Numerical Study of Fractional Quantum Hall Electron-Hole Systems: Evidence of Stable Anyonic Ions

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Spatially separated electron-hole layers in strong magnetic fields are studied by exactly diagonalizing the Hamiltonian of a small number of particles. When the separation between layers is of the same order as the intralayer particle separation, the ground-state energy of the system displays a pronounced cusp as a function of the Landau level degeneracy at an odd denominator fraction $\tilde{\nu}_c$, where $\tilde{\nu}_c \equiv (\nu_e - \nu_h)/(1-\nu_h)$, and ν_e and ν_h are the electron and hole filling factors ($\nu_h \ll \nu_e$). Detailed analysis suggests that the ground state responsible for the cusp consists of stable ions (composed of Laughlin quasiparticles bound to a hole) weakly coupled to an incompressible fluid state of the remaining electrons.

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The strong interparticle correlation of two-dimensional electron systems in a strong perpendicular magnetic field leads to the formation of the Laughlin incompressible liquid state [1] and the fractional quantum Hall effect (FQHE) [2]. Two characteristic features of the incompressible states are the discontinuity in the chemical potential of the system when an electron is added, and the finite energy gap between the ground state and the excited states. Recently, the rapid developments in the magneto-optical experiments [3] in the FQHE regime have prompted great interests in how the properties of incompressible states of an electron system are affected by the presence of a few positively charged free holes (either on the two-dimensional plane or separated from the plane by a finite distance).

In an electron-hole system strong correlations exist between particles of like charge, as well as between particles of opposite charge. For a system with an equal number of electrons and holes ($\nu_e = \nu_h$) in the same two-dimensional layer, the correlations between electrons (or holes), which favor the condensation into the incompressible fluid state, are overshadowed by the strong attractive interaction between electrons and holes. If only the first Landau level is considered, the ground state can be obtained exactly and viewed as a Bose condensed state of noninteracting excitons [4]. The Bose condensed state of $\mathbf{k}=0$ excitons is also believed to be the ground state of the system when the electrons and holes are in two different layers with a small layer separation. As the layer separation is increased, the interlayer correlations become relatively less important, and the system would be expected to undergo a phase transition to either a double FQHE state or an excitonic charge-density-wave state [5].

For a system in which the number of electrons differs from the number of holes ($\nu_e \neq \nu_h$), i.e., for a general electron-hole system, the nature of the ground state is still an unsolved problem. In a recent paper, based on an exact mapping between the electron-hole system and a two-component electron system, MacDonald and Rezayi [6] conclude that in the symmetric case where electrons and holes are in a same layer, the charged electron-hole fluid should exhibit the FQHE when the filling factor of the excess charge $\nu_c = \nu_e - \nu_h$ is a fraction with an odd denominator. This prediction follows from the assumption that for a two-component spin $\frac{1}{2}$ electron system with a negligible Zeeman splitting, the ground state is always maximally spin polarized. In this paper we present a numerical study of a finite-size general electron-hole system with an arbitrary layer separation d. We find that in the symmetric case, neither the chemical potential discontinuity nor the finite energy gap between the ground state and the excited states appears. In the asymmetric case, we discover that when the layer separations are of the order of the magnetic length, strong cusps are obtained at $\nu_c/(1-\nu_h) = p/q$, where q is an odd integer.

Let us first consider the symmetric case where electrons and holes are in the same layer. It has been pointed out by several authors that the electron-hole system in this case has a so-called hidden symmetry [6–8]. One of the simple ways to understand this symmetry is to notice that the commutator of the Hamiltonian of the system $\hat{H}$ and the creation operator of a $\mathbf{k}=0$ exciton $\hat{d}^\dagger(0)$ is proportional to the creation operator itself, that is,

$$[\hat{H}, \hat{d}^\dagger(0)] = E_x(0)\hat{d}^\dagger(0). \tag{1}$$

Here $E_x(0)$ is the binding energy of a single exciton. In the Landau gauge the exciton creation operator $\hat{d}^\dagger(0)$ is given by

$$\hat{d}^\dagger(0) = \frac{1}{N_L^{1/2}} \sum_X a_X^\dagger b_{-X}^\dagger, \tag{2}$$

where $a_X^\dagger$ (or $b_X^\dagger$) is the creation operator of a free electron (hole) in the first Landau level with the wave function $\phi_X^e(\mathbf{r})$ [or $\phi_X^h(\mathbf{r})$], and N_L is the Landau level degeneracy. It follows from Eq. (1) that for a given state of the electron-hole system, the introduction of an extra electron-hole pair with a total momentum $\mathbf{k}=0$ only changes the energy of the system by $E_x(0)$, the single exciton energy. In other words, as far as the energy of the system is concerned, it appears as if a $\mathbf{k}=0$ exciton does not interact with any other particles. This is a truly amazing result since even though a $\mathbf{k}=0$ exciton is charge neutral, the Pauli principle should lead to some sort of exchange and correlation interaction between the electron (hole) participating in the exciton and the rest of electrons (holes) in the system. It turns out that when we act the operator $\hat{d}^\dagger(0)$ on a many-particle state $|\psi\rangle$, we are not creating a bare exciton with the binding energy $E_x(0)$, but a deformed exciton with a reduced binding energy. The reason for this is that not all of the states in the Landau level are available to participate in the exciton state since some of them are already occupied. It is interesting and not difficult to show that the loss in the binding energy of the deformed exciton is exactly compensated by the exchange interaction mentioned above. It is worth emphasizing that the statement that $\mathbf{k}=0$ excitons do not interact with electrons is correct only in the sense of the invariance of energy. Other quantities, such as the pair correlation function between electrons, are drastically altered by the addition of the excitons.

We perform our finite-size calculations in the spherical geometry [9,10]. Electrons and holes are put on a sphere of radius $R^2=S$ with a magnetic monopole at the center, where $2S+1$ is the degeneracy of the first Landau level. The Coulomb interaction between the particles of like charge is taken to be inversely proportional to the chord distance, and the interaction between electrons and holes is modulated by the layer separation d, i.e.,

$$V_{eh}(|\hat{\Omega}_1-\hat{\Omega}_2|)=(R^2|\hat{\Omega}_1-\hat{\Omega}_2|^2+d^2)^{-1/2},$$

where $\hat{\Omega}$ is a unit vector in the radial direction denoting the position of a particle on the sphere. The quantum states of the system are classified by eigenvalues $L(L+1)$ and M of the square of the angular momentum operator $\hat{L}^2$ and its z component $\hat{L}_z$. The effect of the neutralizing background is included by adding a shift [11] of $-N_c^2/2R$ to the calculated energy, where $N_c=N_e-N_h$. In Fig. 1 we have plotted the ground-state energy of a seven-electron, one-hole system at $d=0$ as a function of the parameter S. From the prediction of Ref. [6], one might expect downward cusps to appear at $S=7.5$, 5.5, and 4.5 corresponding to $\nu_c=\frac{1}{3}$, $\frac{2}{5}$, and $\frac{2}{3}$. Our result, however, shows no sign of the discontinuity [12] in slope at these values of S. The assumption that the ground state of the spin $\frac{1}{2}$ electron system onto which the electron-hole system maps is maximally spin polarized is equivalent, in the electron-hole system, to the assertion that the ground state can be obtained from the ground state of a N_c electron system by simply adding N_h excitons of momentum $\mathbf{k}=0$. Finite-size calculations clearly indicate that the ground state for a given S is not necessarily one of these multiplicative states $[\hat{d}^\dagger(0)]^{N_h}|N_c\rangle$, where $|N_c\rangle$ is a quantum state of a N_c electron system with the same S. Also plotted in Fig. 1 is the energy spectrum of the system at $\nu_c=\frac{1}{3}$ ($S=7.5$). As has been noticed by several authors for smaller systems, there is no finite energy gap between the ground state and low-lying excited states. Surprisingly, the lowest energy levels at $L=0$, 1, and 2 corresponding to dressed excitons [8] are almost degenerate. The gap observed in Ref. [8], which separates the multiplicative states from higher states, does not occur for our larger system suggesting that it could be a finite-size effect. Nevertheless, we do find a finite energy gap between the lowest state and the excited state *for a given L* at $L\leq 3$. This gap persists when we increase the size of the system from $N_e=5$ and $N_h=1$ to $N_e=7$ and $N_h=1$.

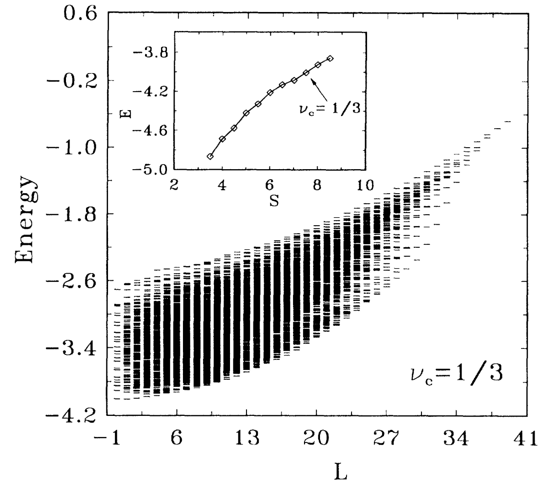

FIG. 1. Energy spectrum of an electron-hole system with $N_e=7$, $N_h=1$, and $d=0$ at $S=7.5$. Inset: Ground-state energy as a function of S for the same system. Note that the Landau level degeneracy $N_L=2S+1$, and $\nu_c=\frac{1}{3}$ corresponds to $S=7.5$.

For the asymmetric case, in which electrons and holes are on two different layers, the ground-state properties, as well as the collective excitations of the system, depend strongly on the interlayer separation. It is conceivable that in a certain range of the layer separation, each hole may bind only one or two quasielectrons, instead of a whole real electron, because of the weaker interlayer interaction. In Fig. 2(a) the ground-state energy of a system of seven electrons and one hole at $d=1.75$ is shown as a function of the Landau level degeneracy S. A pronounced cusp is revealed at $S=8$. Two other weaker cusps (or kinks) appear at $S=6$ and 5. Also plotted in Fig. 2(a) is the ground-state energy of a six-electron system. As can be seen, by adding one electron-hole pair to the six-electron system the cusps (kinks) corresponding to $\nu_c=\frac{1}{3}$, $\frac{2}{5}$, and $\frac{2}{3}$ have all been shifted towards the right by 0.5 of the S value. We have also calculated the

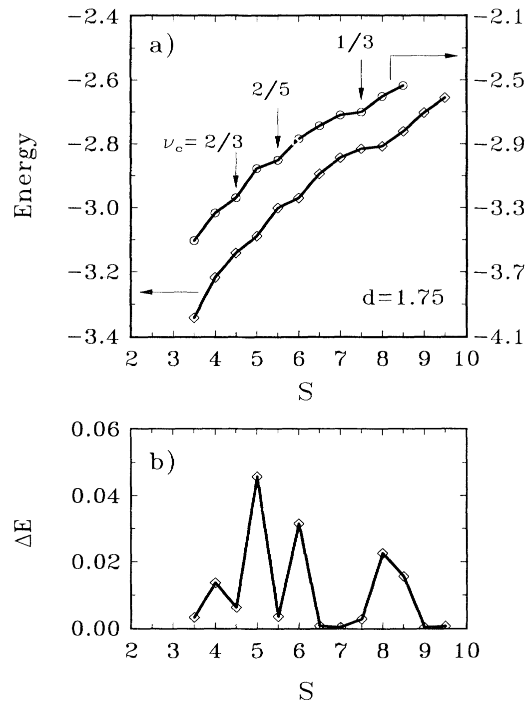

FIG. 2. (a) Ground state energy as a function of S: (diamonds) electron-hole system with $N_e=7$, $N_h=1$, and $d=1.75$; (circles) system with six electrons only. (b) Energy difference between the ground state and the lowest excited state for an electron-hole system with $N_e=7$, $N_h=1$, and $d=1.75$. Note that the peaks here correspond to the cusps (kinks) in (a).

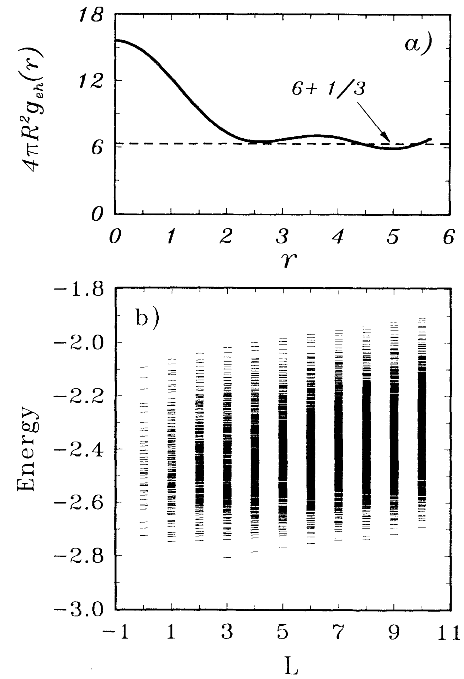

FIG. 3. Electron-hole system with $N_e=7$, $N_h=1$, and $d=1.75$ at $S=8$. (a) Pair correlation function $g_{eh}(r)$ in the ground state ($L=3$). (b) Energy spectrum of the system.

ground-state energy for an eight-electron, two-hole system at the same layer separation. The result shows that the positions of the cusps (kinks) are all shifted in the S value by one unit towards the right relative to those in the six-electron system. In general for a N_e electron and N_h hole system with $d \approx 2$, we would expect that cusps (or kinks) appear at $2S=2S_{N_c}+N_h$, where S_{N_c} is the S value at which an incompressible state occurs for a $N_c = N_e - N_h$ electron system. For the $\nu_c = \frac{1}{3}$ state, we have $2S_{N_c}=3(N_c-1)$ and therefore $2S=3(N_e-1)-2N_h$. This relation suggests that the ground state at $S=8$ in Fig. 2(a) consists of a $\frac{1}{3}$ incompressible liquid of seven electrons and a bound state complex of one hole and two Laughlin quasielectrons. In Fig. 2(b) we show the energy difference ΔE between the ground state and the lowest excited state for several S values. We find that at the positions where the cusps appear in the ground-state energy, ΔE displays strong peaks characteristic of a dissipationless system.

The nature of the bound state complex is evident in Fig. 3(a), where the electron density distribution around the hole $g_{eh}(|\mathbf{r}-\mathbf{r}_h|)$ (averaged over all degenerate states of different M) is plotted with $\mathbf{r}_h$ fixed at the origin (the North pole) for $S=8$. The binding of two quasielectrons to the free hole means the accumulation of an extra $-\frac{2}{3}e$ charge around the position of the hole. Our calculations reveal that the northern hemisphere indeed contains approximately $-\frac{2}{3}e$ more electronic charge than the southern hemisphere, and the value of $4\pi R^2 g_{eh}(|\mathbf{r}-\mathbf{r}_h|)$ appears to approach $7-\frac{2}{3}$ at large $|\mathbf{r}-\mathbf{r}_h|$. The energy spectrum of the system [Fig. 3(b)] consists of a continuous band about $0.06e^2/l$ above the ground state and a few discrete levels between them. If our conjecture on the ground state is correct, these levels may be viewed as excited states of the bound state complex. The cusps at $S=5$ and $S=6$ can also be understood in terms of bound states of the valence band hole and the appropriate number of quasiparticles of the $\nu=\frac{2}{3}$ and $\nu=\frac{2}{5}$ states. Details will be given in a later publication.

In summary, we have studied the spatially separated electron-hole system by exactly diagonalizing the Hamiltonian of a small number of particles. At $d=0$ our result does not show any signature of an incompressible state at $\nu_c=\frac{1}{3}$. For the case of $d \approx 2$, our results reveal that the ground-state energy displays a strong cusp (or kink) at $(\nu_e-\nu_h)/(1-\nu_h)=p/q$ ($\nu_h \ll \nu_e$), and the states responsible to the cusps is an incompressible liquid of electrons at $\nu_e = p/q$ and a dilute gas of bound state complexes consisting of a hole and Laughlin quasielectrons. The implication of these novel states in the optical properties of FQHE's is currently under investigation.

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[12] There is a small wiggle at $S=7$, the position corresponding to $\nu_c=\frac{1}{3}$ shifted towards the left by 0.5 of S. The similar wiggle is also observed for the system with $N_e=8$, $N_h=1$, and $d=0$, and for the system with $N_e=8$, $N_h=2$, and $d=0$. In the latter case the position of the wiggle is shifted towards the left by one unit.