Short Range Exchange Contributions to the Cross Section for $pp \to pp\pi^0$ near Threshold

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The nuclear S-wave pion production is described by the axial charge density operator of the two-nucleon system. It is shown that the short range axial exchange charge operator implied by the nucleon-nucleon interaction enhances the predicted cross section for the reaction $pp \to pp\pi^0$ near threshold by factors 3–5. This suffices to explain most of the underprediction obtained with the single-nucleon and S-wave pion rescattering operator. The result implies that the cross section for $pp \to pp\pi^0$ can provide direct information on the short range components of the nucleon-nucleon interaction.

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Nuclear S-wave pion absorption and production reactions are conventionally described by a single nucleon and a pion rescattering mechanism, according to which the pion scatters off one nucleon by an S-wave collision, and is absorbed (produced) on a second nucleon [1–4]. The key element of this description is a phenomenological effective Hamiltonian

$$H = 4\pi\frac{\lambda_1}{m_\pi}\bar{\psi}\boldsymbol{\phi}\cdot\boldsymbol{\phi}\psi + 4\pi\frac{\lambda_2}{m_\pi^2}\bar{\psi}\boldsymbol{\tau}\cdot\boldsymbol{\phi}\times\boldsymbol{\pi}\psi \tag{1}$$

that describes the rescattering vertex. The two coupling constants λ_1 and λ_2 are determined by the S_{11} and S_{31} pion nucleon scattering lengths a_1 and a_3 as

$$\lambda_1 = \frac{-1}{6}m_\pi\left(1+\frac{m_\pi}{m_N}\right)(a_1+2a_3)\,, \tag{2a}$$

$$\lambda_2 = \frac{1}{6}m_\pi\left(1+\frac{m_\pi}{m_N}\right)(a_1-a_3)\,. \tag{2b}$$

With the values $a_1 \simeq 0.245$ fm and $a_3 \simeq -0.143$ fm of Höhler *et al.* [5], one obtains $\lambda_1 = 0.005$, $\lambda_2 = 0.05$, and with Arndt, Ford, and Roper [6] values $a_1 = 0.23$ fm and $a_3 = -0.11$ fm, one obtains $\lambda_1 = -0.0013$ and $\lambda_2 = 0.047$.

The notable feature of the effective interaction Eq. (1) is the smallness of the coefficient λ_1. The pion rescattering mechanism induced by the second larger term in Eq. (1) has an antisymmetric isospin dependence $\boldsymbol{\tau}^1\times\boldsymbol{\tau}^2$, and hence does not contribute to the $pp \to pp\pi^0$ reaction. The contribution of the S-wave rescattering mechanism to the rate for $pp \to pp\pi^0$ is therefore very small. This is borne out by detailed calculations [7,8], which show that the predicted cross section for $pp \to pp\pi^0$ near threshold is 4–5 times smaller than the empirical values [9] when described by this rescattering amplitude combined with the amplitude for absorption on a single nucleon. We here show that most of this underprediction is removed when the short range two-nucleon mechanisms that are implied by the nucleon-nucleon interaction are taken into account. The result implies that the reaction of $pp \to pp\pi^0$ near threshold may prove useful for discrimination between different nucleon-nucleon interaction models.

The most coherent framework for taking into account the short range exchange contributions is to describe the pion-nucleus interaction by the direct extension of Weinberg's effective pion-nucleon interaction [10] to nuclei:

$$\mathcal{L} = \frac{1}{f_\pi}\mathbf{A}^\mu\cdot\partial_\mu\boldsymbol{\phi}\,, \tag{3}$$

where $\boldsymbol{\phi}$ is the isovector pion field operator and $\mathbf{A}^\mu$ is the isovector axial current of the nuclear system. This extension of the Weinberg Lagrangian appears naturally, e.g., in the Skyrme model [11], and reduces the calculation of matrix elements for nuclear pion production to the construction of the axial current operator, which is formed of single-nucleon and two-nucleon (exchange) current operators.

Near the threshold only the interactions that involve S-wave pions are of significance. These are governed by the charge component of the axial current operator of Eq. (3). The amplitude for $pp \to pp\pi^0$ is then [12]

$$T_{fi} = -i\delta^4(p_1'+p_2'+k-p_1-p_2)\frac{\omega_\pi(k)}{[2\omega_\pi(k)]^{1/2}}\frac{1}{(2\pi)^{1/2}} \times\langle\chi^{(-)}_{\mathbf{p}_1'\mathbf{p}_2'}|\mathbf{A}^0\cdot\hat{\mathbf{z}}|\chi^{(+)}_{\mathbf{p}_1\mathbf{p}_2}\rangle\,, \tag{4a}$$

with

$$\mathbf{A}^0 = \mathbf{A}^0(\mathrm{I}) + \mathbf{A}^0(\mathrm{II})\,. \tag{4b}$$

Here $\mathbf{A}^0(\mathrm{I})$ and $\mathbf{A}^0(\mathrm{II})$ denote respectively the single-nucleon and two-nucleon contributions to the axial charge density operator. In Eq. (4a) $\mathbf{p}_i$ and $\mathbf{p}_i'$, $i=1,2$, are the initial and final nucleon momenta, $\mathbf{k}$ and $\omega_\pi(k)$ are the momentum and energy of the produced pion. The pp scattering wave functions are denoted as $\chi^{(\pm)}$. The single-nucleon contribution is

$$\mathbf{A}^0(\mathrm{I}) = -\frac{g_A}{2}\sum_{i=1,2}\left[\frac{\boldsymbol{\sigma}^i\cdot(\mathbf{p}_i'+\mathbf{p}_i)}{2m_N}\boldsymbol{\tau}^i\right]. \tag{5}$$

The conventional single-nucleon pion production operator is recovered by using the Goldberger-Treiman relation $g_A/2f_\pi = f_{\pi NN}/m_\pi$ in Eqs. (4) and (5).

The axial exchange charge operator is formed of a pion exchange component and a short range component, which can be determined from the nucleon-nucleon interaction [13]. The main pion exchange component corresponds to a mechanism in which the axial field couples to a ρ meson and a pion [14]. This contribution corresponds to the rescattering operator that is obtained with the second term of the effective interaction Eq. (1) [11]. As we mentioned in the beginning of this paper, this rescattering operator cannot contribute to the reaction $pp \to pp\pi^0$ because of its antisymmetric isospin dependence. The only pion component relevant to the present investigation is therefore the (small) rescattering term induced by the first term of the Hamiltonian Eq. (1).

When the nucleon-nucleon interaction is expressed in terms of the Fermi invariants there is a unique axial exchange charge operator that corresponds to each invariant [13]. These can be derived by taking the static limits of the pair terms in the general five-point functions in which the axial field couples to the interacting two-nucleon system. The general expressions for the corresponding two-body-exchange charge operator can then be written as

$$\mathbf{A}^0(\mathrm{II}) = \frac{1}{(2\pi)^3}[\mathbf{A}^0(S)+\mathbf{A}^0(V)+\mathbf{A}^0(T)+\mathbf{A}^0(A)]\,. \quad (6)$$

The most important of these axial exchange charge operators are those that are associated with the scalar (S) and vector (V) components of the nucleon-nucleon interaction. Dropping terms that involve isospin flip, these are [13]

$$\mathbf{A}^0(S) = \frac{g_A}{2m_N^2}[v_S^+(\mathbf{k}_2)\boldsymbol{\tau}^1 + v_S^-(\mathbf{k}_2)\boldsymbol{\tau}^2]\boldsymbol{\sigma}^1\cdot\mathbf{P}_1 + (1\leftrightarrow 2)\,, \quad (7a)$$

$$\mathbf{A}^0(V) = \frac{g_A}{2m_N^2}[v_V^+(\mathbf{k}_2)\boldsymbol{\tau}^1 + v_V^-(\mathbf{k}_2)\boldsymbol{\tau}^2] \times\left[\boldsymbol{\sigma}^1\cdot\mathbf{P}_2 + \frac{i}{2}(\boldsymbol{\sigma}^1\times\boldsymbol{\sigma}^2)\cdot\mathbf{k}_2\right] + (1\leftrightarrow 2)\,. \quad (7b)$$

The expressions for $\mathbf{A}^0(T)$ and $\mathbf{A}^0(A)$ are given in Ref. [13]. In the above equations, the potential coefficients $v_j^{\pm}$ are the isospin independent (+) and dependent (−) potential functions that are associated with the corresponding Fermi invariants. The momentum operators are defined as $\mathbf{P}_i = \frac{1}{2}(\mathbf{p}_i + \mathbf{p}_i')$ $(i=1,2)$. The fractional momentum transfers to the two nucleons are denoted as $\mathbf{k}_1$ and $\mathbf{k}_2$, so that the momentum of the external pion is $\mathbf{k}_\pi = \mathbf{k}_1 + \mathbf{k}_2$. Thus $v_S^+(\mathbf{k})$ could be interpreted as the potential associated with exchange of isospin 0 scalar mesons, the potential $v_V^+(\mathbf{k})$ as that associated with exchange of neutral vector mesons. These potential coefficients can be constructed from the components of complete phenomenological potential models [15], or alternatively by employment of phenomenological meson exchange models. Explicit expressions for these potential functions can be found in Ref. [15]. There is no axial exchange charge operator associated with the pseudoscalar invariant P included in Eq. (6). Such a contribution, which would imply a double counting if a pion exchange term is generated from effective Hamiltonian Eq. (1), is automatically eliminated, if the pseudoscalar invariant $\gamma_5^{(1)}\gamma_5^{(2)}$ is replaced by the on-shell equivalent pseudovector invariant $\gamma_\mu^{(1)}k_\mu\gamma_5^{(1)}\gamma_\nu^{(2)}k_\nu\gamma_5^{(2)}/4m_N^2$ [13,16].

To facilitate comparison with the short range operators in Eq. (6), the pion rescattering amplitude obtained with the effective S-wave interaction Eq. (1) can be cast into the form of an axial exchange charge operator. It is straightforward to show that the contribution of the S-wave pion rescattering to the $pp \to pp\pi^0$ amplitude can be included by adding to Eq. (4b) the following two-body axial charge operator

$$A^0(\pi) = \frac{-1}{(2\pi)^3}\frac{8\pi}{\omega_\pi(k)}\lambda_1\frac{f_{\pi NN}}{m_\pi^2}\frac{\boldsymbol{\sigma}^2\cdot\mathbf{k}_2}{m_\pi^2+\mathbf{k}_2^2}f(\mathbf{k}_2) + (1\leftrightarrow 2)\,. \quad (8)$$

Here $\lambda_1 = 0.005$ of Höhler *et al.* [5] is used in this work. The πNN form factor $f(\mathbf{k}_2)$ is assumed to be of a monopole form with the cutoff mass 1300 MeV/c^2, which is consistent with the Bonn boson exchange model for the nucleon-nucleon potential [17]. In Eq. (8) we do not include the energy of the exchanged pion. This correction, which would be insignificant in the case of the short range operators Eq. (6), would increase the matrix elements of Eq. (8) slightly.

To demonstrate the effect of the short range axial charge operator Eq. (6), we have calculated the cross section for $pp \to pp\pi^0$ near threshold using both the local form of the Bonn potential using the parameters given in Table A.3 of Ref. [17] and the Paris potential [18]. The results are compared with the data of Ref. [9] in Fig. 1. The dot-dashed curve is obtained when only the one-body

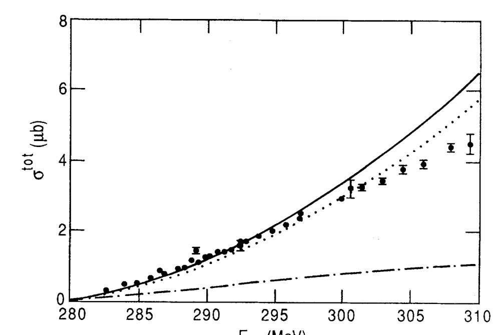

FIG. 1. The solid and dotted curves are respectively the total cross section for $pp \to pp\pi^0$ calculated using the Bonn potential (Table A.3 of Ref. [17]) and the Paris potential [18] model to construct the axial exchange charge operator. The dot-dashed curve is obtained by keeping only the one-body term Eq. (5) and the pion rescattering term Eq. (8) in the calculation using the Paris potential. The data points are from Ref. [9].

term Eq. (5) and pion rescattering term Eq. (8) are taken into account in the calculation using the Paris potential to calculate the initial and final pp scattering wave functions. It underpredicts the data by a large factor, in agreement with previous findings [7,8]. When the short range exchange mechanisms described by the axial exchange charge operators Eq. (6) is taken into account, we obtain the solid curve in Fig. 1. Most of the underprediction is removed. This large enhancement due to the short range axial charge operator is also found to be the case for the calculation using the Bonn potential. For comparison, we also show in Fig. 1 the full calculation (dotted curve) using the Bonn potential. Both predictions do, however, not reproduce the energy dependence of the data in detail. This may be due to the neglect of the energy dependence of the parameter λ_1 in the effective Hamiltonian Eq. (1), of P-wave contributions and πNN three-body scattering in the final state. These corrections should be small, but they can in principle lead to significant contributions through the interference with large amplitudes and thus have significant effects on the predicted energy dependence. The importance of πNN scattering has been pointed out by Blankleider [19] in a unitary three-body calculation. The point here is that the good qualitative agreement with the data is the direct consequence of the short range part of the Bonn and Paris potentials. All parameters that characterize the short range pion production mechanisms are consistent with the employed realistic NN potentials.

It is interesting to explore the difference between the potential models considered. In Fig. 2 we display the relative importance of the different components of the short range operator Eq. (6) for each potential. The scalar (S) term is larger than the vector (V) term for the case of the Bonn potential, while their relative importance is reversed in the case of the Paris potential. In both cases these two short range contributions add coherently and yield almost the same total cross section in the very near threshold energy region. To distinguish the predictions obtained with different potential models, it may be necessary to carry out measurements of spin observables since the scalar and vector terms have different spin dependence [as seen in Eq. (7)].

To conclude, we have shown that the cross section for the reaction $pp \to pp\pi^0$ near threshold can be explained by taking into account the short range two-body contribution to the pion production amplitude, which can be directly derived from the short range part of the nucleon-nucleon interaction. The $pp \to pp\pi^0$ reaction provides a fairly direct way of determining the short range part of the axial exchange charge operator. In combination with the long range pion exchange operator this operator also appears able to explain most of the large enhancement of the effective axial charge that has been found in analyses of first forbidden β transitions in heavy nuclei [13,20]. That there should be a close connection between the matrix elements of nuclear β transitions and pion production

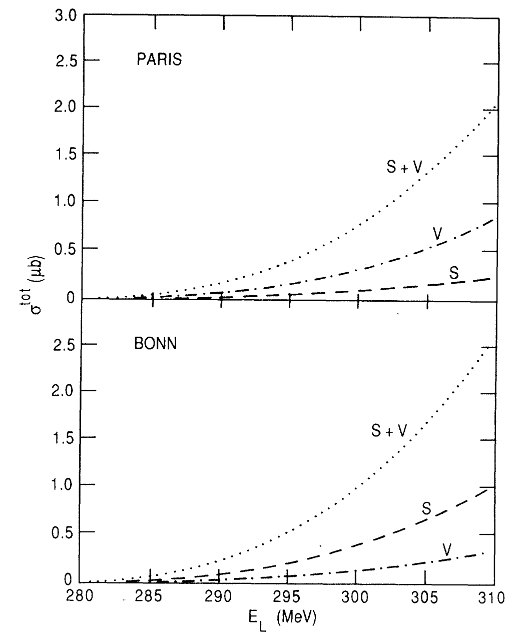

FIG. 2. The contributions to the total cross section for $pp \to pp\pi^0$ from that of scalar (S) and vector (V) exchange contributions to the axial charge density and their coherent sum ($S+V$) of Eq. (7) as calculated using the Bonn potential [17] and the Paris potential [18].

and absorption reactions has of course long been known [21], but the present results represent the first explicit demonstration of this in the case of S-wave pions.

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