Esaki-Tsu Superlattice Oscillator: Josephson-Like Dynamics of Carriers

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We report on a theoretical treatment of the Esaki-Tsu superlattice oscillator emphasizing a profound link between the dynamics of charge carriers in a superlattice and the dynamics of Josephson junctions. Using a balance-equation approach we calculate the oscillator efficiency taking account of the negative effective mass that carriers can have in a miniband and of dissipation. We show that emission of electromagnetic radiation from currently available superlattices can occur due to multiphoton transitions from almost zero up to terahertz frequencies.

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Electrons in a miniband of a semiconductor superlattice, accelerated by an electric field perpendicular to the superlattice layers, can have a negative effective mass leading to a negative differential drift velocity versus electric field behavior. Esaki and Tsu [1] proposed to make use of this nonlinear transport property for generation of electromagnetic radiation (Esaki-Tsu superlattice oscillator) [2]. Recently it has been demonstrated experimentally that negative differential drift velocities can be observed for short-period superlattices [3–5] and are related with negative effective masses [3,6]. In this Letter we report on a theoretical study of the efficiency of a superlattice oscillator. Making use of an analogy between the dynamics of carriers in a miniband and the Josephson junction dynamics, we will show that emission of electromagnetic radiation from superlattices can occur due to multiphoton transitions.

In a superlattice the macroscopic-average state can be derived by averaging the electron quasiclassical velocity and energy with a distribution function that satisfies the Boltzmann equation. With a simplified model of the collision integral the following balance equations can be obtained [7,8]:

$$\dot{V} = eE(t)/m(\varepsilon) - \gamma_V V\,, \tag{1}$$

$$\dot{\varepsilon} = eE(t)V - \gamma_\varepsilon(\varepsilon - \varepsilon_T)\,, \tag{2}$$

where V is the average velocity and ε the average energy of the electrons along the superlattice axis, $\varepsilon_T = \frac{1}{2}\Delta[1 - I_1(\Delta/2kT)/I_0(\Delta/2kT)]$ the thermal energy of the carriers, I_1 and I_0 the modified Bessel functions of first and zeroth order, T the lattice temperature, $\gamma_V = \gamma_\varepsilon + \gamma_{el}$ the relaxation frequency of the average velocity, γ_ε the energy relaxation frequency, γ_{el} the frequency of elastic collisions, $m(\varepsilon) = m_0/(1 - 2\varepsilon/\Delta)$ the energy dependent effective mass of electrons in a superlattice miniband, $m_0 = 2\hbar^2/\Delta d^2$ the effective mass at the miniband bottom, Δ the miniband width, d the superlattice period, and E the electric field along the superlattice axis.

Figure 1 illustrates regions of positive and negative effective mass in momentum and energy space in the tight-binding approximation $\varepsilon(p) = \frac{1}{2}\Delta[1 - \cos(pd/\hbar)]$, where p is the momentum along the superlattice axis. Negative values of the effective mass occur at $\varepsilon > \Delta/2$. There is a singularity of $m(\varepsilon)$ at $\varepsilon = \Delta/2$.

Equations (1) and (2) take into account quasiballistic electron motion within the superlattice miniband but also dissipation. If energy and momentum relaxation are neglected ($\gamma_\varepsilon, \gamma_V \to 0$), the equations lead to the Bloch-Zener equations for dissipationless carrier motion [7]:

$$V(p) = (\Delta d/2\hbar)\sin(pd/\hbar)\,, \tag{3}$$

$$\dot{p} = eE(t)\,. \tag{4}$$

For a static electric field, $E(t) = E_0$, Eqs. (3) and (4) and, consequently, Eqs. (1) and (2) yield a real space localized oscillatory motion (Bloch oscillation) of electrons

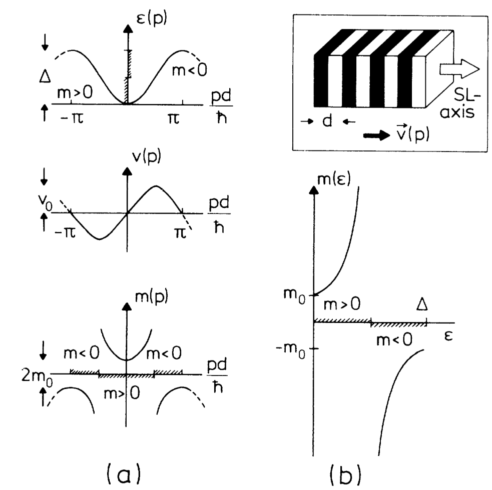

FIG. 1. Energy, velocity, and effective mass of an electron in a superlattice miniband as function of the momentum (a), and effective electron mass as function of the energy (b). Inset: superlattice geometry.

with the Bloch angular frequency $\Omega = eE_0d/\hbar$ due to Bragg diffraction [1,6]. For small fields where the kinetic energy gain of an electron per superlattice period is small compared to the bandwidth, $eE_0d \ll \Delta$, the spatial amplitude $X = \Delta/eE$ of the Bloch oscillation is larger than the period d of the superlattice; for larger fields (and $eE_0d \geq \Delta_G$, where Δ_G is the width of the superlattice minigap) Zener tunneling between minibands may dominate the carrier motion.

The steady-state solution of Eqs. (1) and (2) yields the dc V-E characteristic of the superlattice [7], $V = 2V_p \times (E/E_c)/[1+(E/E_c)^2]$, where $E_c = (\gamma_\varepsilon \gamma_V)^{1/2}/ed$ is the critical field of the negative differential velocity, $V_p = \Delta d I_1(\Delta/2kT)\delta^{1/2}/4\hbar I_0(\Delta/2kT)$ the peak velocity in the V-E characteristic, and $\delta = \gamma_\varepsilon(\gamma_\varepsilon + \gamma_{\mathrm{el}})^{-1}$. In the limiting case of negligible dissipation ($\gamma_\varepsilon \to 0$, $\gamma_\varepsilon \ll \gamma_{\mathrm{el}}$, $\delta \to 0$) the peak velocity tends to zero. For $\gamma_{\mathrm{el}} \to 0$ ($\delta = 1$) and $kT \ll \Delta$ the expressions for V_p and E_c lead to the Esaki-Tsu formula for V [1]. The factor $I_1(\Delta/2kT)/I_0(\Delta/2kT)$ accounts for reduction of the drift velocity at elevated temperatures describing thermal saturation of the miniband transport in a superlattice; this effect has been observed recently [9]. If the experimental data for V_p and E_c are known, our analytical equations permit us to determine γ_ε and γ_V. As an example, we consider a GaAs/AlAs superlattice at $T = 300$ K with $\Delta \approx 110$ meV, $V_p/d \approx 1.4 \times 10^{12}$ s^{-1}, $eE_cd \approx 3.1$ meV [2], and obtain $\delta \approx 0.21$, $\gamma_\varepsilon \approx 2.2 \times 10^{12}$ s^{-1}, $\gamma_V \approx 1.1 \times 10^{13}$ s^{-1}.

Now we analyze the case when a constant bias field E_0 and an alternating electric field E_ω are applied to the superlattice

$$E(t) = E_0 + E_\omega \cos\omega t\,, \tag{5}$$

where ω is the angular frequency of the alternating field considered to be fixed by an external resonance circuit [10]. In this case, the time-dependent drift velocity $V(t)$ is obtained by a simple computer solution of Eqs. (1) and (2). We introduce an oscillator efficiency η by the ratio of the power P_{bias} that is delivered from a bias source and power P_{alt} that is absorbed from the alternating field by [10]

$$\eta(f) = \frac{P_{\mathrm{alt}}}{P_{\mathrm{bias}}} = \frac{\int V(t) E_\omega \cos\omega t\, dt}{\int V(t) E_0\, dt}\,, \tag{6}$$

where the integration is taken over an oscillation period and where $f = \omega/2\pi$ is the frequency of the alternating field. If $P_{\mathrm{alt}} > 0 (\eta > 0)$ energy of the alternative field is absorbed by the superlattice. If $P_{\mathrm{alt}} < 0 (\eta < 0)$ energy is transferred to the alternating field, i.e., the alternating field is amplified and, in principle, oscillation of the system is possible. In the limiting case $f \to 0$ (and small alternating field amplitudes, $E_\omega \to 0$) a negative efficiency ($\eta < 0$) is obtained if the differential drift velocity is negative; then we find (with $E_0 \gg E_c$, $E_\omega \leq E_0$), for the case of a steady-state V-E characteristic, the efficiency

$$\eta(f \to 0) = [1 - (E_\omega/E_0)^2]^{1/2} - 1 \tag{7}$$

that tends to zero for $E_\omega \to 0$.

It should be noted that a careful numerical analysis of Eqs. (1) and (2) shows that driven oscillations of the average velocity $V(t)$ have the period T strictly equal to the period of the external alternating field $T = 2\pi/\omega$. After an initial transient process a stationary state is reached, described by a solution for $V(t)$ that does not depend on the initial conditions for $V(t=0)$, $\varepsilon(t=0)$ and can be presented in a form of a Fourier series containing only the angular frequency ω of the external field and higher harmonics of ω. The higher harmonics play no role because ω is fixed by an external circuit (for a discussion of this point see Ref. [10]). This justifies the definition of the efficiency η given by Eq. (6) where the lower limit of the integration can be arbitrarily chosen.

Figure 2 shows the efficiency that we calculated numerically with the superlattice parameters [2] already mentioned. The frequency range of amplification ($\eta < 0$) extends from almost dc to about the Bloch frequency $f_\Omega = eE_0d/2\pi\hbar$ that lies in the terahertz frequency range. With increasing bias field the $\eta(f)$ curves show fine structure near the subharmonics $f_n = f_\Omega/n$, where n is $2, 3, \ldots$. In the range of absorption ($f \geq f_\Omega$) the power absorbed from the alternating field can exceed ($\eta > 1$) the power absorbed from the bias field [Fig. 2(c)]. The efficiency for absorption and emission increases with increasing bias

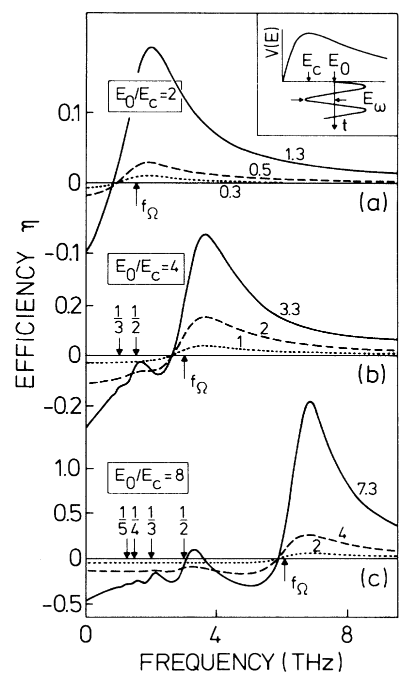

FIG. 2. Superlattice oscillator efficiency for different normalized electric fields E_0/E_c. The range of oscillations ($\eta < 0$) is restricted to frequencies below the Bloch frequency $f_\Omega = eE_0d/2\pi\hbar$. Subharmonics f_Ω/n are indicated by arrows. Above the $\eta(f)$ curves the normalized field strengths E_ω/E_c are shown. Inset: time-dependent electric field diagram.

field (E_0) and increasing strength (E_ω) of the alternating field. Up to almost half of the power absorbed from the bias field can be transferred to the alternating field [Fig. 2(c)].

For a further discussion of our result we make use of the analogy between Eqs. (3) and (4) and the equations governing the dynamics of a superconducting Josephson junction [11] $I=I_c \sin\varphi$ and $\dot\varphi=2eV/\hbar$, where I is the current, I_c the critical current through the junction, V the voltage, and φ the phase difference across the junction. When a carrier is driven by both a constant and a high frequency external field, the solution of Eqs. (3) and (4) is [11]

$$V(t)=V_0\sum_{n=-\infty}^{\infty} J_n\left(\frac{eE_\omega d}{\hbar\omega}\right)\sin\left[\left(\frac{eE_0 d}{\hbar}+n\omega\right)t+\varphi_0\right], \tag{8}$$

where $V_0=\Delta d/2\hbar$, $\varphi_0=p_0 d/\hbar$, $p_0=p(t=0)$, and $J_n(x)$ is the Bessel function of nth order. Equation (8) describes multiphoton transitions (photon assisted tunneling) of free electrons in a superlattice within the framework of the quasiclassical approach. If dissipation is neglected, the steady-state current can flow in a superlattice only if the energy conservation law $eE_0d=n\hbar\omega$ is satisfied. The factor $J_n(eE_\omega d/\hbar\omega)$ gives the probability amplitude of a transition with n-photon emission or absorption. The dependence of the steady-state (time-averaged) free electron velocity on the bias voltage drop per period is schematically shown in Fig. 3(a) demonstrating "Shapiro steps" [11] in a semiconductor superlattice. In Fig. 3(b) we show the "dynamic staircase" —the real space energy diagram illustrating the delocalization of the electron motion in a superlattice due to emission or absorption of photons.

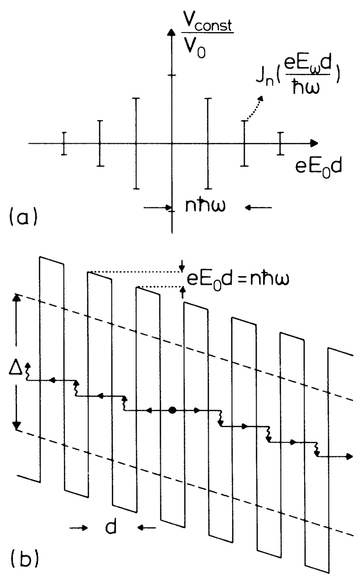

FIG. 3. Time-averaged free electron velocity as a function of the bias voltage drop per period demonstrating "Shapiro steps" in a semiconductor superlattice (a). "Dynamic staircase"—real space energy diagram illustrating the delocalization of the electron motion in a superlattice due to photon emission or absorption (b).

The dynamical behavior of the undamped oscillator delivers a key for a microscopic interpretation of the results for the damped oscillator. As a result of dissipation, interaction of the oscillator with a radiation field is not restricted to sharp resonances, but is a broadband phenomenon. The main slope of the $\eta(f)$ curve (Fig. 2) shows that one-photon emission occurs for $f\le f_\Omega$. The further structure in the curves [see solid curve Fig. 2(c)] is due to 2,3,... photon transitions. For an n-photon transition absorption occurs for $f\ge f_\Omega/n$ and emission for $f\le f_\Omega/n$.

The damped Bloch oscillator is interacting with a radiation field at fixed frequency ω by absorption and emission of n photons ($n=1,2,\dots$) of quantum energy $\hbar\omega$, leading to a net efficiency (η) shown in Fig. 1. At small amplitudes of the alternating field and not too small frequency ($eE_\omega d/\hbar\omega<1$), the probability of emission of a large number of photons is negligible because $J_n(eE_\omega d/\hbar\omega)\to(eE_\omega d/2\hbar\omega)^n/n^2\to 0$ and, therefore, the simulation yields a smooth $\eta(f)$ dependence governed by one-photon transitions. At high amplitudes the probability of transitions with $n\neq 1$ is of the same order as that with $n=1$, resulting in strong modulation of $\eta(f)$. For small frequency ($f\to 0$) the emission is governed by a high order for the multiphoton transitions ($f\to 0$, $n\to\infty$). In this case the resonant peaks of the $\eta(f)$ dependence are smeared out because of the dissipation, and the simulation gives the results described by Eq. (7). We suggest therefore that multiphoton transitions at small frequencies lead to oscillation according to the steady-state Esaki-Tsu theory [1].

It is interesting that Eqs. (3) and (4) that describe electron motion without scattering (dissipation) give a solution for the velocity consisting of harmonics with frequencies that are not commensurate with the external field frequency ω [see Eq. (8)] contrary to the solution of the Eqs. (1) and (2) and, moreover, this solution depends on the initial conditions for the electron momentum P_0. The solution corresponds to the elementary processes of n-photon emission (down motion of electrons) and absorption (up motion of electrons) indicated in Fig. 3 [12]. A comparison with our solution for electron motion with dissipation indicates the fundamental role of dissipation that leads to a discrimination between emission and absorption transitions in Esaki-Tsu oscillators as revealed by our calculations. Actually, the calculations show that the oscillator is able to emit radiation only at frequencies $f<f_\Omega$ ($\hbar\omega<eE_0d$). This is found to be a general result for all investigated material parameters. The reason is that in this case the surplus energy ($eE_0d-\hbar\omega$) of electrons moving "downstairs" can be delivered to the lattice due to electron cooling Conversely, if electrons were able

to emit radiation at frequencies $f > f_\Omega (\hbar\omega > eE_0 d)$, they should have aquired an additional energy $(\hbar\omega - eE_0 d)$ from the lattice in order to keep delocalized motion in real space (see Fig. 3). Of course, in the limit of small dissipation that corresponds to high values of the bias electric field and a high quality of the Bloch oscillations [see Fig. 2(c)], the Bloch frequency f_Ω starts to play a role as the cutoff frequency at which absorption changes to emission. In the limiting case $\eta = 0$ the system neither absorbs nor emits radiation at the Bloch frequency due to equal probabilities of up and down transitions.

Our calculations are restricted to a problem in which there is no spatial variation of the electric field corresponding to the limited space-charge accumulation mode [10]. Finally we would like to stress that the quasiclassical method allows us to treat the superlattice oscillator for $f \le \Delta/2\pi\hbar$ and $E_0/E_c \le \Delta/\hbar(\gamma_\varepsilon, \gamma_V)^{1/2}$. For our particular example we find $E_0/E_c \le 35$ and $f \le 26$ THz, i.e., our calculation suggests that a superlattice oscillator is in principle suitable for generation of radiation up to high terahertz frequencies.

We finally would like to mention that the existence of Bloch oscillations in semiconductor superlattices has recently been demonstrated experimentally [13].

In conclusion, we have shown that amplification of high frequency electromagnetic radiation by an Esaki-Tsu superlattice oscillator can occur due to resonant multiphoton transitions. These we treated quasiclassically taking account of both the negative effective mass of charge carriers in a miniband and dissipation. The oscillator may be suitable for generation of coherent electromagnetic radiation from almost zero to the Bloch frequency, which lies in the terahertz frequency range for currently available superlattices. Our results indicate a profound link between the dynamics of carriers in a superlattice miniband and the dynamics of superconducting Josephson junctions.

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