Direct Current in Mesoscopic Rings Induced by High-Frequency Electromagnetic Field

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Action of an external high-frequency electromagnetic field on free electrons in normal-metal mesoscopic rings threaded by a static magnetic flux is shown to lead to a direct current proportional to the electromagnetic field intensity. The disorder-averaged value of the current is a periodic function of the static magnetic flux and may considerably exceed the corresponding value of equilibrium persistent current.

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During the past five years there has been considerable interest in quantum interference phenomena in small normal-metal samples of intermediate (mesoscopic) sizes, $l \ll L \ll L_\varphi$, where $l = v_F\tau$ is the elastic scattering mean free path and L_φ is the phase-breaking length [1,2]. In such systems, random variations of kinetic coefficients [3,4] and energy spectrum [5] over an ensemble of samples with the same average characteristics (mesoscopic fluctuations) turned out to be observable and universal. More recently [6-8], the idea of mesoscopic fluctuations has been applied to explain the nonvanishing averaged equilibrium current (persistent current) in normal-metal mesoscopic rings threaded by static magnetic flux [9]. The key idea of these works [10] is that in the disconnected rings studied in [9], the number of electrons is conserved rather than the chemical potential, the latter exhibiting mesoscopic fluctuations. The typical value of the averaged persistent current found in [6-8] is of the order of $I \sim c\Delta/\phi_0 \sim e\Delta/\hbar$, where Δ is the mean spacing between discrete energy levels in a ring, and $\phi_0 = hc/e$. However, this value is too small to explain the experimentally observed static magnetization [9]. Other attempts to explain the result in the framework of the free-electron approximation [11-13] also failed to give a value of the persistent current consistent with the experiment [9]. According to the current opinion, this inconsistency may indicate an essential role played by the electron-electron interaction [14]. The same conclusion can be drawn from a very recent experiment [15]. On the other hand, there is a deep connection between interaction and nonlinear phenomena in the external electromagnetic (EM) field. Namely, one can treat the interaction between electrons by integrating the corresponding nonlinear-in-the-EM-field expressions over *quantum* EM fields. Therefore, importance of interaction for mesoscopic phenomena in rings threaded by a static magnetic flux ϕ implies a highly nonlinear susceptibility for a sample consisting of many mesoscopic rings.

In this Letter we will consider the simplest nonlinear effect: dc generation by a high-frequency EM field. The effect we propose belongs to a class of nonequilibrium, kinetic phenomena. That is why we avoid here the term "persistent current" accepted for direct current in the equilibrium state. In contrast to the equilibrium persistent current for free electrons, the nonlinearly generated direct current averaged over a large number of rings (disorder-averaged value) is found to be nonvanishing even in the case where all rings are connected to an electron reservoir, thus keeping the chemical potential fixed. The maximal direct current is estimated to be of the order of e/τ_D, where τ_D is a characteristic diffusion time, i.e., it is much larger than the value of the equilibrium persistent current for noninteracting electrons [6-8].

A general expression for the nonlinear current response of a metal to a high-frequency electric field, $E(t) = \frac{1}{2} \times (\mathcal{E}_\omega e^{-i\omega t} + \text{c.c.})$, contains a dc component:

$$I_0 = \tfrac{1}{2}\{\sigma^{(2)}(\omega,-\omega;\phi) + \sigma^{(2)}(-\omega,\omega;\phi)\}|\mathcal{E}_\omega|^2, \qquad (1)$$

where $\sigma^{(2)}(\omega,-\omega;\phi)$ is the corresponding nonlinear conductivity. The high-frequency electric field $\vec{\mathcal{E}}_\omega$ is assumed to be directed along a ring, i.e., the magnetic flux has a high-frequency component $\phi_1(\omega) = (i\omega)^{-1}\mathcal{E}_\omega L \times e^{-i\omega t}$. However, for the usual experimental geometry for conductance measurements, where the sample has the form of a bar and magnetic flux is absent, the quantity $\sigma^{(2)}(\omega,-\omega;0) = 0$ identically. The reason is that the current I_0 (as any current) should be odd under time-reversal transformations $\omega \to -\omega$, $\phi \to -\phi$. Therefore, it follows from (1) that $I_0(\phi)$ is an odd function, and, hence, by continuity, $I_0(0) = 0$. It is a static magnetic flux that makes the nonlinear conductivity $\sigma^{(2)}(\omega,-\omega;\phi)$ nonzero and yields a dc response to an applied high-frequency electric field.

The effect we study, as well as conductance oscillations with changing magnetic flux in a hollow sample [16], is a particular case of the Aharonov-Bohm effect. Therefore, from general grounds, one can expect the nonlinear conductivity to oscillate as a function of ϕ with a period ϕ_0. However, in a disordered metal, the fundamental harmonic is known to be strongly suppressed [16], and only the first harmonic survives because of a contribution of time-reversal-conjugated trajectories of electrons to a corresponding path integral [17].

For sample sizes and temperatures where the mean level spacing Δ is the smallest energy parameter in a system, the main contribution to the disorder-averaged dc response is made by the diagram shown in Fig. 1 [18]. The corresponding expression for the nonlinear conductivity has the form

$$\sigma^{(2)}(\omega,-\omega;\phi)=-\frac{4e^3D^2}{i\pi\omega L}\sum_k kC_0(k,\omega;\phi)C_0(k,0;\phi)\,, \tag{2}$$

where D is the diffusion coefficient, L is the circumference of a ring, and $C_0(k,\omega;\phi)$ is the particle-particle diffusion propagator ("Cooperon") in the absence of the high-frequency field [16,17]:

$$C_0(k,\omega;\phi)=\left[\frac{2\pi}{\tau_D}\left(m+\frac{2\phi}{\phi_0}\right)^2-i\omega+\frac{1}{\tau_\varphi}\right]^{-1}. \tag{3}$$

In Eq. (3), $\tau_D=(2\pi)^{-1}(L^2/D)\ll\hbar\Delta^{-1}$ is the characteristic diffusion time, and $\tau_\varphi\ll\hbar\Delta^{-1}$ is the phase-breaking time. The necessary conditions for the diffusion approximation, $\tau_D,\tau_\varphi,\omega^{-1}\gg\tau$, are assumed to be fulfilled. The factor $m+2\phi/\phi_0$, where m is an integer, arises because of the quantization of the longitudinal momentum k in a ring [19].

The nonlinear conductivity (2) can be obtained from the usual expression [17] for the contribution of interfering time-reversal-conjugated trajectories to the current response of a quasi-one-dimensional ring,

$$\delta I=-\frac{De^2}{\pi L}\sum_k \mathcal{C}(k;\mathcal{E}_{-\omega})\mathcal{E}_\omega\,, \tag{4}$$

if one formally considers the electron Green's functions in the ladder series, $\mathcal{C}(k;\mathcal{E}_{-\omega})$, to depend on the high-frequency electric field and calculates a linear-in-$\mathcal{E}_{-\omega}$ term (see Fig. 1).

Now, substituting equations (3) and (2) into (1), we have the following expression for the dc component of the

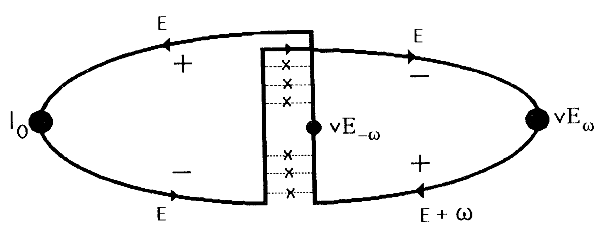

FIG. 1. A diagram for calculating the direct current. Bold lines are retarded (+) or advanced (−) electron Green's functions of the energy indicated; each three-dashed-line set denotes a "Cooperon"; vertices correspond to the interaction with an external electric field ($\mathcal{E}_{\pm\omega}$) or to a current operator (I_0).

current response averaged over an ensemble of many disordered rings:

$$I_0(\omega;\phi)=-\frac{e}{\pi^3\tau_D}\left(\frac{e|\mathcal{E}_\omega|L\tau_D}{h}\right)^2\times\sum_{m=-\infty}^{+\infty}\frac{\kappa_m}{[(\kappa_m^2+\alpha)^2+\bar{\omega}^2][\kappa_m^2+\alpha]}\,, \tag{5}$$

where $\kappa_m=m+2\phi/\phi_0$, $\bar{\omega}=(2\pi)^{-1}\omega\tau_D$, and $\alpha=\tau_D\times(2\pi\tau_\varphi)^{-1}$. As expected, $I_0(\omega;\phi)$ is a periodic odd function of the static magnetic flux ϕ with a period $\phi_0/2$. Therefore, $I_0(\omega;\phi_0/4+\phi)=-I_0(\omega;\phi_0/4-\phi)$, and $I_0(\omega;\phi_0/4)=0$. Because of this property, the function $I_0(\omega;\phi)$ is completely determined by its behavior for $0<\phi<\phi_0/4$. Below we consider only this interval of ϕ. The analysis of the sum (5) shows that in contrast to the equilibrium persistent current studied in [6–8], the disorder-averaged direct current I_0 corresponds to a diamagnetic response to a static magnetic flux for all values of the parameters $\bar{\omega}$ and α.

In deriving the formula (5), nonlinear-in-$|\mathcal{E}_\omega|^2$ terms have been neglected. These terms describe phase breaking by the external EM field in a similar way as for a usual case of conductance of a bar [17]. In the absence of the high-frequency field, the phase difference corresponding to two time-reversal-conjugated trajectories, each having the form of a loop threaded by a static magnetic flux ϕ, is independent of the form of the trajectory and equal to $4\pi\phi/\phi_0$. That is why the interference term survives under disorder averaging to lead to the $\phi_0/2$-periodic oscillations of kinetic coefficients with changing magnetic flux. In the presence of the high-frequency field, an electron acquires an additional phase difference $\delta\varphi$ that depends on a form of the trajectory and thereby is a random quantity. A typical value of $\delta\varphi$ for $\omega\gg 1/\tau_D$ can be found as follows. During a period $2\pi/\omega$ of the EM field, an electron acquires the additional phase $\delta\varphi_i=2\pi e(\hbar\omega)^{-1}|\mathcal{E}_\omega|(\delta l_+-\delta l_-)_i$, where $(\delta l_\pm)_i$ is the displacement of an electron along the ring for the ith positive and negative half period of the field, respectively. For $\omega\tau\ll 1$ the electron motion can be treated using the diffusion approximation, which leads to

$$\overline{(\delta l_+-\delta l_-)}=0,\quad \overline{(\delta l_+-\delta l_-)^2}\sim 2\pi D/\omega\,, \tag{6}$$

where the overbar denotes averaging over different time intervals of length $2\pi/\omega$. The phase $\delta\varphi$ is proportional to the sum of a large number $N=\tau_D\omega$ of independent random variables $(\delta l_+-\delta l_-)_i$. Therefore, its root mean square is given by

$$(\delta\varphi)^2=\overline{(\delta\varphi)_i^2}N=(2\pi e|\mathcal{E}_\omega|L/\hbar\omega)^2\,. \tag{7}$$

A remarkable feature of the phase difference (7) is that it turns out to be independent of disorder. If this phase difference is small, one can neglect higher powers of the EM field and restrict oneself to the quadratic-in-$\mathcal{E}_\omega$ approximation (5). In the opposite case $\delta\varphi\gg 1$, two time-reversal-conjugated trajectories have a random phase difference, and their interference term makes a negligible contribution to the averaged response. Therefore, one can expect a maximum in the dependence of the averaged dc response I_0 on the amplitude of the high-frequency electric field that corresponds to $\delta\varphi \sim 2\pi$. Making use of Eq. (7), we express the flux dependence of the dc response in the limiting case $\omega \gg 1/\tau_D$ in the following way:

$$I_0(\phi) = -\frac{e}{\pi^2 \tau_D}\left(\frac{\delta\varphi}{2\pi}\right)^2 I^+(\phi;\alpha)\,, \tag{8}$$

where the dimensionless function $I^+(\phi;\alpha)$ is obtained by evaluation of the sum in (5),

$$I^+(\phi;\alpha) = \frac{\sin(4\pi\phi/\phi_0)}{\cosh(2\pi\sqrt{\alpha}) - \cos(4\pi\phi/\phi_0)}\,. \tag{9}$$

For lower frequencies $\omega\tau_D \ll 1$, the expression (8) is also formally valid with $\delta\varphi = (e/\hbar)|\mathcal{E}_\omega| L\tau_D$ and $I^+(\phi;\alpha)$ replaced by $I^-(\phi;\alpha) = \frac{1}{2} d^2 I^+/d\alpha^2$. Substituting $\delta\varphi \sim 2\pi$ into (8), we get an estimation for the maximum value of the averaged direct current in a ring. For $\alpha < 1$ and $\phi \sim \phi_0/8$ it is given by

$$I_0^{\max} \sim e/\tau_D\,. \tag{10}$$

An exact value of $I_0^{\max}$ can be found by taking into account the nonlinear corrections of all odd orders to the Cooperon $\mathcal{C}(k,\omega;\phi)$ in (4). This can be done in a way described in [17]. Omitting details of the derivation [20], we give the final result for $\omega \gg 1/\tau_D$, $\tau_D \ll \tau_\varphi$:

$$I_0(\phi) = -\left(\frac{eD}{L^2}\right)\sum_{n=1}^{\infty} I_n\left(\frac{e|\mathcal{E}_\omega|L}{\hbar\omega}\right)\sin\left(\frac{4\pi\phi}{\phi_0}n\right), \tag{11}$$

where $I_n(x)$ is a *universal* function,

$$I_n(x) = \frac{16x^2}{\pi^2}\int_0^{\pi/2} \sin^2 t \exp(-\sqrt{2}nx\sin t)\,dt\,. \tag{12}$$

This function is shown in Fig. 2.

For large values of $2\pi\sqrt{\alpha} = L/L_\varphi(T) \gg 1$ the dc component of the averaged response decreases exponentially. In other words, the effect could be observable only in mesoscopic samples for sufficiently low temperatures T. However, the dependence of I_0 on $\bar{\omega}$ turned out to be not so dramatic. The expressions (7) and (8) show that for high frequencies $1/\tau \gg \omega \gg 1/\tau_D$, i.e., in the region $\omega = 10^{10}$–10^{13} s^{-1}, the dc response does not depend on ω and is proportional to $(e/\tau_D)(\phi_1/\phi_0)^2$, where ϕ_1 is an amplitude of the high-frequency component of the flux. For $\omega \ll 1/\tau_D$, we have $\delta\varphi = 2\pi(\omega\tau_D)(\phi_1/\phi_0)$, so that I_0 decreases as ω^2 (at a fixed value of ϕ_1) with decreasing ω.

In conclusion, we have shown that in a sample consisting of many mesoscopic rings threaded by a static magnetic flux, a *diamagnetic* static magnetization arises as a nonlinear response to an applied high-frequency electric field. The cause of this magnetization is a nonzero ensemble-averaged direct current in the rings. The maximum value of this current is estimated to be of the order of e/τ_D. This value is about 3 orders of magnitude larger than the value of the disorder-averaged equilibrium persistent current obtained within the free-electron approximation.

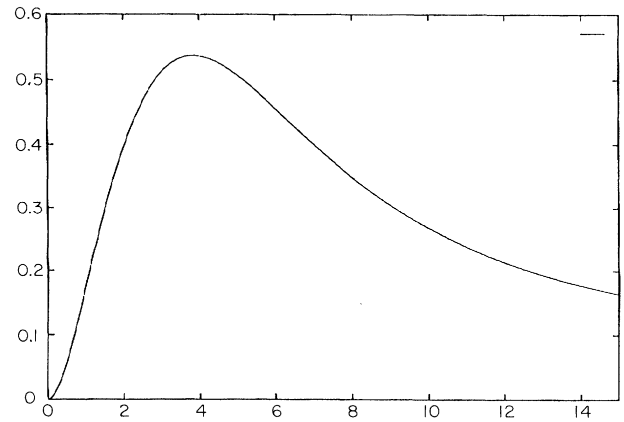

FIG. 2. The amplitude I_1 of the first harmonic of $I_0(\phi)$ in units of eD/L^2 as a function of $x = e|\mathcal{E}_\omega|L/\hbar\omega$.

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[1] Y. Imry, in *Directions in Condensed Matter Physics*, edited by B. Grinstein and G. Mazenko (World Scientific, Singapore, 1986).

[2] B. L. Altshuler, V. E. Kravtsov, and I. V. Lerner, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991).

[3] B. L. Altshuler, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985)].

[4] P. A. Lee and A. D. Stone, Phys. Rev. Lett. **54**, 1622 (1985).

[5] B. L. Altshuler and B. Shklovskii, Zh. Eksp. Teor. Fiz. **91**, 220 (1986) [Sov. Phys. JETP **64**, 127 (1986)].

[6] A. Schmid, Phys. Rev. Lett. **66**, 80 (1991).

[7] B. L. Altshuler, Y. Gefen, and Y. Imry, Phys. Rev. Lett. **66**, 88 (1991).

[8] F. von Oppen and E. K. Riedel, Phys. Rev. Lett. **66**, 84 (1991).

[9] L. P. Levy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).

[10] H. Bouchiat and G. Montambaux, J. Phys. (Paris) **50**, 2695 (1989).

[11] H. Bouchiat, G. Montambaux, and D. Sigeti, Phys. Rev. B **44**, 1682 (1991).

[12] A. Altland, S. Iida, A. Müller-Groeling, and H. A. Weidenmüller (to be published).

[13] K. B. Efetov and S. Iida (to be published).

[14] V. Ambegaokar and U. Eckern, Phys. Rev. Lett. **65**, 381 (1990).

[15] V. Chandrasekhar, R. A. Webb, M. J. Brady, M. B. Ketchen, W. J. Gallagher, and A. Kleinsasser, Phys. Rev. Lett. **67**, 3578 (1991).

[16] B. L. Altshuler, A. G. Aronov, and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 101 (1981) [JETP Lett. **33**, 94 (1981)].

[17] B. L. Altshuler, A.G. Aronov, D. E. Khmelnitskii, and A. I. Larkin, in *Quantum Theory of Solids*, edited by I. M. Lifshits (Mir Publishers, Moscow, 1982); B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).

[18] This result can be rigorously proved by considering the quadratic-in-$|\mathcal{E}_\omega|$ contribution to the density matrix $\hat{\rho}$ and calculating $I_0 = \mathrm{Tr}(\hat{I}\hat{\rho})$.

[19] For thin rings considered here, one can set the transverse momentum to be zero.

[20] A. G. Aronov and V. E. Kravtsov (to be published).