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New Measurement of the Relativistic Doppler Shift in Neon

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We report on a new measurement of the relativistic Doppler shift in neon. We measured the frequency difference between a two-photon transition in a fast beam and a slow beam. The result is compared to the prediction of special relativity. The experiment represents a more than tenfold improvement over other Doppler shift measurements and verifies the time dilation effect at an accuracy level of 2.3 ppm.

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Modern laser spectroscopic techniques have been very successful in improving the accuracy of experiments designed to test the fundamental postulates of special relativity (SR). The isotropy of the speed of light *c* was verified by Brillet and Hall [1] in a Michelson-Morley-type experiment to 2.5×10^{-15}. In a recent improvement of the Kennedy-Thorndike experiment, Hils and Hall [2] have shown the constancy of *c* in different reference frames to a level of 2×10^{-13}.

In addition, the isotropy of the one way speed of light is also under investigation [3], and a limit on the anisotropy factor in the one way speed of light of 9×10^{-8} [4] has been obtained. This anisotropy factor is sometimes considered to test for time dilation as well. However, this interpretation is dependent on the assumptions used to construct test models [5] (which are not fundamental theories of physics) of SR and, in particular, requires the designation of a "preferred" reference frame in space.

The time dilation of SR can be tested directly without resorting to specific test models. Bailey *et al.* [6] measured the lifetime of muons in the CERN storage ring and demonstrated an accuracy of 1×10^{-3}. On the other hand, the transverse Doppler effect is a direct consequence of time dilation, and steady improvements in measurements have been made in the last decade using laser techniques [7]. Thus, Kaivola *et al.* [8] have verified time dilation to 4×10^{-5} using a neon fast beam of $\sim0.0036c$, and Klein *et al.* [9] have achieved 3.2×10^{-5} using Li^+ ions circulating at $0.064c$ in a heavy-ion storage ring.

Our experiment is a new measurement of the relativistic Doppler effect in neon, based on the principle of Ref. [8]. We measure the absolute frequency shift in the two-photon transition of a fast moving neon beam relative to a thermal beam of neon. The experiment verifies time dilation to an accuracy of 2.3×10^{-6} and represents more than tenfold improvement over other measurements.

The experiment was divided into two portions: In the first part the transition frequency of the fast beam two-photon absorption (TPA) resonance was measured relative to a hyperfine component of a nearby I_2 reference line. The details of the fast beam measurement were reported in Ref. [10]. In this paper we report on the second part of the experiment, which deals with measuring the frequency difference between the thermal beam TPA resonance and the same I_2 reference line. We combine the two parts to obtain the relativistic Doppler shift and discuss the systematic effects.

The two-photon transition of interest is between the metastable $1s_5$ and the $4s_1'''$ levels in neon, as shown in Fig. 1. For thermal atoms the TPA is an off resonant process. In the fast beam, the $2p_4$ level was tuned into exact resonance when viewing the laser frequency in the rest frame of the atom. As was discussed in Ref. [8], the classical Doppler effect cancels in TPA but the relativistic transverse shift remains.

The experimental setup for the thermal beam measurement is shown schematically in Fig. 2. Metastable neon atoms were produced by running a dc discharge in a free-jet source [11]. The atoms passed through a skimmer hole and entered an interaction chamber with 1×10^{-6} Torr background pressure. Two counterpropagating laser beams crossed the atoms at a right angle.

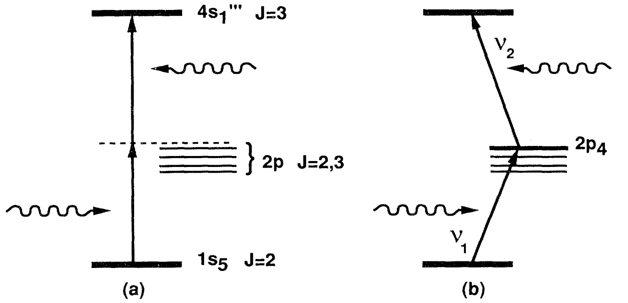

FIG. 1. Partial level diagram of neon. The levels are labeled with Paschen notation. (a) Two-photon transition in the slow beam case is an off resonant process. (b) In the fast beam, an intermediate level is Doppler tuned into resonance.

To enhance the TPA, a Fabry-Pérot cavity was used to provide up to 7.5 W of power in each beam. The cavity also provided a well-defined TEM_{00} mode with a waist radius of 0.23 mm. The cavity length was modulated at 65 kHz and locked to the maximum of the TEM_{00} mode transmission. The interaction region was surrounded by a blackened metal box which shielded both stray light and stray electric fields. Stray magnetic fields were canceled to < 10 mG with three pairs of orthogonal Helmholtz coils.

A cw single mode, 500-kHz linewidth ring dye laser (Coherent 699-21) was used to excite the neon transition. The laser beam was sent through an intensity stabilizer and a Faraday isolator, and mode matched into the cavity. The TPA was monitored by detecting with a photomultiplier the fluorescence resulting from the excitation, in a direction perpendicular to both the atom and laser beams. The long-term frequency stability of the dye laser was obtained by locking the laser to the center of the TPA resonance. The ring dye laser was frequency modulated at 39.5 kHz and the photomultiplier signal was demodulated by a lock-in amplifier to provide the feedback signal for the laser.

A second cw linear dye laser (Coherent 599-21) with 1 MHz linewidth was used for saturation spectroscopy in I_2. The I_2 cell was the same one that was used in the fast beam experiment, and care was taken to duplicate the conditions of that experiment. The laser beam was sent through a single-mode optical fiber to obtain a clean TEM_{00} mode, and expanded to ~ 3.5 mm diameter. A pump beam of 2 mW and probe beam of 0.2 mW were used. The cold finger of the I_2 cell was maintained at $0 \pm 0.1\,^\circ\mathrm{C}$. The laser was frequency modulated at 15 kHz, and third harmonic lock-in detection was used to lock the laser on the center of the I_2 transition.

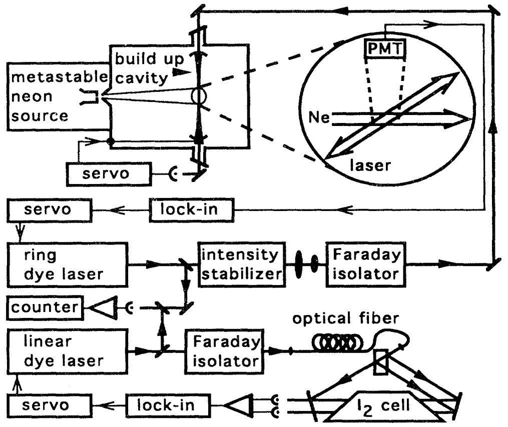

FIG. 2. Schematic of the slow beam experiment. The apparatus includes a two-photon spectrometer consisting of a metastable beam excited by a ring dye laser, a saturation spectrometer consisting of an I_2 cell excited by a linear dye laser, and a heterodyne setup for measuring the frequency difference of the two lasers. PMT: photomultiplier tube.

The frequency difference between the two lasers was measured by sending about 25 μW of each laser for heterodyne detection using an avalanche photodiode (Mitsubishi PD-1002). The diode signal (~ 2.8 GHz, -65 dBm) was mixed with the output from a frequency synthesizer at 2880 MHz (± 1 Hz) and $+6$ dBm. The resulting beat frequency, at about 85 MHz, was amplified and counted with a counter with a 0.5-s averaging time, and stored on a computer.

The beat frequency, taken as a function of the intracavity laser intensity I, used to excite the neon resonance, is shown in Fig. 3. Below 1.8 kW/cm^2, the TPA signal was too weak to effect a good lock. Between 1.8 and 4.0 kW/cm^2, each data point is the average of 2200 readings and each reading was taken with a 0.5-s average time.

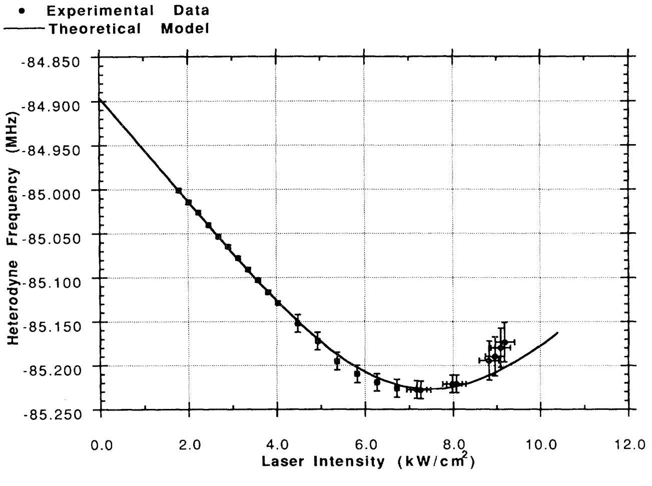

FIG. 3. Measured beat frequency for the slow beam experiment as a function of the laser intensity. The points are experimental results, and the error bars correspond to 1σ uncertainty. The solid line is a fit of the theoretical model to the data. The extrapolation to zero laser intensity yields $-84.8988(34)$ MHz.

Data between 4.0 and 8 kW/cm^2 are the averages of 100 readings. Above 8.2 kW/cm^2, the data are the averages of 20 readings. For higher I, the intensity stabilizer was not used and the points have larger intensity uncertainties. For each I, the data were accumulated on different days over a three-week period to randomize possible errors. The laser did not always have enough power to cover the higher intensity points; thus fewer data were collected, resulting in proportionally larger statistical errors. However, it was found from our model calculations (to be described below) that the high I points have substantially less influence on the zero intensity extrapolation. Thus the larger experimental errors do not present a serious problem.

The measurements must be extrapolated to zero laser intensity. The general shape of the results can be understood in terms of the ac Stark effect and the depletion, or bleaching, of the metastable atoms by the strong laser field. Since only about 10% of the excited atoms return to the original metastable state, there is a bleaching of the metastable density as the atoms traverse the laser beam. In our experiment linearly polarized light was used, and the TPA transition contained two Stark components of $|m_J|=1$ and $|m_J|=2$. These two components shift in opposite directions, with $|m_J|=2$ having the larger shift coefficient and transition probability. At low intensities the net shift is dominated by the ac Stark shift of the $|m_J|=2$ component, and the measured beat frequency decreases approximately linearly with intensity. As laser intensity increases, the $|m_J|=2$ atoms, due to their larger transition probability, are bleached faster before they reach the higher intensity regions of the Gaussian laser beam. This allows the ac Stark shift of the $|m_J|=1$ atoms to have a more pronounced effect, causing the overall shift to turn over.

A rate equation model for the transition was developed for our experiment. The model took into account the natural lifetimes, the ac Stark effect, the Gaussian nature of the laser beam, the bleaching of the metastable atoms, and the detection geometry and efficiency. The equations were numerically integrated to obtain the transition line shape, and the zero crossing point of the derivative of the line shape was computed. The TPA transition probability and the ac Stark shifts are not known precisely, and they were varied as fit parameters. Details of the calculation will be presented elsewhere [12]. A fit to the data, at the 98% confidence level, is shown as the solid line in Fig. 3. The model is in excellent agreement with measurements at low intensities. It is not surprising that the rate equation model does not fit the higher intensity data as well, since coherence effects are ignored.

The data were extrapolated to zero intensity to give a beat frequency value of $-84.8988(34)$ MHz. This value included the statistical errors in the frequency readings and the uncertainty in the model calculations. (When a linear extrapolation of the experimental data to zero intensity was performed for laser intensities below 4.0 kW/cm^2, we found that the result differed by only 0.8 kHz.)

Other possible systematic effects were also investigated. A small stray magnetic field produces a symmetric Zeeman splitting between the m_J states and changes the slope of the ac Stark shift with intensity. This was verified experimentally (by using the Earth's magnetic field as the stray field) and separately with our model. For stray fields below 10 mG the Zeeman splittings for the $|m_J|=2$ and $|m_J|=1$ transitions are 10 and 5 kHz, respectively. For the highest intensity point in Fig. 3 this gives a change in the ac Stark shift of only 0.15 kHz. Thus no correction is applied.

The wave-front curvature effect [13] for our I_2 setup is estimated to be less than 0.2 kHz. We have verified experimentally that the system was insensitive to slight misalignment and for moderate power changes of the pump and probe beams. Temperature variations of the cold finger ($0 \pm 0.1\,^\circ$C) correspond to a maximum pressure shift of ± 0.4 kHz.

To investigate the stability of the laser locks and possible systematic offsets in the electronics, we monitored the beat frequency using two I_2 cells, one for each laser. By exchanging the two cells between the lasers it is possible to deduce the accuracy of the locking electronics. The two cells were monitored over a three-day period, and the lock point accuracy was found to be better than ± 5 kHz.

We now discuss the result of the fast beam experiment [10]. The frequency difference between the I_2 reference and the fast beam TPA resonance was obtained by averaging the data of Ref. [10]. The result, 260.4025(6) MHz, must be multiplied by 2 due to the double passing of the laser beam in the acoustic-optical (AO) frequency shifter. A second AO shifter used as an isolator gave an additional 80.034(1) MHz shift for the TPA resonance. Systematic errors due to the alignment of the laser beams and the fast beam were discussed in Ref. [8], and contributed ± 2.7 kHz to the uncertainty. The voltage stability of the fast beam, ± 70 mV, resulted in an error due to the combined Doppler and ac Stark effect of ± 1.5 kHz. The beat frequencies of the fast beam measurement showed a variation with time, which was most likely caused by the change in the ion source emittance and could result in a slight asymmetry in the ion velocity profile [14]. The effect of this asymmetry was investigated and found to contribute $\leq \pm 3$ kHz.

The experimental results are summarized in Table I. The errors are added in quadrature, and we obtain an experimental value for the transverse Doppler shift $\Delta\nu_{\text{expt}} = -3235.8722(76)$ MHz.

The theoretical value for the transverse Doppler shift predicted by SR may be derived in the following manner. The laser frequency measured in the atomic rest frame is $\nu_{\text{atom}} = \gamma\nu_L(1 - v\cos\theta/c)$, where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the time dilation factor, v is the velocity of the atom, ν_L is the laboratory laser frequency, and θ is the angle between the atom beam and the laser beam directions. In a

TABLE I. Relativistic Doppler shift in neon.

Slow beam experiment	
LO mixing frequency	2880.0000 ± 0.0000 MHz
Beat frequency, slow beam I_2 at zero laser intensity	−84.8988 ± 0.0034
Laser lock accuracy	± 0.005
Fast beam experiment	
Beat frequency, I_2 fast beam	520.8050 ± 0.0012 MHz
Acousto-optic modulator	−80.034 ± 0.001
Alignment uncertainty	± 0.0027
Voltage stability	± 0.0015
Ion source	± 0.003
Experimental Doppler shift	3235.8722 ± 0.0076 MHz
Special relativity prediction	3235.8626 ± 0.0071 MHz

Doppler free two-photon transition in which an atom absorbs one photon from each of the counterpropagating laser beams, the linear $v\cos\theta/c$ terms cancel for the two photons. If ν_1 and ν_2 are the atom rest frame frequencies of the two single-photon transitions as shown in Fig. 1, then the total transition frequency is simply $\nu_0 = \nu_1 + \nu_2 = \gamma(2\nu_L)$. Thus the laser frequency in the laboratory frame is $\nu_L = \nu_0/2\gamma$, showing the redshift due to the transverse Doppler effect.

For the slow beam, the velocity was 830 ± 250 m/s [11], which gives a transverse Doppler shift of 1.9 ± 1.1 kHz. For the fast beam, it is in general not possible to have an adequately narrow velocity distribution. We circumvented the problem by using resonant two-photon spectroscopy to select out a specific velocity group of atoms. The velocity necessary for resonance is determined by the structure of the atomic transition. The laser is collinear with the fast beam, and the Doppler tuning of the intermediate $2p_4$ state shifts the copropagating and counterpropagating laser frequencies to equal ν_1 and ν_2, respectively, in the atom frame, so that $\nu_1 = \gamma\nu_L(1 - v/c)$ and $\nu_2 = \gamma\nu_L(1 + v/c)$. We solve for the unique v/c which ensures resonance and obtain $v/c = (\nu_2 - \nu_1)/\nu_0$. Thus the velocity of the atoms can be calculated from the rest frame transition frequencies. With the neon levels under study, the corresponding beam energy is about 120 keV [10].

We have measured the transition frequencies ν_0 for the $1s_5$-$4s_1'''$ two-photon resonance to be 1011921020.8(1.5) MHz and ν_1 for the $1s_5$-$2p_4$ transition to be 504150972.1(1.8) MHz. Details of the measurement were reported in Ref. [15]. Briefly, these were measured using a temperature-stabilized 1-m quartz confocal interferometer calibrated with I_2 reference lines. The one-photon transition was obtained using saturation spectroscopy in a neon discharge cell, and the value was corrected for pressure shift. Using these values, the fast beam transverse Doppler shift is −3235.8645(70) MHz. The 7.0-kHz uncertainty was obtained by propagating the errors of ν_0 and ν_1, using standard statistical procedures. Note that although the transition frequency measurements have a precision better than 10^{-8}, the final value for the shift, due to its functional dependence on ν_0 and ν_1, has a precision of only 2×10^{-6}. Combining the fast and slow beams, adding their uncertainties in quadrature, the shift predicted by SR for this experiment is $\Delta\nu_{SR} = -3235.8626(71)$ MHz.

The measured result is in good agreement with the prediction of special relativity. The precision of our experiment, ± 7.6 kHz, just resolves the fourth-order Doppler term of 10.3 kHz. The present experiment directly verifies time dilation at an accuracy of 2.3×10^{-6}, which represents a more than tenfold improvement over other experiments.

The present experiment is at its resolution limit. However, using a fast beam with substantially higher energy with an appropriate choice of atomic levels to take advantage of the resonant two-photon process should result in a direct increase of the transverse Doppler shift without corresponding degradation in the resolution. Heavy-ion storage rings [9] or high-energy accelerators are particularly promising in improving the accuracy of this type of experiment in the near future.

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