Critical Behavior of the Hall Coefficient of Si:B

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Measurements between 0.05 and 1 K in magnetic fields small enough to ensure linear response ($B < 1$ T) indicate that the Hall coefficient of Si:B diverges at the metal-insulator transition. This is similar to Ge:Sb and differs from the finite behavior claimed for Si:As and Si:P. Our result may be due to strong spin-orbit effects; it is inconsistent with a recent suggestion that the Hall coefficient is finite in systems with critical conductivity exponent $\mu \approx \frac{1}{2}$.

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The metal-insulator (M-I) transition in doped semiconductors has received a great deal of theoretical [1] and experimental [2] attention over the past decade. Although substantial progress has been made in our understanding of the transition, it is surprising that several rather important issues remain unresolved. Among these, the behavior of the Hall coefficient near the transition is a particularly interesting problem of fundamental importance.

The critical behavior of the longitudinal component of the zero temperature conductivity, $\sigma(0) = \sigma_0[(n/n_c) - 1]^{\mu}$, has been studied in detail in many doped semiconductors as well as in amorphous metal-semiconductor systems. By definition, the transition to the insulating phase occurs at a critical concentration n_c where the zero-temperature conductivity vanishes (the resistivity tends to infinite at 0 K). The critical conductivity exponent μ, which is generally assumed equal to the critical exponent ν that characterizes the divergence of the correlation length, is found to be approximately 1 in most systems. There are a few notable exceptions, namely, amorphous Ar:Ga [3] and all the silicon-based semiconductors, Si:P [4], Si:B [5], Si:As [6], Si:Sb [7], and double-doped Si:P,As [8] where, for reasons that are not yet understood, μ is smaller than 1 and closer to $\frac{1}{2}$. In contrast, the Hall coefficient has been studied in very few materials near the M-I transition and it is not clear under what circumstances it diverges at the critical concentration. Thus, for example, the Hall coefficient has been found to diverge in Ge:Sb [9] and the amorphous systems Bi:Kr [10], Nb:Si [11], Pt:Si [12], and $(Ga_{0.9}Bi_{0.1})_xAr_{1-x}$ [13], while it was found to remain finite in Ar:Ga [3], in In_2O_3 films [14], and in the doped semiconductors Si:As and Si:P [15].

Electron-electron interactions and localization associated with the spatial disorder of the dopant atoms are both important at the transition. Interactions are responsible for a decrease in the density of states at the Fermi energy, while localization causes a decrease in the charge diffusion D. When the transition is driven by interactions, the density of states vanishes at the Fermi energy and the Hall coefficient diverges [16]. On the other hand, at a localization transition the charge diffusion D vanishes, and a scaling calculation for noninteracting electrons by Shapiro and Abrahams [17] predicts that the Hall coefficient should remain finite. Consequently, finite Hall coefficients have been attributed to localization, while diverging coefficients have been attributed to electron-electron interactions. These assumptions have been called into question by recent calculations of Wang, Wang, Kotliar, and Castellani [18], who, contrary to the earlier calculation, claim that the Hall coefficient should diverge at a localization transition or, in effect, that the coefficient $R_H \approx (ne)^{-1}$ probes only the extended states. In light of these recent findings, it is particularly puzzling that R_H is finite in some materials [3,14,15] at the metal-insulator transition.

Two systematic studies of R_H in nominally uncompensated crystalline doped semiconductors have yielded different results. The Hall coefficient was found by Field and Rosenbaum [9] to diverge in Ge:Sb, and it was claimed finite at the transition by Koon and Castner [15] in Si:As. Some early data [19] on Si:P have also been reinterpreted as showing noncritical behavior [15]. It has been suggested [20] that strong spin-orbit scattering may be responsible for the divergence in Ge:Sb and some amorphous systems [13]. Based on the few experimental results currently available, it has also been noted [21] that the Hall coefficient appears to diverge in all materials except those where the conductivity exponent $\mu = \frac{1}{2}$. Support for this conjecture has recently been provided by experiments of Bogerhausen and Micklitz [13], who demonstrated that an increase in spin-orbit scattering through the substitution of 10% Bi for Ga in Ga:Ar results in a change in the conductivity exponent from $\frac{1}{2}$ to 1 and a change in the Hall coefficient from finite to divergent.

The purpose of the experiments described below was to test these conjectures by measuring the Hall coefficient of Si:B. There are strong spin-orbit effects [5] associated with scattering by impurities between the degenerate heavy- and light-hole $J = \frac{3}{2}$ valence bands in p-type Si:B. If spin-orbit effects are indeed important in determining the behavior of the Hall coefficient near the transition, one should observe a diverging R_H. On the other hand, the conductivity exponent of Si:B is closer to $\frac{1}{2}$ than it is

TABLE I. Room-temperature resistivity $\rho(300\ \mathrm{K})$, resistance ratio $R(4.2)/R(300)$, dopant concentration n, and concentration normalized to the critical concentration, $n_c = 4.06 \times 10^{18}\ \mathrm{cm}^{-3}$.

$\rho(300\ \mathrm{K})$ $(10^{-3}\ \Omega\,\mathrm{cm})$	$R(4.2)/R(300)$	n $(10^{18}\ \mathrm{cm}^{-3})$	n/n_c
14.3	1.269	5.22	1.29
14.9	1.436	4.93	1.21
15.5	1.662	4.68	1.15
15.9	1.853	4.53	1.12
16.3	2.124	4.37	1.08
16.9	2.600	4.16	1.02
17.0	2.728	4.11	1.01

to 1, so that the classification scheme that relates R_H to the value of μ would imply the Hall coefficient should remain finite.

Samples were cut from 0.3-mm-thick, 5-cm-diam wafers of Czochralski-grown Si:B obtained from Pensilco (now Puresil). Seven samples were used in these studies for which Table I lists room-temperature resistivities, resistance ratios $R(4.2\ \mathrm{K})/R(300\ \mathrm{K})$, and dopant concentrations using the Thurber scale [22]. All samples were etched in a CP-4 solution to remove any damaged surface layer before electrical contacts were made. Ion implants were necessary to ensure good contact and gold wires were attached to all samples by a special arc discharge technique [23]. The Hall coefficients were measured in an Oxford Model 75 dilution refrigerator at temperatures down to 50 mK in magnetic fields below 1 T. Samples were immersed directly in the ^{3}He-^{4}He mixture in order to achieve good thermal contact and to mount samples free from stress. Measurements were made with a PAR 124A lock-in amplifier at 17.5 Hz at the lowest temperatures and with a AVS-46 ac bridge at temperatures above a few hundred mK. Different excitation currents were used to ensure there was no self-heating. The longitudinal component due to slight misalignment of the contacts was eliminated by either reversing the magnetic field or interchanging current and voltage leads [24], with both methods yielding consistent results.

The Hall coefficient is plotted as a function of temperature in Fig. 1. Measurements were limited to magnetic fields below 1 T to ensure linear response. This is demonstrated in the inset, which shows the Hall resistivity of one sample taken at three temperatures for magnetic fields up to 1 T. We note that earlier measurements [25] in Si:B demonstrated that the critical exponent for the longitudinal conductivity changes in a magnetic field. We stress that the exponent was found to be essentially unaltered from its zero-field value at 1 T, and we therefore expect that measurements of the Hall resistivity in fields below 1 T yield meaningful zero-field extrapolations.

In the absence of electron correlations, a temperature-independent Hall coefficient is expected by scaling theory [17], while electron-electron interactions give rise [26] to square-root corrections in three dimensions. The strong temperature dependence of the data of Fig. 1 indicates that interactions play an important role, particularly very near the transition and at very low temperatures. We assume, then, that the Hall coefficient is given by the sum

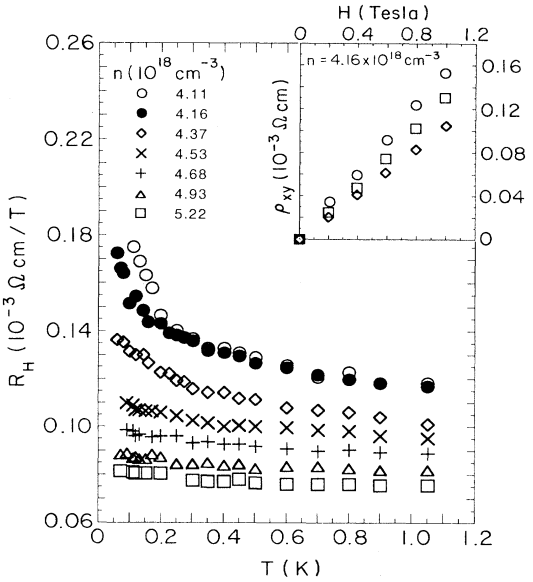

FIG. 1. Hall coefficient of Si:B as a function of temperature for seven dopant concentrations, as labeled. For the sample with $n = 4.16 \times 10^{18}\ \mathrm{cm}^{-3}$, the inset shows that the Hall resistivity is linear with magnetic field up to 1 T at ○—0.05 K, □—0.2 K, and ◇—1.2 K.

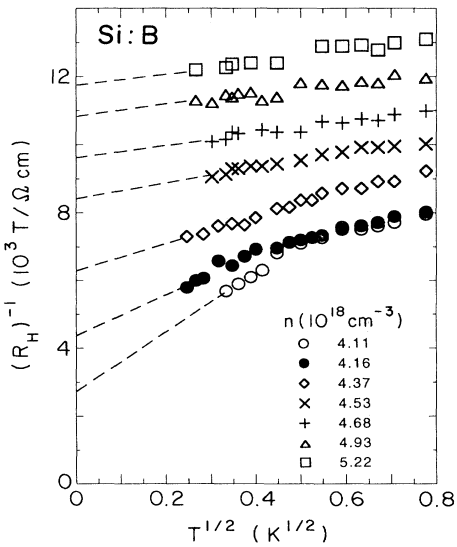

FIG. 2. The Hall number $(R_H)^{-1}$ as a function of $T^{1/2}$ for seven Si:B samples with dopant concentrations as labeled. The dashed lines represent linear regression fits to the data.

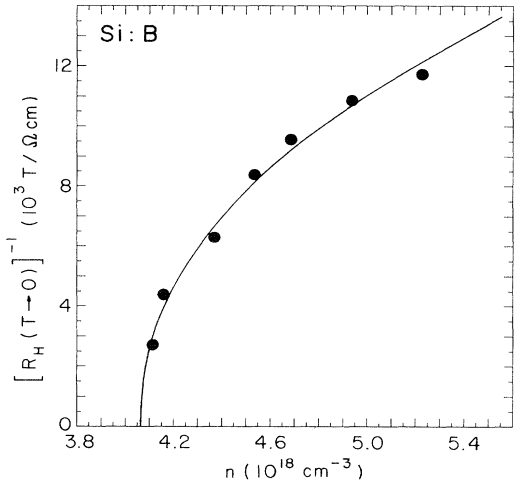

FIG. 3. Zero-temperature extrapolations of the Hall number plotted as a function of dopant concentration. The solid line is a best fit by Eq. (2) with $n_c = 4.06\times10^{18}$ cm^{-3} and $\mu_H = 0.45$.

of two terms

$$[R_H(T)]^{-1} = [R_H(0)]^{-1} + m_H T^{1/2} \quad (1)$$

and plot $[R_H]^{-1}$ as a function of $T^{1/2}$ for Si:B in Fig. 2. The data are consistent with Eq. (1), although the precision of the measurements is insufficient to establish that this is indeed the correct expression. Deviations from this simple form are increasingly evident as the transition is approached. Linear-regression fits by Eq. (1) were carried out using data up to 0.5 K ($T^{1/2}=0.71$) for all samples except the two samples closest to the transition, where the range was restricted to data below 0.2 K ($T^{1/2}=0.45$).

The zero-temperature intercepts, $[R_H(0)]^{-1}$, are plotted in Fig. 3 as a function of impurity concentration. Fits of the data by

$$[R_H(0)]^{-1} = [R_0]^{-1}[(n/n_c)-1]^{\mu_H} \quad (2)$$

yield a critical concentration for Si:B consistent with the value $n_c = 4.06\times10^{18}$ cm^{-3} determined in earlier studies of the longitudinal conductivity [5], a critical Hall conductivity exponent $\mu_H = 0.45$ and a prefactor $[R_0]^{-1} = 21.4\times10^3$ T/Ω cm.

The Hall coefficient of Si:B is quite similar to that found by Field and Rosenbaum [9] in Ge:Sb. In both systems, the coefficient diverges and does so more rapidly than the longitudinal resistivity. Thus, in Ge:Sb the critical Hall conductivity exponent, $\mu_H = 0.69$, is smaller than the exponent for the longitudinal conductivity, $\mu = 0.9$. Similarly, in Si:B, $\mu_H = 0.45$ compared with $\mu = 0.65$. On the other hand, both materials behave quite differently from Si:P and Si:As, where Koon and Castner's [15] measurements indicate that the Hall coefficient remains finite. We note that Si:B has a critical conductivity exponent similar to Si:P and Si:As, or near $\frac{1}{2}$, while the critical exponent for Ge:Sb is near 1. The critical behavior of Si:B is thus inconsistent with the suggestion that the Hall coefficient is finite in systems with critical conductivity exponent $\mu \approx \frac{1}{2}$. It has also been suggested that strong spin-orbit scattering could be responsible for the divergence. Indeed, spin-orbit effects are known [5] to be important in Si:B. We point out, however, that the critical exponent for the longitudinal conductivity of Si:B is not equal to 1, as expected in the presence of strong spin-orbit scattering. We think it is unlikely that spin-orbit effects determine the behavior of the transverse conductivity but not that of the longitudinal transport. Resolution of these questions requires further experimental work on well-characterized materials down to very low temperatures.

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[1] For a theoretical review, see P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985), and references therein.

[2] For an experimental review, see G. A. Thomas, in *Localization and Interaction*, edited by D. M. Finlayson (The Scottish Universities Summer School in Physics, St. Andrews, 1986); Philos. Mag. B **52**, 479 (1985).

[3] Th. Zint, M. Rohde, and H. Micklitz, Phys. Rev. B **41**, 4831 (1990).

[4] T. F. Rosenbaum, R. F. Milligan, M. A. Paalanen, G. A. Thomas, R. N. Bhatt, and W. Lin, Phys. Rev. B **27**, 7509 (1983).

[5] P. Dai, Y. Zhang, and M. P. Sarachik, Phys. Rev. Lett. **66**, 1914 (1991).

[6] P. F. Newman and D. F. Holcomb, Phys. Rev. B **28**, 638 (1983); W. N. Shafarman, D. W. Koon, and T. G. Castner, Phys. Rev. B **40**, 1216 (1989).

[7] A. P. Long and M. Pepper, J. Phys. C **17**, L425 (1984); Solid State Electron. **28**, 61 (1985).

[8] P. F. Newman and D. F. Holcomb, Phys. Rev. Lett. **51**, 2144 (1983).

[9] S. B. Field and T. F. Rosenbaum, Phys. Rev. Lett. **55**, 522 (1985).

[10] M. Rohde and H. Micklitz, Phys. Rev. B **36**, 7572 (1987).

[11] G. Hertel, D. J. Bishop, E. G. Spencer, J. M. Rowell, and R. C. Dynes, Phys. Rev. Lett. **50**, 743 (1983).

[12] P. Lobl, M. Esguerra, F. Kus, E. Luscher, G. Fritsch, M. von Allmen, U. Kambli, and A. Schulte, Solid State Commun. **64**, 1107 (1987).

[13] M. Bogerhausen and H. Micklitz, Ann. Phys. (Leipzig) **1**, 11 (1992).

[14] E. Tousson and Z. Ovadyahu, Solid State Commun. **60**,

407 (1986).
[15] D. W. Koon and T. G. Castner, Phys. Rev. Lett. **60**, 1755 (1988).
[16] B. L. Altshuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys. JETP **50**, 968 (1979)].
[17] B. Shapiro and E. Abrahams, Phys. Rev. B **24**, 4025 (1981).
[18] X. Wang, Z. Wang, G. Kotliar, and C. Castellani, Phys. Rev. Lett. **68**, 2504 (1992).
[19] C. Yamanouchi, K. Mizuguchi, and W. Sasaki, J. Phys. Soc. Jpn. **22**, 859 (1967).
[20] T. G. Castner, Phys. Rev. B **42**, 5317 (1990).
[21] M. Kaveh and N. F. Mott, Phys. Rev. Lett. **68**, 1904 (1992).
[22] W. R. Thurber, R. L. Mattis, Y. M. Liu, and J. J. Filiben, J. Electrochem. Soc. **127**, 2291 (1980).
[23] R. J. Capik, Bell Laboratories internal memo, 1974 (unpublished).
[24] H. H. Sample, W. J. Bruno, S. B. Sample, and E. K. Sichel, J. Appl. Phys. **61**, 1079 (1987).
[25] P. Dai, Y. Zhang, and M. P. Sarachik, Phys. Rev. Lett. **66**, 1914 (1991).
[26] B. L. Altshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980).