Energy Loss Rate of Hot Electrons in a Semiconductor: The Role of Anharmonic Interactions

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The hot-electron energy loss rate (ELR) is studied by using the nonequilibrium Green's function approach. The effect due to the anharmonic interaction is considered. As a result, hot electrons lose their energy primarily by creating pairs of acoustic phonons via LO phonons. When acoustic phonons are kept at the lattice temperature, the nonequilibrium distribution for LO phonons is derived by imposing the steady state condition. We show that our result is able to account for the dramatic enhancement of the hot-electron ELR observed in GaAs/GaAlAs semiconductors at low temperatures.

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During recent years intensive experimental [1–7] and theoretical [8–14] efforts have been devoted to the understanding of the hot electron energy loss rate (ELR) in semiconductors. In the hot-electron energy loss experiments, electrons are heated by an external field. With strong electron-electron interaction, electrons equilibrate among themselves at the temperature T_E before giving off energy to phonon systems. It is believed [1–14] that in semiconductors such as GaAs/GaAlAs, electrons lose energy mainly by emitting longitudinal optical (LO) phonons through the Fröhlich interaction for $T_E > 50$ K, and by acoustic phonons through the deformation potential interaction below $T_E < 15$ K.

The ELR for hot electrons was first calculated by Kogan [8] to describe the energy transfer from hot electrons to LO phonons by employing the second order perturbation theory. It was also studied by Lei and Ting [9] using the Green's function method for hot electrons under a strong electric field. Kogan's formula was modified later by a number of authors [10] to account for the hot-optical-phonon effect by introducing a phenomenological finite energy transfer rate from LO phonons to acoustic phonons. It has been well known that these modified formulas are not able to explain the dramatic enhancement of the ELR observed in the experiment of Shah *et al.* [4] at low temperatures. In several subsequent publications, Das Sarma and co-workers [11] investigated this problem by simply renormalizing the optical phonon Green's function with electron-phonon interactions. The renormalized LO phonon spectrum consists of the low lying electron-hole-like and plasmonlike excitations. By assuming that these electron-hole-like and plasmonlike excitations are kept at the lattice temperature T_L, they found that their ELR [11] exhibits orders of magnitude enhancement at low temperatures over that given by Kogan's formula. However, as Dharma-wardana [12] pointed out recently, the particle-hole-like and plasmonlike excitations exhibited in the LO phonon spectrum of Ref. [11] should be at the electron temperature T_E instead of the lattice temperature T_L. When this correction has been taken care of, the enhancement of ELR at low temperatures disappears. Therefore, the underlying physical reason for the low-temperature enhancement of ELR [4] is still unsettled.

In the following, we shall use the nonequilibrium Green's function approach [15] to study the low-temperature enhancement of ELR. In our model, electrons are interacting with each other and are kept at the electron temperature T_E, electrons and LO phonons are coupled by the Fröhlich interaction, and LO phonons can decay into acoustic phonon pairs through the anharmonic interaction. Acoustic phonons are maintained at the lattice temperature or the heat bath temperature T_L. The distribution function for LO phonons will be determined self-consistently from the energy rate balance equation. By choosing the appropriate anharmonic interaction strength, we show that our results agree quantitatively with the experimental ELR of Ref. [4] on GaAs/GaAlAs quantum wells, especially at low temperatures.

The Hamiltonian for our system can be written as

$$H = \sum_p E_p\, c_p^\dagger c_p + \frac{1}{2}\sum_q v(q)\, \rho_q^\dagger \rho_q + \sum_q \omega_L\, a_q^\dagger a_q + \sum_{q\lambda} \omega_{q\lambda} b_{q,\lambda}^\dagger b_{q,\lambda} + \frac{1}{2}\sum_q [M(q)\, \rho_q^\dagger A_q + M^*(q)\, \rho_q A_q^\dagger]$$
$$+\frac{1}{2}\sum_{q,q',\lambda',\lambda''} [V(q,q',q-q',\lambda',\lambda'')\, A_q B_{q',\lambda'}^\dagger B_{q-q',\lambda''}^\dagger + V^*(q,q',q-q',\lambda',\lambda'')\, A_q^\dagger B_{q',\lambda'} B_{q-q',\lambda''}] \,. \quad (1)$$

Here $\rho_q = \sum_p c_{p+q}^\dagger c_p$ denotes the electron density operator. $A_q = a_q + a_{-q}^\dagger$ and $B_{q,\lambda} = b_{q,\lambda} + b_{-q,\lambda}^\dagger$ are the operators for LO phonons and acoustic phonons, respectively. $\omega_{q\lambda} = c_{s\lambda} q$ and ω_L are the phonon dispersions for acoustic phonons (LA and TA) and LO phonons. λ labels the polarization of the acoustic phonons. $v(q) = e^2/\epsilon_\infty q^2$ is the Coulomb interaction between electrons. The Fröhlich interaction matrix element $|M(q)|^2 = \frac{1}{2} v(q) \omega_L (1 - \epsilon_\infty/\epsilon_0)$ is written in

terms of $v(q)$, ω_L, and ϵ_∞ and ϵ_0, the dielectric constants at high and low frequencies. V is the anharmonic phonon-phonon interaction matrix element involving one LO phonon and two acoustic phonons. Other types of anharmonic interactions describing the collisions among LO phonons are treated phenomenologically in the present work. The direct couplings between electrons and acoustic phonons are neglected, because they do not contribute to ELR except for $T_E < 15$ K.

By using the nonequilibrium Green's function approach [15] and renormalizing the LO phonon Green's function with the electron-phonon interaction [11] and the anharmonic interaction in the random phase approximation (RPA), we have derived the hot-electron (at T_E) energy loss rate (R_L) per electron, and the acoustic-phonon (at T_L) energy gain rate (R_G) per electron, respectively, as

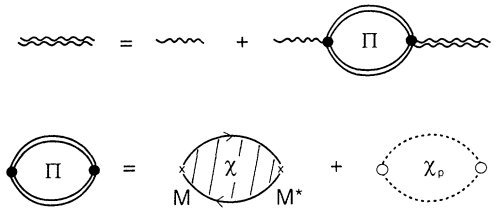

FIG. 1. Feynman diagrams for LO phonon Green's function and LO phonon self-energy. Here single and double wavy lines represent the "bare" $[D_0(q,\omega)]$ and renormalized $[D(q,\omega)]$ LO phonon Green's functions, respectively. LO phonon self-energy $\pi(q,\omega)$ consists of contribution $|M(q)|^2 \times\chi(q,\omega)$ from the Coulomb correlated electron-hole bubble and $\chi_p(q,\omega)$ from the bubble made up by a pair of acoustic phonons (dashed lines).

$$R_L = \frac{1}{N_e}\sum_q \int_0^\infty (d\omega/\pi)\,\omega\,\Gamma_1(q,\omega)\,\{-\mathrm{Im}\,D_0(q,\omega)[n_E(\omega)-n(q,\omega)] + |D_0(q,\omega)|^2\,\Gamma_2(q,\omega)[n_E(\omega)-n_L(\omega)]\}\,\frac{1}{|1-D_0(q,\omega)\pi(q,\omega)|^2} \tag{2}$$

and

$$R_G = \frac{1}{N_e}\sum_q \int_0^\infty (d\omega/\pi)\,\omega\,\Gamma_2(q,\omega)\,\{-\mathrm{Im}\,D_0(q,\omega)[n(q,\omega)-n_L(\omega)] + |D_0(q,\omega)|^2\,\Gamma_1(q,\omega)\,[n_E(\omega)-n_L(\omega)]\}\,\frac{1}{|1-D_0(q,\omega)\pi(q,\omega)|^2}\,. \tag{3}$$

Here N_e is the total electron number. n_E and n_L are the Bose factors at the temperatures T_E and T_L, respectively. $n(q,\omega)$ is the distribution function for LO phonons, to be determined by the steady state condition. $D_0(q,\omega)$ is the retarded Green's function for "bare" LO phonons and $\pi(q,\omega)$ is the total LO phonon self-energy,

$$\pi(q,\omega) = |M(q)|^2\,\chi(q,\omega) + \chi_p(q,\omega)\,. \tag{4}$$

The phonon self-energy contains the contribution $|M(q)|^2\chi(q,\omega)$ from the electron-phonon interaction and the contribution $\chi_p(q,\omega)$ from the anharmonic phonon-phonon interactions. Feynman diagrams for these self-energy contributions are sketched in Fig. 1. Here $\chi(q,\omega)$ is the retarded electron density-density response function. In the random phase approximation, it is given by $\chi_0(q,\omega)/[1-v(q)\chi_0(q,\omega)]$, where $\chi_0(q,\omega)$ is the value for free electron gas. $\Gamma_1(q,\omega)$ and $\Gamma_2(q,\omega)$ are related to the imaginary part of the phonon self-energy by $\Gamma_1(q,\omega) = -2\,\mathrm{Im}|M(q)|^2\chi(q,\omega)$ and $\Gamma_2(q,\omega) = -2\,\mathrm{Im}\chi_p(q,\omega)$.

From the expression of the hot-electron ELR in Eq.(2), we can easily conclude that hot electrons lose their excessive energy in the form of electron-hole pairs and plasmons, which is represented by $\Gamma_1(q,\omega)$, in two distinctive ways: (i) by directly exciting bare LO phonons $[\mathrm{Im}D_0(q,\omega)]$ and (ii) by indirectly emitting pairs of acoustic phonons $[\Gamma_2(q,\omega)]$ via the renormalized LO phonons. It should be pointed out that if $\mathrm{Im}D_0(q,\omega)$ were treated as a δ function, channel (i) could be completely blocked by the presence of the self-energy term $\pi(q,\omega)$ in $|1-D_0(q,\omega)\pi(q,\omega)|^{-2}$, i.e., the first term in Eq. (2) would vanish. Finite contribution to ELR can still be obtained from channel (i), either by broadening $\mathrm{Im}D_0(q,\omega)$ phenomenologically with a collision rate $1/\tau_{pp}$ due to anharmonic interactions other than that given by Eq. (1) or by neglecting $\pi(q,\omega)$ in Eq. (2) which would correspond to the result of second order perturbation theory [8] if the LO phonon temperature is set at T_L. With $1/\tau_{pp}$ in presence, we find that channel (i) is open but still suffers the "hot phonon" effect due to a bottleneck in this electron energy loss channel. This effect has been included here through the LO phonon distribution function $n(q,\omega)$. On the other hand, channel (ii) [the second term in Eq. (2)] does not suffer from any "hot phonon" effect as long as acoustic phonons are maintained at T_L. The first and second terms in Eq. (3), respectively, represent the energy gain rates of acoustic phonons directly from the bare LO phonons and indirectly from electrons via the renormalized LO phonons.

In the steady state, $n(q,\omega)$ is determined by imposing the steady state condition, namely, $R_{L,\mathrm{st}} = R_L = R_G$. This is to say that the ELR from electrons to optical phonons must be identical to that from optical phonons to acoustic phonons. Hence, the steady state LO phonon distribution is found to be

$$n(q,\omega) = \frac{\Gamma_1(q,\omega)\,n_E(\omega) + \Gamma_2(q,\omega)\,n_L(\omega)}{\Gamma_1(q,\omega)+\Gamma_2(q,\omega)}\,. \tag{5}$$

The steady state $R_{L,\mathrm{st}}$ can be written succinctly as follows:

$$R_{L,\mathrm{st}} = \frac{1}{N_e}\sum_q \int_0^\infty d\omega\, \omega\, \Gamma(q,\omega)\, [-\mathrm{Im}\, D(q,\omega)/\pi] \times [n_E(\omega) - n_L(\omega)]\,. \quad (6)$$

Here Γ is given by

$$\Gamma(q,\omega) = \frac{\Gamma_1(q,\omega)\,\Gamma_2(q,\omega)}{\Gamma_1(q,\omega)+\Gamma_2(q,\omega)}\,, \quad (7)$$

and D is the renormalized LO phonon Green's function

$$D(q,\omega) = \frac{2\omega_L}{\omega^2 - \omega_L^2 - 2\omega_L \pi(q,\omega) + i\omega_L/\tau_{pp}}\,. \quad (8)$$

The contribution to the LO phonon self-energy $\chi_p(q,\omega)$ to the lowest order in the anharmonic interaction, according to Fig. 1, can be shown to have the following form:

$$\chi_p(q,\omega) = \sum_{q',\lambda',\lambda''} [\,|\,V(q,q',q-q',\lambda',\lambda'')\,|^2 + V(q,q',q-q',\lambda',\lambda'')\,V^*(q,q-q',q',\lambda'',\lambda')\,] \times \left[\frac{2\omega_{q',\lambda'} n_L(\omega_{q-q',\lambda''})}{(\omega-\omega_{q-q',\lambda''}+i\delta)^2-\omega_{q',\lambda'}^2} + \frac{2\omega_{q',\lambda'}[1+n_L(\omega_{q-q',\lambda''})]}{(\omega+\omega_{q-q',\lambda''}+i\delta)^2-\omega_{q',\lambda'}^2} + \frac{2\omega_{q-q',\lambda''} n_L(\omega_{q',\lambda'})}{(\omega+\omega_{q',\lambda'}+i\delta)^2-\omega_{q-q',\lambda''}^2} + \frac{2\omega_{q-q',\lambda''}[1+n_L(\omega_{q',\lambda'})]}{(\omega-\omega_{q',\lambda'}+i\delta)^2-\omega_{q-q',\lambda''}^2}\right]. \quad (9)$$

The following anharmonic interaction matrix element [16]

$$V(q,q',q-q',\lambda',\lambda'') = -i\,\frac{2\gamma}{c_{s\lambda'}}\sqrt{\frac{\omega_L \omega_{q',\lambda'}\omega_{q-q',\lambda''}}{3NM}}\,\delta_{\lambda',\lambda''} \quad (10)$$

will be adopted for the numerical calculation. Here γ is the Grüneisen parameter, describing the strength of the anharmonic phonon-phonon interaction. M is the mass per unit cell. $c_{s\lambda}$ is the acoustic phonon sound speed. N is the number of unit cells. We note that a more realistic anharmonic interaction model could be employed for ELR calculation. But the present choice renders ELR calculation more manageable, while retaining the essential physics of hot-electron relaxation in semiconductors.

The expression for the LO phonon self-energy $\chi_p(q,\omega)$ is simplified by introducing the F function,

$$F(q,\omega,\lambda') = \frac{1}{3N}\sum_{q'} \omega_{q'\lambda'}[1+2n(\omega_{q'\lambda'})]\frac{2\omega_{q-q',\lambda'}^2}{(\omega+\omega_{q',\lambda'}+i\delta)^2-\omega_{q-q',\lambda'}^2}\,. \quad (11)$$

When $\chi_p(q,\omega)$ is written in terms of the F function, it becomes

$$\chi_p(q,\omega) = 8\gamma^2\omega_L \sum_{\lambda'} \frac{1}{Mc_{s\lambda'}^2}[F(q,\omega+i\delta,\lambda') + F(q,-\omega-i\delta,\lambda')]\,. \quad (12)$$

In deriving the above equation, we have used the symmetric properties of the phonon-phonon interaction matrix element in q',λ' and $q-q',\lambda''$. For the Debye model, all the F functions with different λ are related. This is because different acoustic phonons can be scaled into each other ($\omega_{LA} = c_{sl}q$ and $\omega_{TA} = c_{st}q$ for $q < q_D$).

We have studied the LO phonon self-energy $\chi_p(q,\omega)$ and $|\,M(q)\,|^2\,\chi(q,\omega)$ for semiconductor GaAs/GaAlAs. For the sake of simplifying the numerical calculation, the wide quantum well [4] made up by GaAs/GaAlAs semiconductors has been approximated as a three-dimensional system. The parameters employed here correspond to those of GaAs and they are tabulated in Table I. The Grüneisen parameter is known to be of order unity ($\gamma \sim 1$) for most solids [17]. In the hot-electron energy loss experiment [4], the lattice temperature $T_L = 1.8$ K is much smaller compared to the Debye temperature. Therefore, the lattice temperature will be set to zero in our numerical computation. We find that $\mathrm{Im}\chi_p$ exhibits maxima at $2\omega_{TD} < \omega_L$ and $2\omega_{LD} > \omega_L$, together with some low lying excitations created by the anharmonic interaction. The phonon self-energy due to the electron-phonon interaction has been studied in Refs. [11] and [12]. It is known that $\mathrm{Im}\chi(q,\omega)$ has peaks at the plasmon and electron-hole excitations.

The hot-electron ELR ($= R_{L,\mathrm{st}}$) can be calculated according to Eq. (6) or Eqs. (2) and (5). Since $1/\tau_{pp}$ corresponds to the collision rate among LO phonons which is originated from the anharmonic interaction other than that given by Eq. (1), such a collision process usually involves the excitation of several LO phonons and thus needs much higher energy to accomplish. At low temperature we expect $1/\tau_{pp} \approx 0$. Under this condition, we need only to consider the process in which the excessive energies of the hot electrons in the form of electron-hole pairs and plasmons [$\Gamma_1(q,\omega)$] are lost to create pairs of acoustic phonons via the renormalized LO phonons as the intermediate states. $\Gamma_2(q,\omega)$ corresponds to the spectrum for a pair of acoustic phonons whose excitation energy varies continuously from zero up to $2\omega_{TD}$ and $2\omega_{LD}$. The presence of these low energy excitations

TABLE I. Table for parameters used in the numerical calculations ($a_0^* = \epsilon_\infty \hbar^2/m^* e^2$).

ϵ_∞	10.91	ϵ_0	12.91
c_{sl}	5.29×10^3 m/s	c_{st}	2.48×10^3 m/s
ω_L	37.0 meV	ω_{LD}	31.45 meV
ω_{TD}	14.75 meV	M	$135M_p$
m^*	$0.067m_e$	N_e	1.5×10^{17} cm^{-3}
E_F	15.4 meV	$k_F a_0^*$	1.41

in $\Gamma_2(q,\omega)$, where hot electrons transfer their energy, together with the renormalized LO phonons as the intermediate states [$|D(q,\omega)|^2$] which show strong resonances or peaks near ω_L and plasmon frequency ω_p, is essential for the enhancement of ELR at low temperatures. Now we present our numerical results for $\gamma = 0.2, 0.5$, and 1.0 in Fig. 2, together with the experimental data of Ref. [4]. By choosing the appropriate anharmonic interaction strength such as $\gamma \sim 0.5$, we find that our theoretical calculation can fit the experimental measurement quantitatively over a wide temperature range. It should be noted that $\gamma \sim 0.5$ is reasonable because most of the solids are found to have Grüneisen parameters of order of unity [17].

Finally we wish to emphasize again that in our energy loss channel, the energy is always transferred from the electron system $\Gamma_1(q,\omega)$ (at T_E) to create pairs of acoustic phonons $\Gamma_2(q,\omega)$ (at T_L). On the other hand the energy loss channel of Ref. [11], in which the energy is transferred from the electron system to create particle-hole and plasmonlike excitations $\Gamma_1(q,\omega)$ (assumed at T_L) in the coupled optical modes, is completely absent from our approach. In this aspect, our conclusion is consistent with that of Dharma-wardana [12]. In a recent Comment [13] on Ref. [12], Das Sarma and Korenman tried to argue that their energy loss mechanism is still valid if the anharmonic *collision time* τ due to the decay of a LO phonon into acoustic phonons is set to zero. A close examination shows that such an argument is not self-consistent because they considered only the effect of τ ($\rightarrow 0$) on the distribution function of LO phonons, but completely neglected the $1/\tau$ ($\rightarrow \infty$) term in the dressed LO phonon propagator. For example, the result of Ref. [13] can never be achieved from Eq. (6) if $\Gamma_2(q,\omega)$ [$=\mathrm{Im}\chi_p(q,\omega) = 1/\tau$] is set to be infinitely large.

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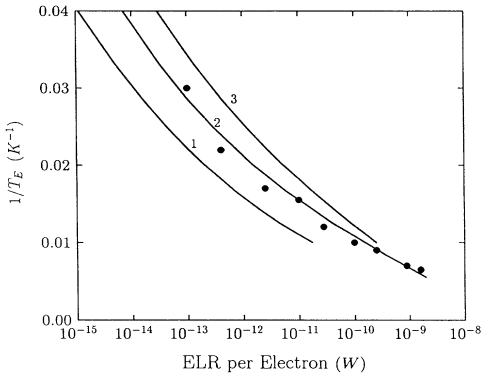

FIG. 2. ELR as a function of electron temperature. Here solid curves represent the results of the present theory, and black dots are the experimental data of Shah *et al.* (Ref. [4]). Curves 1, 2, and 3 correspond to $\gamma = 0.2, 0.5$, and 1.0.

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