 1$, so $\int_0^1 dx\, g_1^{e\Lambda}(x) = \frac{1}{18}$. A more sophisticated analysis makes use of information coming from flavor SU(3) octet axial charges from hyperon β decay:

$$\int_0^1 dx\, g_1^{e\Lambda}(x) = \tfrac{1}{18}(2\Sigma - D). \tag{2}$$

F and D are invariant matrix elements in β decay, presently estimated to be $F=0.47\pm0.04$ and $D=0.81\pm0.03$, and Σ is the flavor-singlet quark spin operator. The assumption $\langle N|\bar{s}\gamma_\mu\gamma_5 s|N\rangle = 0$ [1] gives $\Sigma = 3F-D$ and predicts $\int_0^1 dx\, g_1^{e\Lambda}(x) = 0.022\pm0.014$. The most reliable prediction is obtained by using the proton data to supply the necessary information on Σ,

$$\int_0^1 dx\, g_1^{e\Lambda}(x) = \int_0^1 dx\, g_1^{ep}(x) - \tfrac{1}{18}(2D+3F). \tag{3}$$

This sum rule is on the same footing as the Bjorken sum rule [4] except that it relies on the full SU(3)-flavor symmetry while Bjorken's requires only isospin invariance. Using the EMC analysis $[\int_0^1 dx\, g_1^{ep}(x) = 0.126\pm0.018]$ we find $\int_0^1 dx\, g_1^{e\Lambda}(x) = -0.042\pm0.019$, far from the naive quark model or the $\langle N|\bar{s}\gamma_\mu\gamma_5 s|N\rangle = 0$ prediction.

While the quark model identifies the Λ spin with the spin of the s quark, the above analysis suggests that the actual situation might be more complex. This becomes more obvious if one uses SU(3) to decompose the last (most reliable) prediction (3) into its Δu, Δd, and Δs contributions, yielding

$$\begin{aligned} \Delta u^\Lambda = \Delta d^\Lambda &= \tfrac{1}{3}(\Sigma - D) = -0.23\pm0.06\,, \\ \Delta s^\Lambda &= \tfrac{1}{3}(\Sigma + 2D) = +0.58\pm0.07\,, \end{aligned} \tag{4}$$

as opposed to the naive expectation $\Delta u^\Lambda = \Delta d^\Lambda = 0$ and $\Delta s^\Lambda = 1$. Unfortunately, no Λ targets are available for deep-inelastic scattering experiments and it thus seems impossible to actually measure $\int_0^1 dx\, g_1^{e\Lambda}(x)$.

In this Letter, we instead show how to measure the polarized fragmentation functions for the decay of quarks into a Λ—an experimental program that, as we will see below, seems to be realistic. The program we describe requires measurement of total inclusive Λ production in e^+e^- annihilation at various energies: off, near, and on the Z^0 peak. Provided Λ's can be reconstructed and their polarization measured in the usual fashion through the self-analyzing decay $\Lambda \to p\pi^-$, the necessary fragmentation functions should be easy to measure.

There is a potentially important background from the process $e^+e^- \to \Sigma^0 + X$ followed by $\Sigma^0 \to \Lambda\gamma$. Λ's produced in this way are *not* part of the $q \to \Lambda$ fragmentation function which includes strong interaction processes alone. (Note, Λ's produced by strong decays of hyperon resonances, $Y^* \to \Lambda X$, *are* properly included in the Λ fragmentation function.) A precise experiment would be required to veto events in which a prompt photon accompanies the produced Λ.

Fortunately, even though excluding secondary Λ's would help to reduce systematic errors this is not crucial since the Σ^0 multiplicity in $e^+e^- \to$ hadrons is typically about a factor 3.5 smaller [5] than the Λ multiplicity. Furthermore, the Λ's from $\Sigma^0 \to \Lambda\gamma$ are depolarized by a factor $\frac{1}{3}$ compared to the initial Σ^0's, which is important here because we are only interested in the helicity asymmetric cross section. Combining the multiplicity suppression with the depolarization effect we arrive at only a 10% contamination for the helicity asymmetric Λ production cross section when secondary Λ's from Σ^0 decay are not vetoed.

In the parton model, the differential cross section for

$e^-e^+ \to h+X$ is obtained by summing over the cross sections for $e^+e^- \to q\bar{q}$, weighted with the probability $d_q^h(z,Q^2)$ that a quark with momentum $(1/z)P$ fragments into a hadron h with momentum P [6],

$$\frac{d^2\sigma^h}{d\Omega\, dz}=\sum_q \frac{d\sigma^q}{d\Omega} d_q^h(z,Q^2)\,. \tag{5}$$

Here $q=k_{e^-}+k_{e^+}$, $Q^2=q^2=s>0$, and $z=2P\cdot q/Q^2$. For a field theoretic definition of the fragmentation function $d_q^h(z,Q^2)$, see, for example, Refs. [7–9]. The more sophisticated treatment is equivalent to the parton model for our purposes so we use the parton model language henceforth. In the naive parton model the fragmentation functions depend only on the scaling variable z. However, similar to deep inelastic structure functions, fragmentation functions in QCD also show logarithmic evolution with Q^2 [8,9]. Energy conservation requires

$$\sum_h \int_0^1 dz\, d_q^h(z,Q^2) z = 1\,. \tag{6}$$

In the following we concentrate on polarized fragmentation functions,

$$\begin{aligned}\Delta\hat{q}(z) &= d_{q(L)}^{\Lambda(L)}(z) - d_{q(L)}^{\Lambda(R)}(z)\\ &= d_{q(L)}^{\Lambda(L)}(z) - d_{q(R)}^{\Lambda(L)}(z)\,,\end{aligned} \tag{7}$$

$$\begin{aligned}\Delta\hat{\bar{q}}(z) &= d_{\bar{q}(L)}^{\Lambda(L)}(z) - d_{\bar{q}(L)}^{\Lambda(R)}(z)\\ &= d_{\bar{q}(L)}^{\Lambda(L)}(z) - d_{\bar{q}(R)}^{\Lambda(L)}(z)\,,\end{aligned} \tag{8}$$

defined as the probability that a left-handed quark of flavor q fragments into a left-handed Λ (with momentum fraction z) minus the probability that the left-handed quark fragments into a right-handed Λ. The interpretation of the antiquark fragmentation function (8) is similar. For simplicity we suppress the Q^2 dependence of the d_q^h. In order to avoid confusion with similar observables in the context of polarized deep-inelastic scattering, we put a caret on all fragmentation asymmetries. For a measurement of these helicity asymmetric fragmentation functions one needs to know both the polarization of the initial state (quark) and the final state (baryon). In the case of the Λ baryon the final state polarization can be easily determined because the (weak) decay $\Lambda \to \pi^- p$ violates parity. In the rest frame of the Λ the decay distribution of the proton is [10]

$$I(\theta) = (1/4\pi)(1+a\cos\theta)\,, \tag{9}$$

($a=0.642\pm0.013$) [5] where θ is the angle between the momentum of the outgoing proton (in the rest frame of the Λ) and the spin of the Λ. For a more detailed discussion of this "self-analyzing" decay we refer to the literature [10].

We now consider Λ production via photons and Z^0's. To exploit e^-e^+ annihilation via photons one has to start from polarized e^- (or e^+) in order to fix the polarization of the quarks. Using (5), as well as the shorthand notation for the asymmetries (7) and (8), one thus finds for the helicity asymmetric cross section assuming $e^+e^- \to \gamma \to q\bar{q}$

$$\frac{d^2\sigma(e^-(L)e^+ \to \Lambda(L)X)}{d\Omega\, dz} - \frac{d^2\sigma(e^-(L)e^+ \to \Lambda(R)X)}{d\Omega\, dz} = \frac{\alpha^2}{2s}\cos\theta \sum_q Q_q^2[\Delta\hat{q}(z)+\Delta\hat{\bar{q}}(z)] = \frac{\alpha^2}{2s}\cos\theta\left(\frac{5}{9}[\Delta\hat{u}(z)+\Delta\hat{\bar{u}}(z)]+\frac{1}{9}[\Delta\hat{s}(z)+\Delta\hat{\bar{s}}(z)]\right), \tag{10}$$

where L,R denotes the helicity of the e^- and the Λ (e^+ unpolarized; polarization of X not measured) and θ is the angle between the momenta of the incoming e^- and the outgoing Λ in the c.m. frame ($\cos\theta = \hat{\mathbf{k}}_{e^-}\cdot\hat{\mathbf{P}}_\Lambda$). Here we have made use of isospin symmetry of the fragmentation functions which implies for a Λ

$$\Delta\hat{d}(z) = \Delta\hat{u}(z)\,, \tag{11}$$

$$\Delta\hat{\bar{d}}(z) = \Delta\hat{\bar{u}}(z)\,. \tag{12}$$

(This is truly isospin symmetry, as distinct from the "isospin symmetry of the sea" often discussed in connection with the Gottfried sum rule.)

At higher energies, where the Z resonance as well as γZ interference are relevant it is not necessary to start from a polarized e^-e^+ state because the parity-violating coupling of the fermions favors certain helicity states. In the standard electroweak theory, combined with parton model assumptions, one obtains

$$\begin{aligned}\frac{d^2\sigma(e^-e^+ \to \Lambda(L)X)}{d\Omega\, dz} - \frac{d^2\sigma(e^-e^+ \to \Lambda(R)X)}{d\Omega\, dz} &= \frac{\alpha^2}{2s}\sum_q \chi_1(-Q_q)\{a_q v_e[\Delta\hat{q}(z)-\Delta\hat{\bar{q}}(z)](1+\cos^2\theta)+2a_e v_q[\Delta\hat{q}(z)+\Delta\hat{\bar{q}}(z)]\cos\theta\}\\ &\quad +\chi_2\{(v_e^2+a_e^2)v_q a_q[\Delta\hat{q}(z)-\Delta\hat{\bar{q}}(z)](1+\cos^2\theta)+2v_e a_e(v_q^2+a_q^2)[\Delta\hat{q}(z)+\Delta\hat{\bar{q}}]\cos\theta\}\,,\end{aligned} \tag{13}$$

where [5]

$$\chi_1 = \frac{1}{16\sin^2\Theta_W\cos^2\Theta_W}\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+\Gamma_Z^2 M_Z^2}, \quad (14)$$

$$\chi_2 = \frac{1}{256\sin^4\Theta_W\cos^4\Theta_W}\frac{s^2}{(s-M_Z^2)^2+\Gamma_Z^2 M_Z^2}. \quad (15)$$

$M_Z = 91.17$ GeV and $\Gamma_Z = 2.49$ GeV are the mass and width of the Z. $v_e = 4\sin^2\Theta_W - 1$ and $a_e = -1$ are the vector and axial vector couplings of the electron to the Z. Here we adopt the conventions of the Particle Data Group [5] where the coupling of a fermion to the Z boson is given by $-(g/2\cos\Theta_W)\bar{\psi}\gamma^\mu(v - a\gamma_5)\psi Z_\mu$. The couplings of the quarks to the Z are ($v_u = 1 - \frac{8}{3}\sin^2\Theta_W$, $v_d = v_s = -1 + \frac{4}{3}\sin^2\Theta_W$, $a_u = 1$, and $a_d = a_s = -1$. Both e^- and e^+ are unpolarized and the L,R denotes the helicity of the Λ. Again using isospin symmetry for the fragmentation functions and inserting the explicit expressions for the axial and vector couplings, we find

$$\frac{4s}{\alpha^2}\left[\frac{d^2\sigma(e^-e^+\to\Lambda(L)X)}{d\Omega\, dz} - \frac{d^2\sigma(e^-e^+\to\Lambda(R)X)}{d\Omega\, dz}\right]$$
$$= \chi_1\Big(\{c_1[\Delta\hat{u}(z) - \Delta\hat{\bar{u}}(z)] + c_2[\Delta\hat{s}(z) - \Delta\hat{\bar{s}}(z)]\}(1+\cos^2\theta) + \{c_3[\Delta\hat{u}(z) + \Delta\hat{\bar{u}}(z)] + c_4[\Delta\hat{s}(z) + \Delta\hat{\bar{s}}(z)]\}\cos\theta\Big)$$
$$+\chi_2\Big(\{c_5[\Delta\hat{u}(z) - \Delta\hat{\bar{u}}(z)] + c_6[\Delta\hat{s}(z) - \Delta\hat{\bar{s}}(z)]\}(1+\cos^2\theta)$$
$$+ \{c_7[\Delta\hat{u}(z) + \Delta\hat{\bar{u}}(z)] + c_8[\Delta\hat{s}(z) + \Delta\hat{\bar{s}}(z)]\}\cos\theta\Big). \quad (16)$$

With $x_W = \sin^2\Theta_W = 0.2325 \pm 0.0008$ [5], the c's are given by

$$c_1 = -2v_e = 0.1400 \pm 0.0064,$$
$$c_2 = -\tfrac{2}{3}v_e = 0.0467 \pm 0.0022,$$
$$c_3 = 4(1 - \tfrac{20}{9}x_W) = 1.9333 \pm 0.0071,$$
$$c_4 = 4(\tfrac{1}{3} - \tfrac{4}{9}x_W) = 0.9200 \pm 0.0014,$$
$$c_5 = 8(1 - 4x_W + 8x_W^2)(1 - 2x_W) = 2.1505 \pm 0.0074,$$
$$c_6 = 4(1 - 4x_W + 8x_W^2)(1 - \tfrac{4}{3}x_W) = 1.3868 \pm 0.0028,$$
$$c_7 = -16v_e(1 - 2x_W + \tfrac{20}{9}x_W^2) = 0.7337 \pm 0.0343,$$

and

$$c_8 = -8v_e(1 - \tfrac{4}{3}x_W + \tfrac{8}{9}x_W^2) = 0.4133 \pm 0.0193.$$

In principle, Eqs. (13) and (16) are sufficient to determine all four independent fragmentation functions [$\Delta\hat{u}(z)$, $\Delta\hat{\bar{u}}(z)$, $\Delta\hat{s}(z)$, and $\Delta\hat{\bar{s}}(z)$] of a Λ separately. For example, the charge-conjugation-even combinations $\Delta\hat{q}(z) + \Delta\hat{\bar{q}}(z)$ [which are proportional to $\hat{G}_1(z)$] and the charge-conjugation-odd combinations $\Delta\hat{q}(z) - \Delta\hat{\bar{q}}(z)$ [which are proportional to the parity-violating fragmentation function $\hat{X}_1(z)$ [11]] may be distinguished by means of their behavior under $\theta \to \pi - \theta$. The contribution from the three relevant quark flavors ($u = d$, s) can be disentangled by varying the invariant mass and thus emphasizing annihilation via photons or Z's or the interference term independently. However, there is one practical limitation to this program: c_1, c_2, c_7, and c_8 are all proportional to the vector coupling of the Z to an electron, $v_e = 4x_W - 1$, which is very small because x_W is very close to $\frac{1}{4}$. This does not limit the possibility of measuring the charge-even fragmentation functions because annihilation via photons and the γZ-interference term are sufficient for this purpose. Furthermore, other numerical factors compensate for the smallness of $1 - 4x_W$ in c_7 and c_8. However, since charge-odd terms do not contribute to annihilation via photons, the smallness of c_1 and c_2 may restrict accurate measurements of the charge-conjugation-odd terms to the linear combination

$$c_5[\Delta\hat{u}(z) - \Delta\hat{\bar{u}}(z)] + c_6[\Delta\hat{s}(z) - \Delta\hat{\bar{s}}(z)]. \quad (17)$$

Alternatively, one can start from polarized e^- for energies near the Z resonance, where one finds

$$\frac{4s}{\alpha^2}\left[\frac{d^2\sigma(e^-(L)e^+\to\Lambda(L)X)}{d\Omega\, dz} - \frac{d^2\sigma(e^-(L)\to\Lambda(R)X)}{d\Omega\, dz}\right]$$
$$= 2\sum_q Q_q^2[\Delta\hat{q}(z) + \Delta\hat{\bar{q}}(z)]\cos\theta + \chi_1(-Q_q)(v_e + a_e)\{a_q[\Delta\hat{q}(z) - \Delta\hat{\bar{q}}(z)](1+\cos^2\theta) + 2v_q[\Delta\hat{q}(z) + \Delta\hat{\bar{q}}(z)]\cos\theta\}$$
$$+\chi_2(v_e + a_e)^2\{v_q a_q[\Delta\hat{q}(z) - \Delta\hat{\bar{q}}(z)](1+\cos^2\theta) + (v_q^2 + a_q^2)[\Delta\hat{q}(z) + \Delta\hat{\bar{q}}(z)]\cos\theta\}$$
$$= 2\{\tfrac{5}{9}[\Delta\hat{u}(z) + \Delta\hat{\bar{u}}(z)] + \tfrac{1}{9}[\Delta\hat{s}(z) + \Delta\hat{\bar{s}}(z)]\}\cos\theta$$
$$+\chi_1\Big(\{\tilde{c}_1[\Delta\hat{u}(z) - \Delta\hat{\bar{u}}(z)] + \tilde{c}_2[\Delta\hat{s}(z) - \Delta\hat{\bar{s}}(z)]\}(1+\cos^2\theta) + \{\tilde{c}_3[\Delta\hat{u}(z) + \Delta\hat{\bar{u}}(z)] + \tilde{c}_4[\Delta\hat{s}(z) + \Delta\hat{\bar{s}}(z)]\}\cos\Big)$$
$$+\chi_2\Big(\{\tilde{c}_5[\Delta\hat{u}(z) - \Delta\hat{\bar{u}}(z)] + \tilde{c}_6[\Delta\hat{s}(z) - \Delta\hat{\bar{s}}(z)]\}(1+\cos^2\theta) + \{\tilde{c}_7[\Delta\hat{u}(z) + \Delta\hat{\bar{u}}(z)] + \tilde{c}_8[\Delta\hat{s}(z) + \Delta\hat{\bar{s}}(z)]\}\cos\theta\Big), \quad (18)$$

where

$$\tilde{c}_1 = 4(1-2x_W) = 2.1400 \pm 0.0064\,,$$

$$\tilde{c}_2 = \tfrac{1}{3}\tilde{c}_1 = 0.7133 \pm 0.0021\,,$$

$$\tilde{c}_3 = 2\tilde{c}_1(1-\tfrac{20}{9}x_W) = 2.0687 \pm 0.0138\,,$$

$$\tilde{c}_4 = 2\tilde{c}_1(\tfrac{1}{3}-\tfrac{4}{9}x_W) = 0.9844 \pm 0.0045\,,$$

$$\tilde{c}_5 = \tfrac{1}{4}\tilde{c}_1^3 = 2.4501 \pm 0.0119\,,$$

$$\tilde{c}_6 = \tfrac{1}{2}\tilde{c}_1^2(1-\tfrac{4}{3}x_W) = 1.5800 \pm 0.0119\,,$$

$$\tilde{c}_7 = 2\tilde{c}_1^2(1-2x_W+\tfrac{20}{9}x_W^2) = 6.0004 \pm 0.0428\,,$$

and

$$\tilde{c}_8 = \tilde{c}_1^2(1-\tfrac{4}{3}x_W+\tfrac{8}{9}x_W^2) = 3.3800 \pm 0.0236\,.$$

Note that $\tilde{c}_1$ and $\tilde{c}_2$ are not small compared to $\tilde{c}_3$ and $\tilde{c}_4$, making it easier to extract the charge-odd asymmetries for u and s quarks separately. As far as disentangling the contributions from the various quark flavors is concerned, the situation is better in annihilation then in deep-inelastic scattering off nucleons. The fragmentation into Λ's allows the measurement of four linearly independent, spin-dependent observables at leading twist (actually six, if one makes use of isospin symmetry). Equivalent measurements in deep-inelastic scattering off nucleons would require the combinations of electromagnetic as well as charged current data from polarized protons and neutrons—a very difficult challenge.

Unfortunately, very little is known theoretically about fragmentation functions—especially the helicity-odd fragmentation functions discussed here. For example, there are no sum rules known, since the moments of fragmentation functions are not related to hadron expectation values of local operators. Also there is little guidance from theory on the z dependence of the polarized fragmentation functions. In a naive quark model for the Λ, one would expect $\Delta\hat{s}(z)$ to be positive, while all other fragmentation functions $[\Delta\hat{\bar{s}}(z)$, $\Delta\hat{u}(z)$, and $\Delta\hat{\bar{u}}(z)]$ should vanish. However, our experience with polarized deep-inelastic structure functions suggests that this picture will most likely be modified. The existence of such a straightforward experimental program to measure the flavor dependence of polarized fragmentation functions should spur the theoretical community to consider these quantities.

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