Shear Damping of Drift Waves in Toroidal Plasmas

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An important conclusion of earlier work using the ballooning representation is that shear damping of plasma drift waves may be suppressed in a torus. This application of the formalism requires that the diamagnetic frequency have a maximum and implies that drift modes can exist only at this maximum. Here we show that there is a far more general class of torodial drift modes. Shear damping is less well suppressed in these new modes, but they extend over a much larger fraction of the plasma radius. They may therefore have significant implications for plasma transport.

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A well-known instability of magnetized plasmas is associated with electrostatic drift waves. These have short wavelength across the magnetic field, long wavelength parallel to it, and frequency close to the electron diamagnetic frequency ω^*. In this Letter we describe a new type of drift wave in a torus, which may be important in interpreting anomalous transport.

In a cylindrical plasma, or in a plane slab, drift waves experience "shear damping" [1]. A mode $\sim\exp[i(nz/L-m\theta)]$ is centered on the resonant surface $m-nq(r)=0$ $[q(r)=rB_z/LB_\theta]$ and the damping is essentially due to the fact that energy is "radiated" from this resonant surface (eventually to be lost by ion Landau damping or other processes). This damping may stabilize the modes in a cylinder.

It was first shown by Taylor [2] that in a *toroidal* system, coupling between modes of different m could result in drift modes in which the shear damping was suppressed. This was later confirmed by calculations [3,4] using the "ballooning representation" [5–7]. However, the conclusion that these are the only, or the most important, drift modes may have been premature. We have now found, within the ballooning representation, a new class of toroidal drift modes. These differ from the modes considered hitherto in that their shear damping is less well suppressed, that they extend over a greater fraction of the plasma radius, and that they occur much more generally.

The ballooning representation can be regarded as a transform from the periodic domain $0\le\theta<2\pi$ to an infinite domain $-\infty<\eta<\infty$, with the periodicity condition replaced by conditions on the behavior at infinity. It allows one to exploit the existence of two disparate radial length scales at high toroidal mode number n—the equilibrium scale length r and the separation of mode resonant surfaces $1/nq'(r)$. The perturbation is expressed as

$$\phi=A(q)\exp\{i[n\zeta-nq(\eta-\kappa)-\Omega t]\}\,, \qquad (1)$$

where κ is a free parameter, $q(r)$ is used as radial coordinate, and ζ is the toroidal angle. Then an expansion in the small parameter $1/n$ leads in lowest order to a differential equation in the extended coordinate η alone, with q appearing only as a parameter. The condition that its solution be well behaved at infinity leads to a "local" eigenvalue $\omega(q,\kappa)$ periodic in κ. This does not itself determine the frequency of the toroidal mode, but if the local eigenvalue can be expanded about a stationary point, where $\omega(q,k)=\omega_0$, then the higher-order calculations establish that a toroidal mode indeed exists with frequency close to ω_0. If the damping is small compared to the real frequency, the stationary point is near a maximum in the diamagnetic frequency $\omega^*(q)$. Consequently, this type of toroidal mode can occur only near a maximum of ω^*. (This is reminiscent of the plane slab situation where drift modes without shear damping also exist only at maxima of ω^* [8].)

Thus the conventional theory of toroidal drift waves is actually restricted to one special type. We will show that there is another type, unrelated to a maximum in ω^*.

A model for drift waves in a circular large-aspect ratio torus is

$$\left[\frac{1}{(nq')^2}\frac{\partial^2}{\partial x^2}-\frac{1}{s^2}-\sigma^2\left(\frac{\partial}{\partial\theta}+inq'x\right)^2-\epsilon\left(\cos\theta+\frac{is}{nq'}\sin\theta\frac{\partial}{\partial x}\right)+[\Omega^*(x)-\Omega]\right]\phi(x,\theta)=0\,, \qquad (2)$$

where x denotes the distance from some rational surface and s is the shear rq'/q. This is equivalent to the model used in Ref. [9] but with the dependence on mode number n made explicit. The other parameters are defined in Ref. [9] and are independent of n. The first two terms in Eq. (2) arise from a finite Larmor radius and the third from ion sound (these terms lead to shear damping in the cylinder). The fourth term, proportional to the inverse aspect ratio ϵ, is the effect of toroidal coupling and $\Omega^*(x)-\Omega$ represents the difference between the eigenmode frequency and the local diamagnetic frequency.

In the ballooning transformation [5] we write

$$\phi(x,\theta)=\sum e^{-im\theta}\int e^{+im\eta}\hat{\phi}(x,\eta)d\eta\,. \tag{3}$$

Then $\hat{\phi}$ satisfies Eq. (2) in the extended domain $-\infty<\eta<\infty$ and need not be periodic. In order to unify the description of situations both with and without a maximum in ω^*, it is convenient to write $\hat{\phi}$ in the WKB form

$$\hat{\phi}(x,\eta)=\xi(x,\eta)\exp\{-inq'[x\eta-S(x)]\}\,. \tag{4}$$

To lowest order in $1/n$, ξ must satisfy an ordinary differential equation in η at each x,

$$\left[\sigma^2\frac{d^2}{d\eta^2}+\frac{1}{s^2}+(\eta-k)^2+\epsilon[\cos\eta+s(\eta-k)\sin\eta]-[\Omega^*(x)-\Omega]\right]\xi_0(k,\eta)=0\,, \tag{5}$$

where $k=dS/dx$. The eigenvalue for this equation defines a function $\Omega(x,k)$, periodic in k. In the present simple model, $\Omega(x,k)=\Omega^*(x)+g(k)$, where $g(k)$ incorporates the shear damping and is periodic in k. [Note also that $\xi(\eta+2\pi,k+2\pi)=\xi(\eta,k)$.]

If the diamagnetic frequency $\Omega^*(x)$ has a maximum at x_0 and $g(k)$ is stationary at k_0, we can write

$$\Omega=\Omega_0+\tfrac{1}{2}[(x-x_0)^2\Omega^*_{xx}+(k-k_0)^2g_{kk}] \tag{6}$$

(where we choose k_0 to correspond to a minimum rather than a maximum in the shear damping). This defines two branches $k^+(x,\Omega)$, $k^-(x,\Omega)$ which provide WKB solutions

$$\alpha\exp\left[inq'\int^x k^+dx'\right],\quad \beta\exp\left[inq'\int^x k^-dx'\right], \tag{7}$$

valid except near the turning points where $k^+=k^-$, $dk/dx=\infty$. To obtain a global solution one selects the WKB forms which decay as $x\to\pm\infty$ and extends each around the appropriate turning point. Matching the two forms in their common region of validity leads to the usual WKB eigenvalue condition

$$\frac{nq'}{2\pi}\int(k^+-k^-)dx'=\text{integer}+\tfrac{1}{2}\,, \tag{8}$$

where the integration is between turning points. The important properties of this mode are (i) the radial wave number k is restricted to a small range $O(1/n^{1/2})$ around k_0, where the damping is minimal; (ii) although the mode encompasses many resonant surfaces, it is localized around x_0 and extends only over a fraction $O(1/n^{1/2})$ of the plasma radius.

We now turn to the situation when $\Omega^*(x)$ does not have a maximum, e.g., when it is essentially linear. Then, instead of the form (6) we have

$$\Omega(x,k)=\Omega_0+\lambda x+g(k)\,, \tag{9}$$

and $k(x)$ now has an infinite number of (periodic) branches. The construction of the WKB solution is now quite different from the previous case. It is impossible to construct a global solution involving only a *single* WKB term as $x\to\pm\infty$, because solutions on all branches of $k(x)$ are coupled to each other when the solutions are extended around turning points. The construction therefore involves all branches and will be discussed elsewhere. For the present purpose it is simpler to obtain the solution by an alternative method using a wave number representation for $\hat{\phi}$ [10–12],

$$\hat{\phi}(x,\eta)=\int\hat{\xi}(p,\eta)\exp\{-inq'[x(\eta-p)-\hat{S}(p)]\}dp\,. \tag{10}$$

This can be related to the more conventional form of ballooning representation (4) by the transformation

$$\xi(x,\eta)\exp[inq'S(x)]\to\int\hat{\xi}(p,\eta)\exp\{inq'[px+\hat{S}(p)]\}dp\,, \tag{11}$$

so that, to lowest order in $1/n$, $\hat{\xi}(p,\eta)$ satisfies Eq. (5) with $k(\equiv dS/dx)\to p$ and $x\to -d\hat{S}/dp$. The eigenvalue now defines a function $\Omega(d\hat{S}/dp,p)$ and the eigenfunction retains the property $\hat{\xi}(p+2\pi,\eta+2\pi)=\hat{\xi}(p,\eta)$. Consequently, the representation (10) will be periodic in η if $\exp(inq'\hat{S})$ is periodic in p. Hence, if we impose this condition, $\hat{\phi}(x,\eta)$ will *itself* describe a periodic mode and there is no need to invoke the ballooning representation: We simply put $\eta\to\theta$. This accords with the observation of Dewar and Glasser [13] that the ballooning transformation may be regarded merely as a device to introduce an infinite covering space for θ.

Using (10) we obtain, in place of (9),

$$\lambda\frac{d\hat{S}}{dp}-[g(p)+\Omega_0-\Omega]=0\,, \tag{12}$$

and the condition that $\exp[inq'\hat{S}(p)]$ be periodic is satisfied if

$$\frac{nq'}{2\pi\lambda}\int_0^{2\pi}[g(p)+\Omega_0-\Omega]dp=\text{integer}\,. \tag{13}$$

The integral (13) represents the area between the constant $\Omega(x,k)$ curve and the x axis over one period of k. This is a surprising result since it depends on the arbitrary origin of the x coordinate. The interpretation is that there are eigenmodes with frequencies corresponding to all values of $\Omega^*(x)$ and the eigenvalue condition (13) relates the frequency of any mode to its location. In the present model, where λ and Ω_0 are real, all these modes have the same damping given by

$$\text{Im}\,\Omega=\frac{1}{2\pi}\text{Im}\oint g(p)dp\,. \tag{14}$$

These new modes, which do not require a maximum in the Ω^* profile, are quite different from those described hitherto. Their radial wave number is not restricted to be near a minimum in the local damping. Instead all values contribute, as shown by Eq. (14), and the damping corresponds to an average of the local value. (A similar averaging occurs for modes in rotating plasmas [14].) The radial structure of the modes is also different: Because the variation of $g(p) \sim O(\epsilon)$ they extend over a fraction $O(\epsilon)$ of the plasma radius, instead of $O(1/n^{1/2})$. (This is reminiscent of toroidal Alfvén eigenmodes [15, 16].)

In conclusion, the conventional theory of toroidal drift modes describes modes which have minimum shear damping. However, these modes may have only limited relevance: They occur only where the diamagnetic frequency ω^* is a maximum and their eigenfunction spans only $O(1/n^{1/2})$ of the plasma radius. In this Letter we have shown that there is another class of drift waves in a torus which has markedly different characteristics. These new modes experience more shear damping (closer to that in a plane slab or cylinder), but they can occur throughout the plasma, not just at maxima of ω^*, and their eigenfunctions span $O(\epsilon)$ of the plasma radius.

Although these results are based on a simple model we expect the general features to be valid for more complex models (and indeed they may be relevant to other toroidal modes). They may be important for the interpretation of anomalous transport in tokamaks. Conventional calculations (such as Ref. [3]) correctly identify the onset of drift wave instability only in cases where ω^* has a maximum. Even in that case this threshold may not correspond to the onset of large anomalous transport, because the unstable mode is confined to the vicinity of the point of maximum ω^*. On the other hand, the new modes can occur anywhere, whatever the profile of ω^*. Furthermore, because of their extended character one might speculate that they will lead to large anomalous transport. If so, plasma profiles might be determined by a marginal stability criterion—but one defined by stability of the new modes, not the conventional ones. Since the threshold for the new modes will be closer to that in a cylinder, this might justify the use of marginal profiles calculated with cylinder-like damping to interpret toroidal experiments [17,18].

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