Extension of the Parker Bound on the Flux of Magnetic Monopoles

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An extension of the Parker bound on the flux $\mathcal{F}$ of magnetic monopoles leads to a stronger bound than obtained previously. The survival and growth of a small galactic seed field requires $\mathcal{F} \leq 10^{-16}(m/10^{17}\ \text{GeV})\ \text{cm}^{-2}\ \text{s}^{-1}\ \text{sr}^{-1}$. This new limit rules out the possibility that monopoles much lighter than $10^{17}\ \text{GeV}/c^2$ can provide the closure density of the Universe.

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The possibility that magnetic monopoles may exist in the Universe has long intrigued both theorists and experimentalists. Dirac [1] first showed that magnetic monopoles could be accommodated within electromagnetic theory if their magnetic charge, g, is given by an integer multiple of $\hbar c/2e$. 't Hooft and Polyakov [2] later showed that magnetic monopoles arise as topological defects in gauge theories; in particular, monopoles are a generic feature of grand unified theories (GUTs). The mass of GUT monopoles is usually set by the scale of unification, thought to be $\sim 10^{15}$ GeV or so; $m \sim m_{\text{GUT}}/\alpha_{\text{GUT}} \sim 10^{17}$ GeV [3].

In the standard cosmology, the number of magnetic monopoles produced in a GUT phase transition is far too large to be compatible with the observed energy density of the Universe: the "monopole problem" [3]. In inflationary models, massive entropy production reduces the monopole abundance within the observable Universe to an exponentially small value [4]. At present, no clear prediction exists for the expected density of monopoles in the Universe. Astrophysics, however, can provide clues for experimentalists about what monopole flux to expect.

In the last ten years the experimental search for GUT monopoles has intensified. Their large masses ($\sim 10^{17}$ GeV) and small velocities ($v \sim 10^{-3}c$) alerted experimentalists to the possibility that previous methods based on ionization by monopoles might not have been sensitive to superheavy slowly moving monopoles. Initially, small induction experiments were tried [5]. However, once it was shown that scintillators could respond to slow GUT monopoles [6], detectors were built with sensitivities to fluxes approaching the astrophysical bounds. With the largest such detector coming on line [7], it is appropriate to reconsider these bounds.

Astrophysical bounds on the magnetic monopole flux fall into three classes: (1) bounds based on the mass density of monopoles either locally or in the Universe, (2) bounds based on monopole catalysis of nucleon decay in neutron stars and white dwarfs, and (3) bounds based on the monopole energy drain from astrophysical magnetic fields. While flux limits based upon monopole catalysis of nucleon decay are the most stringent [8], it is not obligatory that monopoles catalyze nucleon decay.

The original Parker bound, $\mathcal{F} \leq 10^{-16}\ \text{cm}^{-2}\ \text{s}^{-1}\ \text{sr}^{-1}$, was obtained by requiring survival of today's magnetic field in the Galaxy, $B \sim 3 \times 10^{-6}$ G [9]. This bound was reexamined and shown to be mass dependent [10]. In this Letter, we strengthen the Parker bound by considering the evolution of a much smaller seed field early in the history of our Galaxy, $B \sim 10^{-20} - 10^{-11}$ G. This smaller field also had to survive the flux of monopoles traveling through the Galaxy.

The time evolution of the magnetic field in the Galaxy is determined by competition between dynamo action, turbulent dissipation, and (possible) dissipation by a flux of magnetic monopoles. Although the details can be quite complicated [9,11,12], we obtain a good estimate of the behavior of the magnetic field strength B through the equation

$$\frac{dB}{dt} = \gamma B - \alpha B^2 - \frac{Fg}{1 + \mu/B}, \tag{1}$$

where all quantities have been written in dimensionless form: B is the magnetic field strength in units of 3×10^{-6} G (the present day galactic field strength); γ is the growth rate of the field due to the galactic dynamo in units of $10^{-8}\ \text{yr}^{-1}$ (the galactic rotation rate); t is time in units of 10^8 yr; and α represents the action of turbulent dissipation in units of $(300\ \text{G yr})^{-1}$. The final term represents the dissipation of the magnetic field due to a flux of magnetic monopoles; here F is the flux in units of $1.2 \times 10^{-16}\ \text{cm}^{-2}\ \text{sr}^{-1}\ \text{s}^{-1}$ and g is the magnetic charge in Dirac units (we take $g = 1$). There is a net kinetic energy gained by monopoles passing through the Galaxy resulting in a net drain of energy from the magnetic field [9]. The particular form for the dissipation term depends on the quantity $\mu \equiv m_{17} v^2/\ell g$ where m_{17} is the mass of the monopole in units of 10^{17} GeV, ℓ is the coherence length of the magnetic field in units of 1 kpc, and v is the velocity of the monopoles as they impinge upon the magnetic region of the Galaxy, in units of $10^{-3}c$ [10]. We expect a massive monopole to acquire this velocity due to gravitational acceleration by the Galaxy during infall. Light monopoles ($\mu \ll B$) will be accelerated to higher velocities by the galactic field, while "heavy monopoles" ($\mu \gg B$) do not have their velocities changed significantly. At our position immersed in the magnetic region of the Galaxy, we expect light monopoles to be moving much faster than $10^{-3}c$, while heavy monopoles should be moving at about $10^{-3}c$.

By defining

$$V(B) = \tfrac{1}{3}\alpha B^3 - \tfrac{1}{2}\gamma B^2 + FB - F\mu \ln[\mu + B], \qquad (2)$$

we rewrite Eq. (1) as

$$\frac{dB}{dt} = -\frac{dV}{dB}, \qquad (3)$$

where we have implicitly assumed that the parameters γ, α, and μ all vary sufficiently slowly that they can be taken to be constants.

The behavior of $B(t)$ is determined by V: The extrema of V are the fixed points for the evolution of B; further, maxima are unstable fixed points and minima are stable fixed points. If the monopole flux exceeds the critical value $F_c = (\mu\alpha + \gamma)^2/4\alpha$, then V has a single extremum, a minimum at $B = 0$. In this case, the field strength evolves toward zero for all initial conditions. Thus $F > F_c$ is not allowed. In the opposite limit, $F < F_c$, the potential has three extrema at $B = 0, B_+$, and B_-, where B_+ is of order the present strength of the galactic field. Two possibilities for the shape of the potential exist: (1) if

$$F < \gamma\mu, \qquad (4)$$

then V has a maximum at $B = 0$ and the field will evolve toward the minimum at $B = B_+$ [see Fig. 1(a)]. Thus, condition (4) represents a sufficient (but not necessary) condition on the monopole flux for the survival of the galactic field. (2) If $F > \gamma\mu$, the potential has minima at both $B = 0$ and $B = B_+$ and a maximum at $B_- > 0$ provided that $\gamma > \mu\alpha$ [see Fig. 1(b)]. In this case, the field evolves to the value B_+ provided that the initial seed field strength B_0 is sufficiently large,

$$B_0 > B_- = \frac{1}{2\alpha}\{(\gamma - \mu\alpha) - \left[(\gamma - \mu\alpha)^2 - 4\alpha(F - \gamma\mu)\right]^{\frac{1}{2}}\}. \qquad (5)$$

This latter condition implies a flux limit of the form

$$F < (\mu + B_0)(\gamma - \alpha B_0) \le \frac{(\mu\alpha + \gamma)^2}{4\alpha}. \qquad (6)$$

Notice that, in general, $\gamma \gg \alpha B_0$. Thus, a good approximation to this bound is $F < (\mu + B_0)\gamma$.

To summarize, in order for the galactic magnetic field to grow to its present strength, the flux F of magnetic monopoles must obey either bound (4) or bound (6). We also note that with $\alpha = 0$, V can have no stable fixed point (for $B > 0$). The physical significance of this result is that a flux of monopoles cannot (by itself) regulate the magnetic field strength in the Galaxy.

The flux limits derived above depend on the value of the seed magnetic field. For γ of order unity, a seed field larger than 10^{-20} G is required to produce the currently observed field strength [13,14]. The origin of the seed field is unknown, though several generation mechanisms have been proposed [15,16]. Based on these proposed mechanisms, we consider a range $10^{-20}\text{ G} \le B_0 \le 10^{-11}$ G. Our new monopole flux limit is shown as a function of mass in Fig. 2 where we have chosen a realistic upper limit for the seed field strength, $B_0 = 10^{-12}$ G, and have set all other parameters to unity. For $m \le 10^{17}$ GeV, our bound is tighter than the previous Parker bound. A simple analytic estimate of our flux limit is

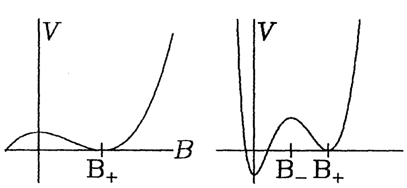

FIG. 1. Sketch of V for $F < (\gamma + \mu\alpha)^2/4\alpha$. (a) For monopole flux $F < \mu\gamma$, there exists a single minimum (for positive field strength) at B_+. (b) For monopole flux $F > \mu\gamma$, there exist two minima ($B = 0, B_+$). In this case, initial field strengths of $B_0 > B_-$ are required to ensure evolution to $B = B_+$.

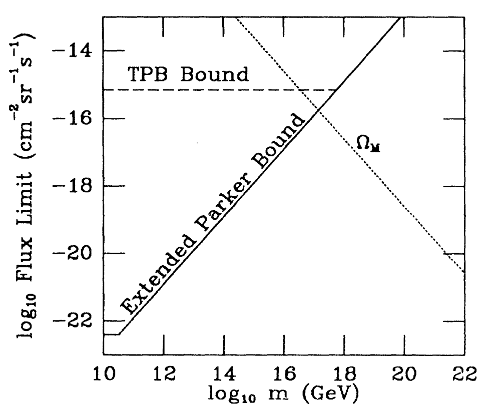

FIG. 2. Monopole flux limits as a function of the monopole mass m in GeV. The line labeled TPB Bound shows the modified Parker bound obtained in Ref. [8]. The solid lines show the extended Parker bounds of this paper. The line labeled Ω_M represents the bound obtained by assuming monoples are uniformly distributed throughout the Universe but do not "over close" the Universe. If the monopoles are clustered with galaxies, this closure bound becomes weaker by a factor of 10^5.

$$\mathcal{F} \lesssim 1.2 \times 10^{-16} \left(\frac{m}{10^{17}\,\mathrm{GeV}}\right) \mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}, \qquad (7)$$

for $m \gtrsim 3 \times 10^{11}$ GeV$(B_0/10^{-11}$ G), and

$$\mathcal{F} \lesssim 3 \times 10^{-22}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}(B_0/10^{-11}\,\mathrm{G}), \qquad (8)$$

for $m \lesssim 3 \times 10^{11}$ GeV$(B_0/10^{-11}$ G). Larger values of B_0 would only increase the level of the "flattened" portion of the limit at low mass.

In passing, we note that one can also obtain a flux limit by requiring that some sort of seed magnetic field must survive during the formation of the Galaxy. In this case, the amplification of the field is due to flux freezing, which produces an effective growth rate given by the collapse time of the protogalaxy. The dissipation term due to a flux of monopoles has the same form as before, although the size of the coherence length ℓ ($\propto 1/\mu$) of the field will be different. We also note that the monopole flux depends on the redshift because $F = nv/4\pi$ and the monopole number density n scales with redshift as $n \sim (1+z)^3$. We can thus derive a bound on the flux of magnetic monopoles by considering the survival of a seed field during protogalactic collapse. Although this bound can be tighter than that obtained from our extension of the Parker bound, we stress that the uncertainties involved are very severe. Indeed, one could consider the implications of the survival of a seed field at earlier and earlier epochs, but with each additional "step" back, more uncertainties arise.

The bound presented here is more stringent than previous Parker bounds for monopole masses below about 10^{17} GeV, and it has been obtained using conservative assumptions. *In principle*, Parker-type bounds can be evaded if monopoles participate in the maintenance of the galactic magnetic field through coherent oscillations [10,17]; in this circumstance the kinetic energy gained by monopoles is returned back to the field a half cycle later. However, it seems unlikely that monopole oscillations can maintain the necessary spatial and temporal coherence in the face of galactic inhomogeneities and their gravitational velocity dispersion [18]. Moreover, such scenarios cannot explain the present field strength through the growth of a very small seed field.

Figure 3 shows the current experimental situation where the most stringent experimental flux limits (90% confidence level) have been plotted versus velocity. Indirect searches involving techniques such as etched nuclear tracks or catalysis that require assumptions other than the electromagnetic interaction of the monopole have been omitted. The combined limit for all searches based on magnetic induction has been obtained from Ref. [19]. The Baksan result [20] and the current MACRO result [21] are based on scintillation and together define the best limits to date for astrophysically interesting monopole velocities. The MACRO experiment [7] is just now becoming fully operational and will approach a sensitivity of $10^{-16}\,\mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\,\mathrm{s}^{-1}$ in a little over five years.

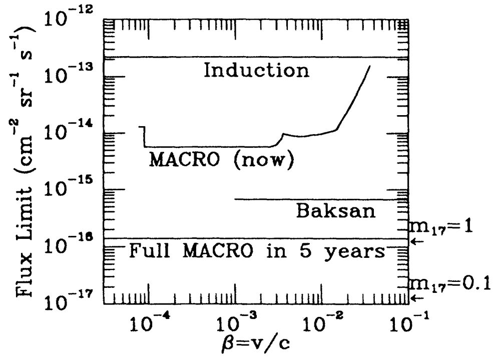

FIG. 3. Direct experimental monopole flux limits plotted as a function of the observed monopole velocity v for an isotropic flux (solid lines). The extended Parker flux limits have also been shown for monopole masses of 10^{17} GeV and 10^{16} GeV. The maximum monopole flux allowed by the extended Parker bound and the closure bound obtains for 10^{17} GeV.

As noted previously, "heavy" monopoles ($\mu \gg B$) should be moving at speeds of order $10^{-3}c$ within the Galaxy, while light monopoles ($\mu \ll B$) move significantly faster, having been accelerated by the galactic magnetic field. To compare our flux limit with the experimental results, one must specify the monopole mass. For monopole masses $< 10^{17}$ GeV the extended Parker bound is more restrictive than the closure bound (see Fig. 2) and thus rules out the possibility that monopoles much lighter than 10^{17} GeV provide closure density for the Universe.

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