Solitary Wave Spikes Emitted from a Microwave Free-Electron Laser

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(Received 1 February 1993)

Narrow, high-power "spikes" can be emitted by a 24-GHz free-electron laser (FEL). Spikes, ~450 psec long, may occur randomly or in a sequence which is not the mode-locked period of the resonator. The slippage of the FEL is varied by changing the diameter of the drift tube. The slippage is zero when the electron axial speed is the same as the wave group velocity. Spiking was observed with and without slippage. Measurements of the FEL spectrum are reported and we compare the spike width with a solitary-wave theory. The experiment rules out the sideband instability or superradiance as a cause for this spiking.

PACS numbers: 41.60.Cr

When the free-electron laser (FEL) is operated under conditions of high power, it is possible that the emission can break up into narrow pulses of high-intensity radiation which are called "spikes." This feature has been observed in numerical simulations [1-3] as well as experiments [4,5] where the radiation field is high enough to make the synchrotron period shorter than the undulator length. While these observations relate to the nonlinear state of saturated power in an oscillator, narrow pulses of mode-locked radiation also have been observed by Jerby, Bekefi, and Wurtele [6] under conditions where the radiation field is still in the linear regime of exponential growth; these spikes may have been initiated by pulses in the electron beam current. A model has been proposed by Warren, Goldstein, and Newnam that connects the spiking to the sideband instability [3]; this instability has been studied [7,8] experimentally in a millimeter-wave FEL device at Columbia. The latter experiments were all conducted under conditions where there is finite "slippage," which depends on the difference between the group velocity (v_g) of light waves and the axial speed ($v_\parallel$) of the electrons. Spikelike pulses of radiation are also believed to occur from superradiance, which itself requires slippage of the radiation pulse off a pulse of bunched electrons in an FEL [9-12].

We report an experiment where the FEL can be operated at nearly zero slippage under conditions where the electron beam pulse is essentially infinitely long; our finding is that spiking persists. A model of the nonlinear state of the FEL has been developed by Cai and Bhattacharjee [13] and Bonifacio, Maroli, and Dragan [14]; in the former work, it is shown that the radiation field satisfies a Ginzburg-Landau (GL) equation which has solitary-wave solutions, among them being singularities which resemble spikes and which we compare with our data. In this Letter we show by an experiment and a numerical study of the GL theory that spiking survives the zero-slippage condition where the sideband instability should become stable [15].

Our experiment was done using a Pulseline accelerator, which delivers a 150-nsec, 600-800-kV pulse to a diode (field emission cathode) immersed in a guiding field ~1 T. The experiment is operated as a microwave FEL near 24 GHz, using a bifilar helical undulator with period of 4 cm. The FEL operates (Fig. 1) as either a 24-GHz amplifier using signal from a magnetron, or as an oscillator (at high gain the device oscillates due to reflection at the end of the drift tube). A spectrometer was used to

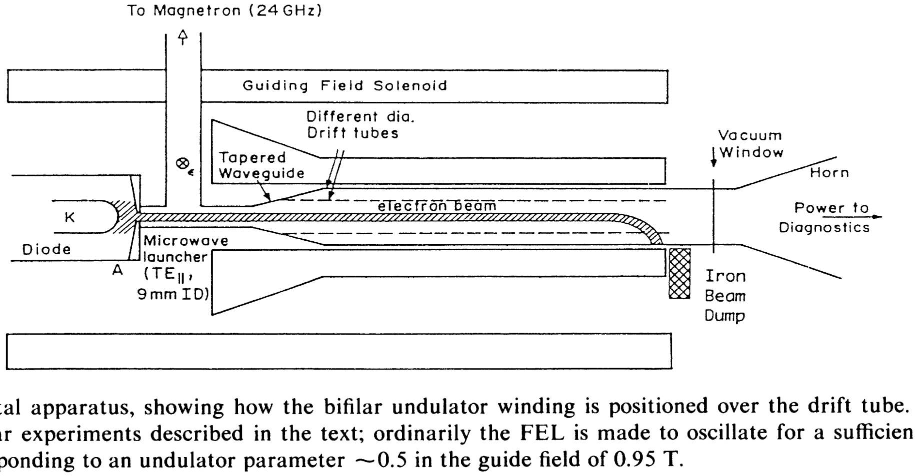

FIG. 1. Experimental apparatus, showing how the bifilar undulator winding is positioned over the drift tube. The magnetron is used only for particular experiments described in the text; ordinarily the FEL is made to oscillate for a sufficiently large undulator field, > 0.8 kG, corresponding to an undulator parameter ~0.5 in the guide field of 0.95 T.

monitor radiation from 19 to 35 GHz emitted from the FEL. In order to change the slippage, we use drift tubes having different inside diameter. The larger drift tube (i.d. 24 mm) is for "conventional" FEL operation with finite slippage at a beam energy of 620 kV, while zero slippage can be obtained at 22 GHz for a 710-kV beam with a drift tube i.d. of 17 mm. The larger drift tube had a ratio of electron parallel velocity to wave group velocity of 0.91 under operating conditions. For a twenty-period undulator, this gives a slippage time ~300 psec at 24 GHz for the larger tube; however, in the smaller drift tube the slippage time varies from zero to ~100 psec across the band of frequencies that we observe to be emitted.

Calibration of the undulator magnetic field was done for each configuration, as the penetration of the field from the pulsed capacitor bank which powers the undulator depends on the diffusion of field through the stainless steel walls of the drift tube. The undulator has a 10-cm entry zone where the helical field gradually increases. The undulator is pulsed to produce a field of 0.9 kG, which causes a transverse beta of 0.22 (undulator parameter, 0.5) in the guiding field. The microwave coupler is designed to launch a TE11 mode into the drift tube from the rectangular waveguide that connects to the magnetron. The diode voltage of the accelerator is adjusted to amplify radiation at 24 GHz. Most of the data were taken with the FEL in an oscillator mode, with the magnetron off. The FEL output signal lasts about 100 nsec and reaches a power of approximately 10 MW.

Figure 2 shows portions of two typical shots where spiking appears. The data were taken with a Tektronix SCD 5000 Transient Digitizer, which has a maximum acquisition rate of 200 GS/sec and an analog bandwidth of 4.5 GHz. Data obtained from several shots permit a measurement of the spike FWHM, 450 ± 100 psec. Spikes may occur singly with random spacing, or in a periodic array (as in Fig. 2) throughout the duration of the saturated FEL power pulse. The periodicity is not related to the round trip travel time of radiation in the drift tube, which is 7 nsec.

Spiking was observed under all conditions of operation, for near zero and finite slippage, for spectra which showed sidebands, and for spectra which showed more nearly coherent emission obtained by using a resonant reflector. Spikes are emitted from the oscillator even when a strong "seed" signal (~20 kW, 24 GHz) is injected into the system; this signal causes the oscillator to emit a much narrower band of frequencies centered on 24 GHz. The average spike width was not found to depend on the size of the drift tube.

The spectrum of the radiation was obtained with a grating spectrometer; the 24-GHz magnetron was used to

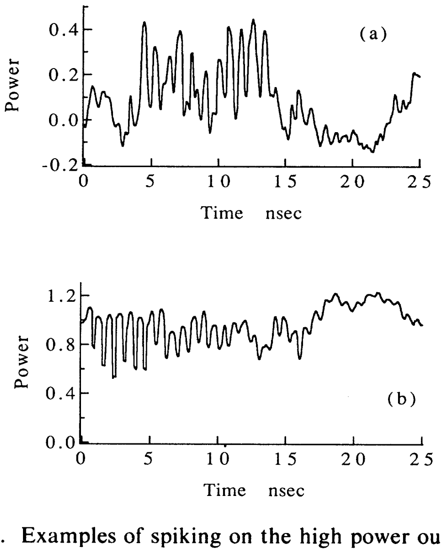

FIG. 2. Examples of spiking on the high power output of the FEL. (a) Operating near 24 GHz as an oscillator with a Bragg reflector at nearly zero slippage (710 kV, 17-mm drift tube); (b) oscillator with much larger slippage (620 kV, 24-mm drift tube). Spiking occurs from 4 to 13 nsec in (a), and from 0 to 11 nsec in (b). Power scale is in arbitrary units. Note FEL power level in (a) is lower than in (b). These portions of the laser pulse are taken following saturation.

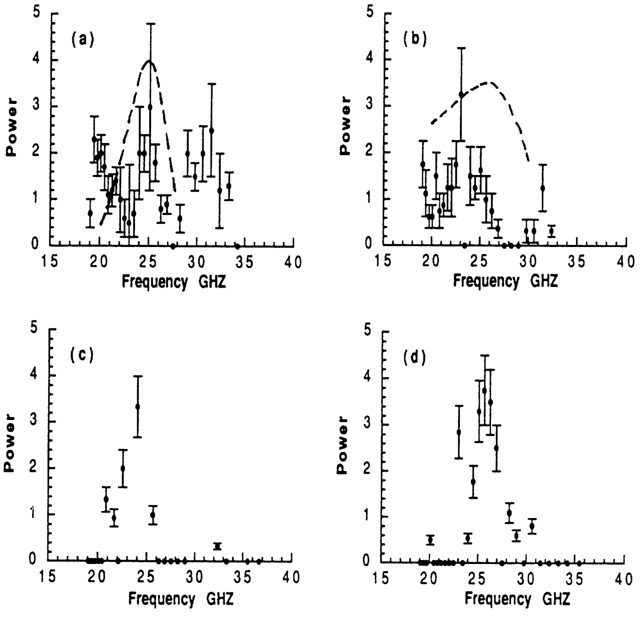

FIG. 3. Power spectrum (arbitrary units). (a) 24-mm i.d. drift tube FEL oscillator (V=620 kV); (b) 17-mm i.d. drift tube FEL oscillator (V=710 kV); (c) same as (b), but with a 20-kW signal injected at 24 GHz; (d) same as (b), but with a Bragg reflector installed at the output of the FEL. (a) and (b) also show the single-pass unsaturated power gain (dashed curve), calculated using the code described in Ref. [8]; this shows the spectral zone over which appreciable power output can occur. "Error bars" indicate the range over which the experimental data fluctuate.

calibrate the wavelength scale. Several spectra are shown in Fig. 3: 3(a) is the output of the oscillator operated with a 24-mm drift tube (620 kV), while 3(b)-3(d) are spectra of the 17-mm drift tube (710 kV). In Fig. 3(a) there appear what might be sidebands, which have about the correct displacement from the 24-GHz carrier, ~ ± 15%, considering the synchrotron period and the slippage [8]. The carrier is identified by the beam energy for which the magnetron signal at 24 GHz shows maximum gain. In Fig. 3(b) we find the sidebands are much less prominent. The gain spectrum becomes very wide when the "tangency" condition ($v_\parallel = v_g$) applies, as indicated by a numerical calculation of the single pass power gain in the small signal limit (dashed line). In Fig. 3(c) a high-power "seed" signal is injected into the oscillator, and we find that the oscillation now occurs at the same frequency as the input, with no sidebands apparant (at tangency condition, the sideband instability is suppressed [15]). Finally, in Fig. 3(d) we show the spectrum of the 17-mm drift tube FEL oscillator under conditions when the output power is reflected by a 26-GHz Bragg reflector [16], showing a much more coherent [17] operation than in Fig. 3(b); there appear to be no sidebands of the type found in Fig. 3(a). Spiking nevertheless has been observed under all conditions. The detectors in the spectrometer average over the spiking phenomena, and therefore the spectra that we show here describe the FEL output in some "average" sense, so that a Fourier transform of these spectra would not necessarily result in a spike pulse.

We now compare the observed spiking with the prediction of a solitary wave theory [13] applicable to our experiment. This theory represents the gain and dispersive behavior of the FEL signal by a Ginzburg-Landau equation:

$$\frac{\partial A}{\partial z} = i\lambda_0(\omega_0)A - v_g^{-1}\frac{\partial A}{\partial t} - \frac{i}{2}(\alpha_1 + i\alpha_2)\frac{\partial^2 A}{\partial t^2} + i\beta|A|^2 A \,. \qquad (1)$$

In the above, A represents the amplitude of the electromagnetic wave, α_1 is the group velocity dispersion, α_2 is the gain dispersion, $\lambda_0(\omega_0)$ is the optimum eigenvalue of the signal growth at frequency ω_0, and β is a complex coefficient which determines the signal saturation and is obtained by a WKB analysis of the reduced FEL equations in Ref. [13]. Two types of analytic solutions were obtained for the solitary-wave type of pulse: a periodic array of very narrow spike singularities and a broader isolated spike [see Eqs. (17) and (16), respectively, of Ref. [13]]. In the case of our experiment, the first solution predicts spikes of FWHM ~7 psec and spacing ~22 psec, which is too fast for our apparatus to record. The broader spike is predicted by the theory (see below) to have FWHM T~330 psec, which compares with the experimental measurement of FWHM (450 ± 100 psec). This broader spike solution has a power profile varying as $\sim 1/\cosh^2(1.76t/T)$, which we have found to be an acceptable representation of a typical observed spike in Fig. 2(a). In Ref. [13], it was pointed out that the analytic solitary-wave solutions come in two families, but it is not possible to determine which class of solutions will actually occur. This may depend on the initial conditions of the experiment, for example, irregularities of the electron beam current.

We have programmed the solution of the GL equation to study the evolution of spikes from different initial conditions. The coefficients in Eq. (1) are determined using a numerical simulation [8] to obtain $\lambda(\omega)$ (thus the waveguide and the beam filling factor are accounted for), and for the experimental data appropriate to Fig. 2(a), $v_g/c = 0.91$, $\mathrm{Im}(\lambda_0) = 0.084\ \mathrm{cm}^{-1}$, $\alpha_1 = 2.2 \times 10^{-23}\ \mathrm{sec/cm}^2$, $\alpha_2 = 7.2 \times 10^{-24}\ \mathrm{sec/cm}^2$ [the experimental value for $\mathrm{Im}(\lambda_0)$ is $0.05\ \mathrm{cm}^{-1}$]. The beta ($\beta_r = 20\ \mathrm{cm}^{-1}$, $\beta_i = 4.7\ \mathrm{cm}^{-1}$) is obtained from $\lambda(\omega)$. If one injects a small amplitude initial spike that resembles the analytic solution with the correct phase, it will grow to several MW intensity, maintaining a width of 330 psec throughout the growth as shown in Fig. 4(a). If one chooses an initial spike that has the correct amplitude profile, but not the correct phase, then as it grows and reaches saturation, it

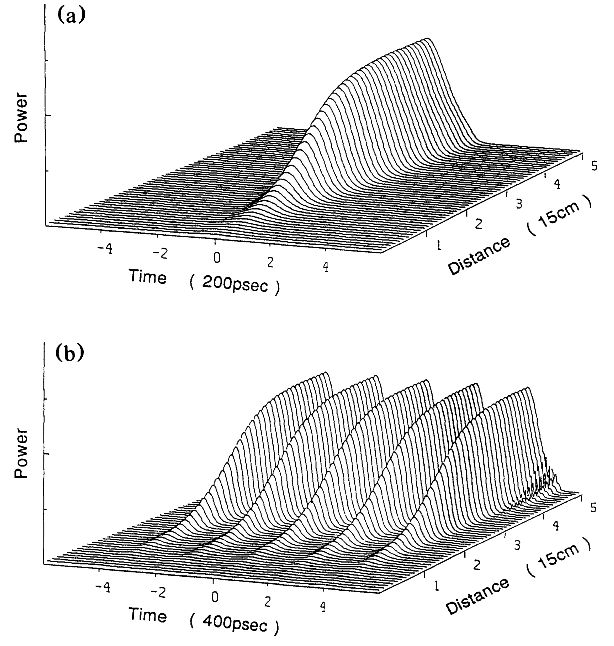

FIG. 4. (a) Growth and saturation of a solitary pulse along the electron beam of the microwave FEL using a numerical solution of the GL equation; parameters are appropriate to the data shown in Fig. 2(a) and give a pulse 330 psec FWHM. (b) A group of spikes, 330 psec wide, spaced 800 psec apart, obtained from the GL equation, similar to those found in Fig. 2. The vertical scale is the same in (a) and (b) and the calculation extends to 75 cm.

will become broadened. Thus it is possible for certain irregularities of the excitation that satisfy the initial conditions to grow into spikes in accord with the analytical solution while others may not. The experiment also showed sequences of closely spaced spikes, which we simulated with a string of initial pulses each of which has the amplitude and phase of the exact solution of the GL equation. If the spacing of the spikes is a few FWHM times, each pulse evolves almost independently, maintaining its width. When the spacing is narrowed excessively, in deep saturation there appear additional pulses of FWHM ~10 psec from the interference (our microwave detector cannot resolve such a short pulse). In Fig. 4(b), the spike spacing is 800 psec, as found in the data of Fig. 2, but if the spacing is reduced to one FWHM, the individual spike feature disappears.

We have found that the FEL has the capability to produce narrow pulses of high-power radiation in both the linear as well as the nearly saturated regime, even in the case of almost zero slippage and very long electron pulse. Thus we rule out the sideband instability and superradiance in connection with the spiking we have observed. The Ginzburg-Landau equation model of the FEL shows that spike pulses that have amplitude and width which agree with our experimental observations can evolve. Although the model also predicts the existence of periodic spiking solutions, thus far we are unable to quantitatively account for the interpulse spacing observed experimentally. Injection of a single solitary pulse at the input of a traveling-wave FEL amplifier as described above would result in a solitary wave with interesting features.

The authors acknowledge the help of Michael Cecere with the Transient Digitizer system, discussions with Professor A. Bhattacharjee, and the cooperation of Dr. Mary Potasek in connection with the program of the Ginzburg-Landau equation. This research was supported by the ONR.

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