Vortex Nucleation in Superfluid ^{4}He Probed by ^{3}He Impurities

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The critical velocity for the onset of phase slips in the flow of superfluid ^{4}He through a micro-orifice has been found to be strongly affected at temperatures below ~0.1 K by minute quantities of ^{3}He impurities, in the few parts per 10^9 range. It is shown that this effect yields essential information on the vortex nucleation mechanism and in particular the local superfluid velocity at which vortices are formed, which is 22 m/s in these experiments.

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The phenomenon of local breakdown of superfluidity by flow through small orifices or by fast-moving objects has been very actively studied in recent years, both in ^{4}He [1-4] and in ^{3}He [5]. It is signaled by the onset of dissipation at a certain critical velocity v_c. Two intrinsic mechanisms for v_c have been clearly identified, the creation of elementary excitations, rotons in ^{4}He, when the Landau criterion for superfluidity is violated [3], and the nucleation of vortices. This last mechanism, on which we focus here [6], is still poorly understood. However, it is gradually becoming apparent that, in a number of cases [1-4,7-9], these vortices may appear at the solid boundary in the form of half rings with a radius r of a few coherence lengths and with a self-velocity opposing and nearly balancing the local flow velocity u_s. Such a conjecture accounts well for the vortex nucleation rate and the critical velocity values (~50 to 60 m/s) observed in the propagation of negative ions [8]. In orifice flow, it predicts the correct temperature dependence of v_c, but, as only the mean value of the velocity v_s over the orifice cross section is measured, the knowledge of the local velocity u_s at the nucleation site, necessary to fully characterize the half-ring model, is lacking. We report here on the profound effect on v_c of minute traces of ^{3}He impurities and show that this effect brings essential information on u_s.

We have used the same apparatus and experimental techniques as in Ref. [9] to measure the critical velocity of several ultradilute solutions of ^{3}He in ^{4}He with ^{3}He concentrations x_2 ranging from 3 ppb to 100 ppb (10^9). The gaseous samples have been manufactured by mixing pure (0.9 ± 0.1 ppb) ^{4}He [10] obtained in the laboratory [9] and commerical helium with $x_3 = 100$ ppb (± 6 ppb) [10]. The amount of mixture just necessary to slightly overfill the experimental cell was metered and condensed into the cold apparatus. The cold part was then isolated by closing a superfluid tight valve located on the mixing chamber of the dilution refrigerator. The liquid helium sample thus had no free surface. The ^{3}He impurities appeared to settle down rapidly. Their influence on the critical velocity showed no time dependence.

However, it became apparent during the course of the measurements that the operating procedure used to condense the gaseous mixture into the experimental cell had a bearing on the results. When condensation took place rapidly into a cell maintained below 1 K, the effect of ^{3}He appeared to be less pronounced than when the liquid sample was cooled slowly through the λ transition. This effect is illustrated in Fig. 1 for a nearly pure sample and for the 45 ppb sample. The turn-down of v_c is seen to occur at a lower temperature in the quenched state than in the annealed state for the 45 ppb sample, and to be completely suppressed for the nearly pure sample.

The influence of annealing can be understood as fol-

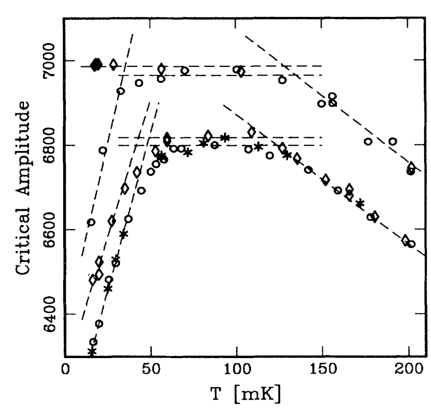

FIG. 1. Critical resonator amplitude in instrumental units, proportional to the critical velocity vs temperature for two samples: top curve, nearly pure (x_3 estimated to be ~7 ppb) in the quenched (◇) and annealed (○) states; bottom curve, shifted downward by 200 units, 45 ppb quenched (◇), annealed once (○), and annealed a second time (*). The dashed straight lines are linear fits underlining the quantum plateaus, the thermal activation regime at high temperature, and the effect of ^{3}He at low temperature. The small shifts in the plateau levels upon annealing are within experimental reproducibility and may not be a real effect.

lows. The ^{3}He quasiparticles are known to condense onto vortex cores, as predicted theoretically by Rent and Fisher and Ohmi and Usui [11], and as established experimentally by Ostermeier and co-workers [12]. Their condensation energy onto vortices in the bulk is $\mathcal{L}_{3b} \sim 3$ K. Upon rapid condensation below the λ point, the superfluid is stirred in a violent way. Vortices are formed in large number and are pinned at preferential pinning sites, located in the sintered copper heat exchanger linking the cell to the nuclear demagnetization stage [13]. These remanent vortices in turn trap ^{3}He quasiparticles, thus reducing the apparent x_3 in the bulk and the net effect of ^{3}He on v_c. Annealing effectively destroys remanent vortices and the trapped ^{3}He quasiparticles are released. From the magnitude of the effect shown in Fig. 1, and assuming that vortex cores are decorated by a string of ^{3}He atoms a distance a_3 apart, we estimate the remanent vortex line length L necessary to accommodate the ^{3}He atoms to be $L \sim x_3 a_3 N_A V/V_4$, N_A being Avogadro's number, V the volume of the cell (11.1 cm^3), and V_4 the molar volume of pure ^{4}He at low pressure ($\sim$27.5 cm^3). Taking $a_3 \sim 3.55$ Å and $x_3 \sim 30$ ppb (estimate from the drop in v_c shown in Fig. 1), we find $L \sim 2600$ km. This seemingly large vortex length only amounts to roughly one vortex per microcavity in the sinter [13]. No effect of annealing was seen on the 0.9 ppb (pure) sample. This may indicate that a small amount of vorticity [14] survives the annealing process, or else that a tiny fraction of ^{3}He remains buried in the disordered solid layer at the wall. We note that the magnitude of L is significantly smaller, in the quenched state as well as in the annealed state, than that quoted by Awschalom and Schwarz [15].

The full effect of ^{3}He impurities on v_c, for annealed samples, is shown in Fig. 2 for x_3 ranging between 0.9 and 100 ppb. This effect takes place in the quantum regime for the nucleation of vortices [9], that is, on the plateau of v_c at low temperature. This marked influence of ^{3}He on v_c, which was first observed in more concentrated mixtures [16–19], is seen in Figs. 1 and 2 to possess three remarkable features: (i) The effect develops in full even at a concentration, $x_3 = 3$ ppb, for which the mean distance between impurities is of the order of 2500 Å. (ii) The drop in v_c by at least 10% signals a profound alteration of the nucleation rate. (iii) There is little or no change, for these low concentration [18] and to experimental accuracy, in either the quantum plateau level, the crossover temperature T_q, or the high temperature slope of v_c: The phase-slip onset mechanism is left practically unaltered at high temperature.

The propagation of negative ions is also known to be strongly affected by minute quantities of ^{3}He [20], the propagation velocity being reduced by 7% and the nucleation rate being increased by $\sim 10^3$ at 23 bars. However, the influence of ^{3}He on ions is already significant at temperatures as high as 0.5 K and its origin is thought [20,21] to arise from the trapping of ^{3}He atoms by the predominantly hollow ion bubble. The physical situation with ions appears to differ from that of orifice flow which we think is as follows.

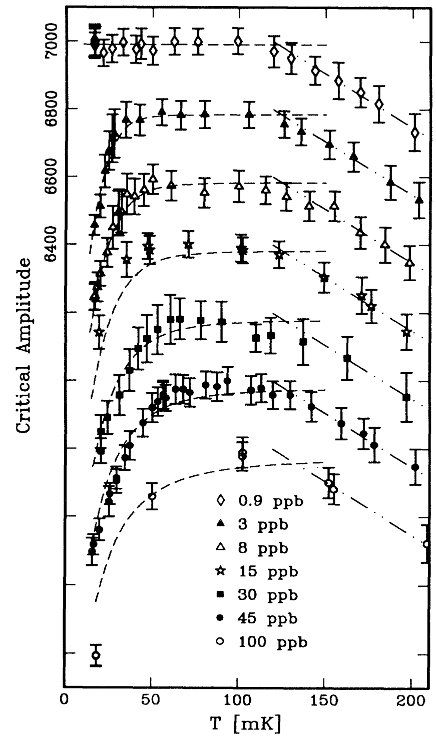

FIG. 2. Critical amplitudes vs temperature, as in Fig. 1, for "pure" ^{4}He and 6 mixtures, shifted downward with respect to one another by 200 units. The dashed curves are computed as explained in the text. The dash-dotted straight lines are fits to the high-temperature data.

According to the views expressed in Ref. [9], vortices are nucleated very close to the wall of the orifice at a site of spatial extent V_n where the velocity u_s is the highest. An asperity or some other defect constitutes such a site, of a typical size in the 10 Å range. A specific form of the energy barrier opposing nucleation can be constructed [9] by assuming that the vortex grows in the shape of a half ring from zero length at the wall into a full blown half ring with a radius r of 15 to 30 Å surviving in the local velocity field u_s.

We first note that the ^{3}He quasiparticle energy is decreased by the superflow velocity u_s. This is due in one part to the Bernoulli effect. The local pressure is decreased in regions of high flow velocity and the binding energy of ^{3}He increases. For another part, the backflow of superfluid about the moving quasiparticle interacts with the imposed superflow velocity field as shown by Baym [22] and the net result is

$$\delta\epsilon = -\tfrac{1}{2}[(1+\alpha)m_4 + (1 - m_3/m^*)m_3]u_s^2 . \qquad (1)$$

The Bardeen-Baym-Pines parameter α describes the increase in the fluid volume under the substitution of a ^{4}He atom by a ^{3}He atom, m_4 and m_3 are the bare masses of the two isotopes and m^* the effective mass of ^{3}He quasiparticles in solution. The nucleation site thus acts as an atom trap in which the number density is [23] $n_3 = n_{30} \times \exp\{\delta\epsilon/k_B T\}$: When the local superfluid velocity u_s is high and the temperature is sufficiently low, the ^{3}He quasiparticles collect in large number by the nucleation site.

We then note that the binding energy of a ^{3}He atom to a full blown vortex filament in the bulk, $\mathcal{L}_{3b}$ [12], is reduced close to the wall by two effects: (i) Healing of the superfluid wave function decreases the vortex energy [24]; hence its ability to attract ^{3}He. (ii) The net effect of the van der Waals attraction on both ^{4}He and ^{3}He yields a predominantly repulsive effect of the wall on ^{3}He (however, see [25]).

These effects vary rapidly with the distance from the wall. The nascent vortex core energy is little affected close to the wall and progressively decreases when the vortex grows outward. The energy barrier E_a opposing the nucleation of vortices is modulated accordingly. In the presence of one ^{3}He atom, we write the change in E_a as $\mathcal{L}_3 - E_k$ as, for small vortices of length πr, the ^{3}He atom is confined to the core and acquires a zero-point kinetic energy $E_k = \hbar^2/2m_3 r^2$ [26], evaluated for the vortex radius r_m at the top of the barrier [9]. The nucleation rate in the pure ^{4}He case is given by

$$\Gamma_p = \frac{\omega_0}{2\pi}\left\{864\pi\frac{E_a}{\hbar\omega_0}\right\}^{1/2}\exp\left\{-\frac{36}{3}\frac{E_a}{\hbar\omega_0}\right\}, \qquad (2)$$

in which ω_0 is the attempt frequency, given in terms of the crossover temperature between the quantum and the thermal regime T_q by $\hbar\omega_0 = 2\pi k_B T_q$ [9]. Neglecting the change in the prefactor, i.e., to logarithmic accuracy, we may write the nucleation rate with one trapped ^{3}He as $\Gamma_1 = \Gamma_p \exp\{36(\mathcal{L}_3 - E_k)/5\hbar\omega_0\}$.

When T is reduced, the number density n_3 in the atom trap formed in the nucleation volume increases very rapidly. The probability of finding one ^{3}He atom in the volume $V_n p_1 = n_3 V_n \exp(-n_3 V_n)$ (for Boltzmann's statistics) also increases rapidly at first, but the probabilities p_j of finding j ^{3}He atoms in V_n, $(n_3 V_n)^j \exp(-n_3 V_n)/j!$ soon take over. Upon the addition of a jth ^{3}He atom on the vortex core and under the assumption that the ^{3}He atoms trapped on the core behave like a one-dimensional ideal Fermi gas [27], the barrier energy is lowered by an additional quantity $E_j = \mathcal{L}_3 - \hbar^2 n^2/2m_3 r^2$ [27], with $n = j/2$ for even j and $n = (j+1)/2$ for odd j. To logarithmic accuracy again, the nucleation rate becomes $\Gamma_j = \Gamma_{j-1}\exp\{-36E_j/5\hbar\omega_0\}$. The total rate is obtained by adding the rates of the processes with $0, 1, \ldots, j, \ldots,$ ^{3}He atoms multiplied by the probabilities p_j:

$$\Gamma = e^{-n_3 V_n}\Gamma_p + \sum_1^N p_j \Gamma_j . \qquad (3)$$

From Eq. (3), we can proceed to compute numerically the critical velocity u_s as in Refs. [2] and [9]. We have used the limiting form of $E_a(u_s)$ close to the lability point at which metastability disappears, which is $E_a = \frac{2}{3}E_J(1 - u_s^2/u_{c0}^2)^{3/2}$. Parameters E_J and u_{c0} are obtained from the high-temperature behavior, as in [9].

The determination of u_s, Eq. (3), involves four *a priori* unknown parameters, $\delta\epsilon$, $\mathcal{L}_3$, E_k, and V_n. To these must be added the unknown concentration of ^{3}He, x_0, which is not released by annealing, as mentioned above. Of these five parameters, the first two have an exponentially strong influence on the results and are robustly fixed by the adjustment to the experimental data: $\delta\epsilon$ primarily governs the temperature of the downturn of v_c and $\mathcal{L}_3 - E_k$, the slope with which v_c drops at low temperature. The other three parameters are more loosely determined even though six data sets, corresponding to different concentrations, are fitted. The curves in Fig. 2 are obtained with $\delta\epsilon = 0.204$ K, $\mathcal{L}_3 = 1.68$ K, $E_k = 0.4$ K, $V_n = 40^3$ Å^3, and $x_0 = 1.5$ ppb.

With the help of Eq. (1), $\delta\epsilon$ gives access to the value of the local superfluid velocity u_s, which, with $\alpha = 0.28$ and $m^* = 2.27 m_3$, turns out to be 22 m/s. This value is determined primarily by the requirement that the ultradilute ^{3}He impurities are attracted by a weak potential well at the nucleation site. It lies outside the range of 6 to 12 m/s considered as likely in our previous work [9]. We note that these last values were estimated from the high-temperature slope of $v_c(T)$ and rely more heavily on the nucleation model validity than the present work. The values of the remaining parameters, with E_k yielding $r_m = 4.5$ Å, and $\mathcal{L}_3$ corresponding to about a half of the binding energy in the bulk, i.e., a mean reduction of ρ_s of a factor 2, are consistent with the model features.

In conclusion, the study of the effect of ^{3}He on the phase-slip critical velocity directly probes the superfluid velocity at the nucleation site, barring the unlikely existence of shallow bound states close to the substrate [25]. The value for u_s obtained here is quite consistent with the nucleation of half-ring vortices on a favorable asperity. This model appears to yield a satisfactory description, at the semiquantitative level, of the critical velocity for the onset of phase slips in ^{4}He.

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[27] This assumption is never founded for a one-dimensional gas with nonzero interaction. It is made here for the sake of simplicity.