Arching Effect Model for Particle Size Segregation

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Starting from 2D experimental observations and using topological arguments, we search for the set of stable positions when a large disk is raised step by step among a 2D array of small monodisperse disks. We show and confirm experimentally that larger disks should move upward continuously via the arching effect whereas smaller disks may only climb up intermittently. We find the critical diameter ratio for a continuous ascent in 2D and 3D. Our model sheds a new light on the decisive role played by vaults in particle size segregation.

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Particle size segregation is commonly observed in many industrial contexts where the process of particulate is prevalent (see [1] and references therein). It occurs during the shaking or the shearing of mixtures of polydisperse granulates. Size segregation chiefly concerns pharmaceuticals, powder metallurgy, coal and mineral processing, solid state chemistry, geophysics, etc. Here, we are concerned with the problem of segregation under shaking of a single (or more) larger particle(s) in an environment of smaller ones. It is still a challenge to understand the basic mechanism involved in the rise of the bigger particle and recently the problem has received much attention since computer models of shaking exhibiting segregation have been put forward in 2D [2–8] as well as in 3D [4]. Each of these models has a recipe that mimics granular dynamics through a piling procedure. The model of Rosato *et al.* is based on a Monte Carlo shaking procedure [3] and the method of Jullien, Meakin, and Pavlovitch involves a series of deposition and relaxation procedures [4,8]. Their models explore some topological properties of the disturbance created by the bigger particle in the field of smaller particles in the spirit of the earlier intuition of Brown [5]. In particular, the void distribution is modified during the shaking, allowing the sifting of smaller particles downward preferentially. Because of the piling procedures, these models are expected to describe situations where large relative motion between the grains is possible. Though they do indeed exhibit segregation, it is still unclear whether the real granular dynamics [6] and the segregation effect are correctly reproduced.

Our approach comes from a different perspective. From the experimental recognition that segregation may occur even in the case of high densities (almost constant contact between the grains), and that the ascent of the intruder, as observed in our experiments, might occur either intermittently or in a continuous regime, we put forward a simple and rather intuitive tentative explanation of the segregation process which allows us to understand these experimental features. It is essential to note here that our model by no means concerns the often observed situation where the intruder is carried upward by a convective flow of the particulate which takes place when the excitation amplitude is far above the threshold (see Ref. [7]). Here we examine a situation where the surrounding array is allowed to relax around the intruder after each upward motion. Therefore we work in a quasistatic limit, just above the threshold of excitation, i.e., when the geometry of the relaxed array is expected to be of predominant importance. Under these circumstances, we stress the importance of the vaults as a leading mechanism that may create a bias in the relative upward motion of a large particle. We use topological arguments sustained by experimental observations to look for the set of stable positions when a larger disk is raised by infinitesimal steps in the surrounding network of smaller monodisperse particles. Our analysis exhibits two different regimes for the upward motion, depending on the diameter ratio Φ of the disks being larger or smaller than a critical diameter ratio Φ_c. A series of experiments on vibrated sets of metallic beads and disks in 2D support our analysis of the problem.

In the following, we investigate the segregation phenomenon in the case of a model granular medium. We work in the spirit of Ref. [7] and we restrict ourselves to a 2D situation where we consider a set of monodisperse small disks of radius r surrounding a larger disk of radius R (called the intruder). This is a situation we model experimentally using a set of corrugated aluminum beads (see Ref. [7]) with $2r = 1.5$ mm and a cylinder the diameter of which can be chosen to be of any size $2R$. We design the width of the cylinder to be 1.3 mm and three steel beads of diameter 1.5 mm are mounted in it in order to adjust to the width of the cell and thus ensure low friction with the frontal boundaries. This experimental situation is reported in Figs. 1(a) and 1(c). It exhibits two fundamental features: First, the intruding disks create a long-range upward triangular perturbation in the small-disk array. Second, the intruder [Fig. 1(a)] may not be in contact with the underlying row of small disks and is rather supported by the two lateral walls lying at an angle of 60° which corresponds to the static angle of repose of a 2D monodisperse disk piling.

Now we model this situation and focus on these features according to the following procedure: We look for the set of successive stable positions of an intruding

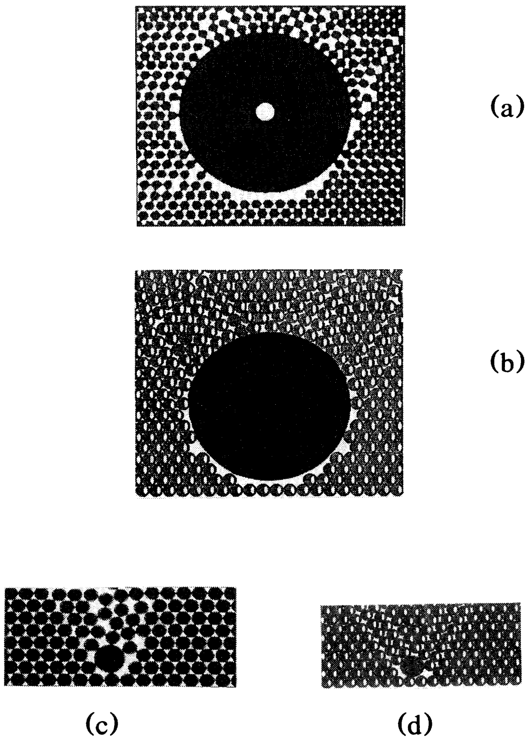


FIG. 1. Various configurations of a disk in an assembly of monodisperse smaller disks. (a) and (c) have been photographed in 2D real experiments whereas (b) and (d) are the results of computer simulation. Note that arching effects are clearly observed in (a) and (b). On the contrary, the smaller disk is supported by the underlying network [(c) and (d)].

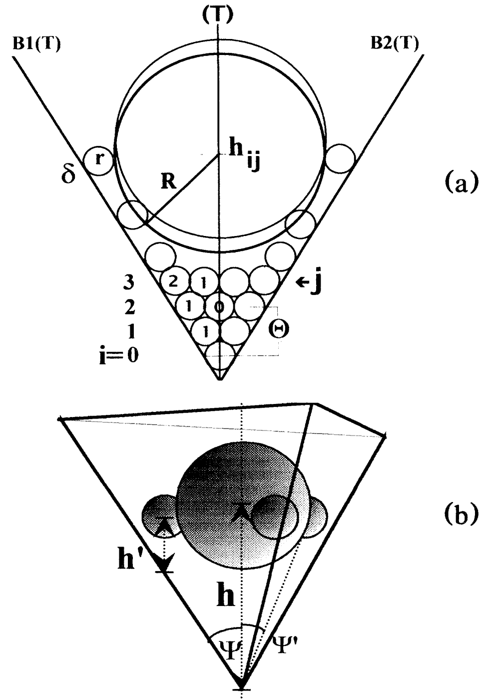


FIG. 2. Schematic diagram describing the vault effect. (a) Triangular (2D) and (b) tetrahedral (3D) schemes used to calculate thresholds for the arching effect.

disk when we lift it up, step by step, in a surrounding array made of smaller disks [Fig. 2(a)]. According to the experimental observation the perturbation in the arrangement of small disks is confined within the triangular domain (T) with an opening angle of 60°. The triangular boundaries of (T), $B1(T)$, and $B2(T)$ in Fig. 2(a) are assumed to act as *rigid* structures. To simplify, we consider a situation where the distribution of disks is symmetrical with respect to the vertical axis, which is not necessarily the case in the real situation (Fig. 1).

Now, considering Fig. 2(a), we search for the successive stable positions of the intruder when we lift it up, step by step, in the small-disk array delimited by the triangular domain (T). Because of the natural periodicity of the problem, it is necessary and sufficient to raise the intruder over a period $\Theta = 2r\sqrt{3}$ (3.46 BU or bead units). A careful observation of the successive situations encountered during the ascent indicates that the intruder may find stable positions according to two distinct processes. First, it may naturally lie on underlying beads belonging to the array. Second, it may be clamped in a vault situation by a mediator made up of a couple of small disks stuck between the boundaries $B1(T)$ and $B2(T)$ as shown in Fig. 2(a). The vault effect will occur when the couple of small disks are at a position below the center of the intruder, which is the minimal condition we consider here for a vault to take place. In the following, we will consider successively these two possibilities and identify them by labeling the height h of the center of the intruder (relatively to 0) by the indexes s for support or v for vault. If we label the rows (i) and the columns (j) of small disks as in Fig. 2(a), the index j will run over the range $k=[(-1)^{i+1}+1]/2$ to $\mathrm{Int}[(i+1)/2]$, where Int stands for the integer truncate of the argument. We find the height of the center of the intruder with respect to the supporting disk (i,j) to be

$$h_{ij}^{S}=[(R+r)^2-r(2j-k)^2]^{1/2}+2r(1+i\sqrt{3}/2)\,. \quad (1)$$

Now, we organize the set of stable supporting positions by increasing height for a progressive ascent of the intruder. However, as mentioned above, the rising intruder may encounter stable positions through the arching effect. When this situation happens, the intruder is at a stable position, just maintained by a vault and a hole is left at the bottom. Every small variation in the height will be stable until the mediator disks become unstable and fall down in the hole. Starting from a configuration when the intruder is in contact with the lateral boundaries $B1,2(T)$ so that $h^{v1}=2r\Phi$, the intruder will meet another arching condition if a couple of small disks can enter the space left between $B1,2(T)$ and the intruder. The condition for starting a new vault is then, approximately $h^{v2}=\sqrt{3}(R$

$+\delta)\cong r\sqrt{3}(\Phi+2)$, where δ is the space between the walls and the intruder. Note that we approximated δ to be $2r$. In a real configuration, the support for the vault is provided by either a small disk protuberance or a hole between adjacent disks, instead of the ideal walls $B1(T)$ and $B2(T)$. This simplification will be justified *a posteriori* by the result of computer simulations. Thus, the fraction of vault configurations S over a period Θ as function of the diameter ratio is

$$S=1-\frac{h^{v2}-h^{v1}}{\Theta}=\frac{2-\sqrt{3}}{2\sqrt{3}}\Phi\cong 0.077\Phi . \quad (2)$$

A continuous ascent via the arching effect is obtained for $S=1$; therefore, the critical diameter for a continuous ascent is approximately

$$\Phi_c^{2D}\cong 12.9 .$$

Now we extrapolate the preceding analysis to the calculation of the critical diameter ratio Φ_c^{3D} in 3D using an extension of the previous simple geometrical arguments and a rough topological reasoning. Let us consider Fig. 2(b) where we have extended the 60° angle concept to a 3D configuration by searching for the simplest polyhedron compatible with the 3D piling and, since we are looking for the threshold quantity, where the Φ_c^{3D} would be maximum. Compared to higher-order polyhedrons, the tetrahedron fit these requirements. Now we look for the diameter of small beads which can enter the tetrahedron along the lines as in Fig. 2(b) which defines h, h', Ψ, and Ψ'. We have

$$h-h'=\frac{R-r}{\sin\Psi}=\frac{R+r}{\tan\Psi'} ,$$

where

$$h=3R,\quad h'=3r,\quad \Psi=\arccos(\tfrac{1}{3}) ,$$

and

$$\Psi'=\arctan(\sqrt{2}/2) .$$

So the critical diameter ratio for continuous ascent is guessed to be of the order of

$$\Phi_c^{3D}=\frac{3+\sqrt{2}}{3-\sqrt{2}}\cong 2.78 .$$

Note that this value is remarkably close to the 2.8 value obtained by Jullien, Meakin, and Pavlovitch [4]. We will discuss the relationship between these findings further in the text.

Now we are back to the 2D case. Using relation (1), we calculate the set of ascent diagrams for different diameter ratios Φ. The result is reported in Fig. 3. The stable positions of the large disk are the solid lines, the dashed lines in this figure represent the unstable configurations.

In order to illustrate the preceding considerations, we set up a simple computer simulation to produce the successive configurations of the piling after each vertical upward motion of the intruder. We use the following procedure. We first lay a regular row of small disks on a horizontal line. Then we pin a larger intruding disk at a height h above the lowest row. Then we introduce small disks in the piling, one after the other and at the lowest possible site. Because of the possibility of multiple second choices, we have checked that filling the pile from one side, or nearest to the center, or at random does not introduce any difference in the results. Dealing with this simple algorithm based only on geometry and gravitation, we were able to reproduce completely the ascent diagram reported in Fig. 3. Note that, although the algorithm does not make use of the triangle approximation, the computer results do not separate significantly from the theoretical ones. Pictures of piles obtained through this simulation are reported in Figs. 1(b) and 1(d).

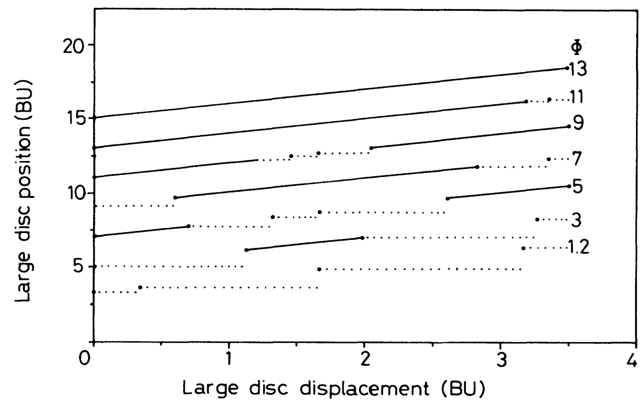
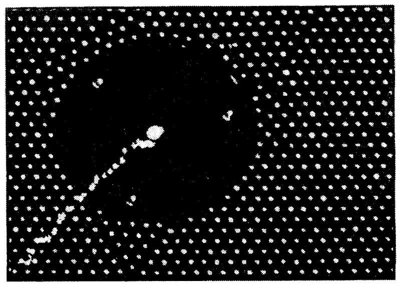


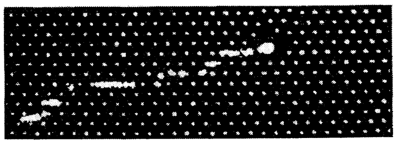
FIG. 3. Ascent diagrams of various disks of diameter ratio Φ. Solid lines represent stable configurations whereas dashed lines depict the unstable ones. The same lines fit to theory and simulation outputs.

Several important consequences may be deduced from the arching effect model. First, it concerns the threshold for upward motion in the case of a vault, which should be infinitely low. In other words, since the intruder can be clamped at any upper position, it always responds positively to any upward solicitation due to infinitesimal fluctuations. In a real experiment it would correspond to an agitation threshold slightly larger than the gravity acceleration. We find this to occur throughout the *whole* period Θ only when the diameter ratio Φ is equal to or larger than the critical diameter ratio $\Phi_c^{2D}=12.9$ ($\Phi_c^{3D}=2.78$ in 3D). Another consequence is that large disks move up in a continuous motion at low amplitude of excitation whereas small ones require a higher amplitude of excitation or should wait for a larger fluctuation to occur; thus their motion is expected to be intermittent.

We tested experimentally these findings with two disks having a small ($\Phi=2$) and a large ($\Phi=16$) diameter ratio. In order to track the trajectories of the intruding disks, we use an image processing device. We decorated the center of the intruders with a bright white round mark that could selectively be recorded by a motorized camera which was driven linearly in time and horizontally in parallel to the container. We stroboscope at the frequency of sinusoidal excitation. We place the disks at a sufficiently large depth to be independent of the natural rolls induced by the boundaries in the upper side (see Ref. [7]). Figures 4(a) and 4(b) *have been obtained by*



(a)



(b)

FIG. 4. Experimental observation under vibrations at 15 Hz of the continuous upward motion of a large steel disk [(a): $\Phi=16$] and of the intermittent ascent of a small steel disk [(b): $\Phi=2$] in a vibrated set of monodisperse aluminum beads. In (a) the acceleration is 10% above threshold Γ_c and it is 20% above threshold Γ_c in (b). The horizontal time scale is the same for both photographs [total 1 h for (b)].

superimposing two photographs. The first photo is the trajectory of the center of the intruder. The second photo is a picture of the cell at the end of the experiment. This photo superposition shows the continuous ascent of the large disk obtained at an acceleration 10% above Γ_c whereas the ascent of the small disk is slower, intermittent, discretized by the lattice rows, and obtained at an acceleration 20% above Γ_c where $\Gamma_c \cong 1.1g$ (g acceleration due to gravity) is the acceleration threshold as defined in [7]. Note that these experiments, to the best of our knowledge, are the first evidence for the existence of a segregation in a dense 2D monodisperse piling. In addition, they display both the continuous and intermittent regimes as predicted in the present work. These experimental results convey substantial support for the finding of a critical diameter ratio by Jullien, Meakin, and Pavlovitch [4,8]. Furthermore our model and experiments provide a clear definition of this quantity which turns out to determine a change in the ascent process (from intermittent to continuous) of the intruder.

Now, we discuss briefly our results in the light of the recently published 3D computer experiments [4,8]. We first remark that the authors noticed that the presence of a hole beneath the intruder is essential to its upward motion, which, in the present context, is nothing else than an evidence for arching support. This corroborates our approach to the problem. Then we ask the question why the piling algorithm used there has no evidence for the intermittent ascent of the small disks which we find in our model and in our experiments. A tentative answer to this question may be found in the following considerations: Contrary to [4,8], the arching model gives information on the effect of the amplitude on the relative motion of the intruder with respect to the small disks piling. In particular, we observe that the upward motion of small intruders, which is not always supported by vaults, can only be obtained by *inducing large upward steps* which allow them to cover distances corresponding to plateaus in Fig. 3. Since the piling procedure in [4,8] is deterministic, and once the configuration has reached a plateau, the intruder cannot climb any further and is stuck at a certain level as can be seen in Fig. 2(a) in Ref. [4]. Our arching effect model explains also why simulations in [4] exhibit large fluctuations for intruders smaller than Φ_c. We find that the ascent of a small intruder depends strongly on the initial condition [see Eq. (1) and Fig. 3]. Moreover it is essential to note that, in our model, avalanches are a *consequence* of the climbing and *not a cause*, which is rather to be searched for in the vault clamping of the intruder.

Actually, if the arching effect model allows us to understand the mechanism which may determine the irreversible "diodelike" biased motion of the intruder, it does not give any information about the motor which drives the process. It is still unknown whether the intruder is lifted up via inertial differential motion of the disk, or via long-range friction interactions such as those responsible for the building up of the "2D sandpile," [7] or via some other process yet to be discovered. A series of quantitative 2D experiments dealing with particle size segregation is now in progress in our laboratory. Results will be published very soon. Investigation of this problem using dynamical simulations now available would be of great interest here. Moreover, let us note that the arching effect model can be extended to polydisperse systems. The problem is to know whether the basic features of this model (existence of a perturbation cone and a threshold for continuous vault configurations) remain valid in this more complex situation.

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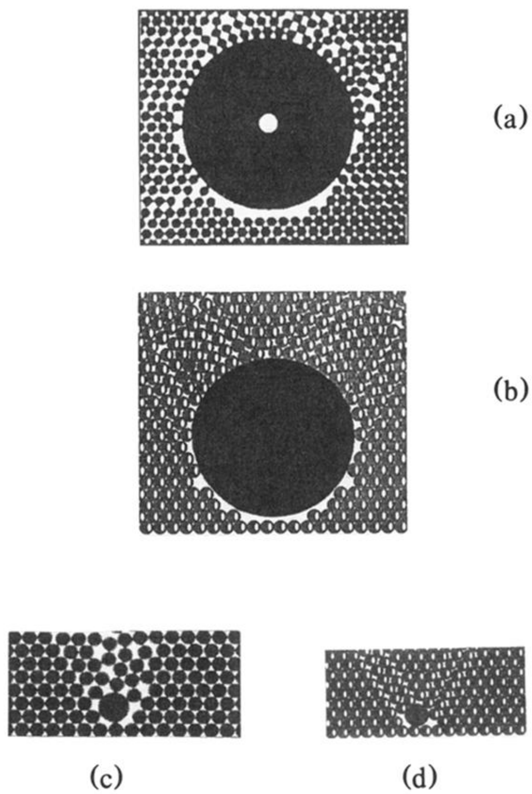

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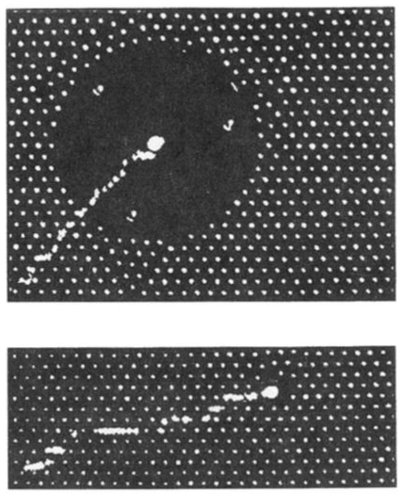

(a)

(b)

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