Nonperturbative Evidence for Nondecoupling of Heavy Fermions

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We investigate, using a $1/N$ expansion, the behavior of a parameter in the scalar-fermion sector of the standard model that shows perturbative nondecoupling as the fermion becomes heavy. This low-energy parameter is related to the S parameter defined through W_3-B mixing. We obtain the leading $1/N$ contribution to this parameter that, if expanded perturbatively, collapses to its constant one-loop result. Nonperturbatively, we find that as the mass of the fermion approaches the triviality scale the behavior of the parameter is nonuniversal and shows nondecoupling.

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When the mass of a particle is generated by a coupling constant, there are physical effects at low energy that do not vanish as the particle mass becomes very heavy. These so-called nondecoupling effects [1] are crucial in that they provide a window into the physics of energies higher than is currently available. This is evidenced by the current restrictions placed on the top quark and Higgs boson masses in the standard model [2] due to precision measurements [3].

Nondecoupling effects have also raised an important issue regarding the attempts to formulate chiral theories on the lattice [4]. One of the main problems in this program is the inevitability of the existence of the unwanted fermion doublers as required by the Nielsen-Ninomiya theorem [5]. In some of the approaches to this problem, one generates masses of the order of the cutoff scale of the theory for the unwanted fermions using an effectively Yukawa-like coupling. It has been pointed out that at one-loop order, this procedure leaves behind nondecoupling effects, so it is unlikely to be equivalent to the model without the unwanted fermions in the low-energy theory [6]. (Other possible problems have also been pointed out, some previously [4,7].)

To date, nondecoupling effects have been studied within perturbation theory mostly to one- and, on few occasions, to two-loop order. As the mass of the particle becomes heavier, of course the perturbation theory becomes less reliable. It is, therefore, essential to study these issues nonperturbatively and it is necessary to do so when the mass of the particle is of the order of the cutoff scale. Such a study will enable us to determine how these parameters behave outside the perturbative regime and establish the limits of validity of perturbation theory. Also, there can be, and will be, important qualitative effects that do not arise within perturbation theory, as we shall see.

Let us consider a version of the standard model with spontaneous breakdown of a *global* $SU(2)_L\times U(1)_Y$ symmetry in which gauge couplings have been turned off. The effective Lagrangian for the Nambu-Goldstone bosons with heavy fermions integrated out can be written as [we use the spacelike signature $(-+++)$ for the metric]

$$-\mathcal{L}_\chi=\tfrac{1}{2}Z_3(\partial_\mu\chi^3)^2+Z_+|\partial_\mu\chi^+|^2+\text{ interactions}. \qquad (1)$$

In general this Lagrangian will be nonlocal and Z_3,Z_+ will be momentum-dependent functions (in momentum space) amenable to a nonperturbative calculation in the Yukawa coupling. Let us define the parameter $\tilde{S}$ by

$$\tilde{S}\equiv-2\pi v^2\frac{d}{d(p^2)}Z_3(p^2)\bigg|_{p^2=0}, \qquad (2)$$

where v is the corresponding vacuum expectation value that signals the spontaneous breakdown of the symmetry. Obviously, $\tilde{S}$ is "gauge invariant" in the sense that it cannot depend on any gauge-fixing parameters. To one loop we can compute, for instance, the contribution of a doublet of massive fermions to $\tilde{S}$, and this yields $\tilde{S}=2\times1/(12\pi)$. It is mass independent and, in particular, independent of the amount of the mass splitting. The two Yukawa couplings cancel out in the definition of $\tilde{S}$ because the derivative pulls out two inverse powers of the fermion mass. So $\tilde{S}$ shows perturbative nondecoupling, and as a matter of fact, it counts the number of heavy fermions that have obtained their masses through the mechanism of spontaneous symmetry breaking.

When we turn the gauge couplings on, the Lagrangian of Eq. (1) induces an interaction between the gauge bosons when the Nambu-Goldstone bosons are "eaten." To lowest order in the gauge couplings g,g' (which are known to be small), this interaction can be expressed as

$$-\mathcal{L}_\chi=\frac{1}{2}Z_3\left(\partial_\mu\chi^3-\frac{v}{2}(gW_\mu^3-g'B_\mu)\right)^2+Z_+\left|\partial_\mu\chi^+-\frac{gv}{2}W_\mu^+\right|^2+\text{ other terms}, \qquad (3)$$

where Z_3,Z_+ may be nonperturbative in the Yukawa coupling. We then discover that $\tilde{S}$ is the contribution to lowest order in g,g' of the longitudinal part of the gauge bosons to the S parameter as defined by Peskin and

Takeuchi [8],

$$S \equiv -\frac{16\pi}{gg'}\frac{d}{dp^2}\Pi_{W^3B}\bigg|_{p^2=0}, \tag{4}$$

which characterizes the amount of W^3-B mixing and is a measurable quantity. For instance, the one-loop contribution of a heavy degenerate doublet to S and $\tilde{S}$ are identical, namely, $S=\tilde{S}=1/(6\pi)$ [9]. However, S and $\tilde{S}$ are not identical in general since, for instance, in the case of a nondegenerate heavy doublet, $\tilde{S}$ remains the same but S receives an extra logarithmic contribution,

$$S=\frac{1}{6\pi}\left(1-Y_L\ln\frac{m_U^2}{m_D^2}\right),$$

where Y_L is the hypercharge of the left-handed doublet and $m_{U,D}$ are the masses of the up- and down-type fermions in the doublet.

In this paper, we shall study the nonperturbative behavior of $\tilde{S}$ when the Yukawa coupling (or the fermion mass) becomes very large. We choose $\tilde{S}$ because it has the same perturbative characteristics as S as far as nondecoupling is concerned—which is what we are interested in studying—but allows a much simpler $1/N$ type of nonperturbative treatment than S. Moreover, it seems quite reasonable to us that $\tilde{S}$, being determined by the dynamics of the symmetry-breaking sector, captures the essence of the nondecoupling phenomena found in the S parameter. After all, it is because of the spontaneous symmetry breakdown that, at least perturbatively, nondecoupling occurs.

Among the presumably trivial theories, chiral Yukawa theories have been extensively studied on the lattice [4,6,7,10] and we hope that our simple calculation will be useful in future numerical analyses.

One might hope that nonperturbative effects could make $\tilde{S}$ vanish (just like they tend to stop the perturbative growth of the ρ parameter [11]). However, we find within the $1/N_F$ expansion that as the mass approaches the cutoff the parameter $\tilde{S}$ does *not* vanish and is cutoff dependent; in other words, it is nonuniversal.

The version of the standard model we want to study using the $1/N_F$ expansion has the following Lagrangian:

$$\begin{aligned}-\mathcal{L}_\phi &= \partial_\mu\phi^\dagger\partial_\mu\phi+\lambda(\phi^\dagger\phi-v^2/2)^2\\ &\quad+\bar{q}_L\not{\partial}q_L+\bar{U}_R\not{\partial}U_R+y(\bar{q}_L\phi U_R+\bar{U}_R\phi^\dagger q_L),\end{aligned} \tag{5}$$

where ϕ is in an N_F-dimensional irreducible representation of $\mathrm{SU}(N_F)$. The scalar field develops a vacuum expectation value $\langle\phi\rangle=(v/\sqrt{2},0,0,\ldots,0)^T$ that breaks the symmetry of the Lagrangian from $\mathrm{U}(N_F)$ down to $\mathrm{U}(N_F-1)$ and gives mass to the U fermion. We define the χ^0 and χ^- of Eq. (1) as $\phi=((v+H+i\chi^0)/\sqrt{2}, i\chi^-,\ldots)^T$, where ϕ has N_F components. There are one massive real scalar H, with tree-level mass $\sqrt{2\lambda}v$, and $2N_F-1$ Nambu-Goldstone bosons. Within the fermion sector, q_L and U_R are an $\mathbf{N}_F$ and a $\mathbf{1}$ of $\mathrm{SU}(N_F)$, respectively. We can think of the q_L field as $q_L\equiv(U_L D_L^1, D_L^2,\ldots,D_L^{N_F-1})^T$.

To study the model nonperturbatively, we use the $1/N_F$ expansion by keeping y^2N_F, λN_F, and v^2/N_F fixed as we take N_F to infinity. In this limit, the leading quantum corrections only contribute to the propagator for the Higgs field, H, and the U fermion. The scalar sector and the fermion sector can be solved independently. Except for a trivial shift, v remains unrenormalized so the remaining renormalizations are only those of λ and y. We refer the reader to [11-13] for details. Let us only mention that, to leading order in $1/N_F$, the U propagator reads

$$\begin{aligned}S_U(p)&=\{i\not{p}[A_{R,\mathrm{bare}}(p^2)P_R+P_L]+y_{\mathrm{bare}}v\}^{-1},\\ A_{R,\mathrm{bare}}(p^2)&\equiv 1-\frac{y_{\mathrm{bare}}^2N_F}{2(4\pi)^2}\ln\frac{p^2}{s_{\mathrm{bare}}^U},\end{aligned} \tag{6}$$

where P_L,P_R are projection operators onto the left-, and right-handed fields and s_{bare}^U denotes a regulator-dependent quantity. The Yukawa coupling is renormalized according to

$$y^2(s_0)=\frac{y_{\mathrm{bare}}^2}{1-y_{\mathrm{bare}}^2N_F/(32\pi^2)\ln s_0/s_{\mathrm{bare}}^U} \tag{7}$$

with an arbitrary renormalization scale s_0. This coupling diverges at a scale $s_{\mathrm{triv}}^U\equiv s_0\exp\{32\pi^2/y^2(s_0)N_F\}$, which we identify with the physical cutoff scale in the theory, the triviality scale. This quantity has the generic form of a nonperturbative effect. The mass m_U and the width Γ_U of the fermion are determined from the location of the pole of the full fermion propagator in the complex plane. For convenience, we choose the renormalization scale at the mass scale, $s_0=|m_U-i\Gamma_U/2|^2$, in what follows. In this convention, since the cross sections need to be finite at least at the scale of the mass of the fermion, $y^2(s_0)$ has to be finite and positive within the physical region. In Fig. 1 we show the fermion mass and width as well as the triviality scale as a function of the Yukawa coupling $y^2(s_0)$ evaluated at a scale s_0. The mass of the fermion is smaller than $5.0(v^2/N_F)^{1/2}$ when the coupling constant $y^2(s_0)$ is positive and finite.

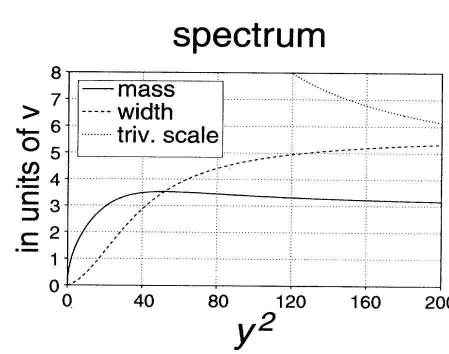

FIG. 1. The plot of the triviality scale s_{triv}, the mass m_U, and the width Γ_U, against the renormalized coupling constant $y^2(s_0)$.

This $1/N_F$ generalization of the standard model is largely dictated by simplicity. For instance, it would be of interest to also study the case where custodial symmetry is unbroken, perhaps using the large-N limit of [14]. However, the model in this case seems substantially more complicated.

The leading-order corrections to the two-point function of the neutral Nambu-Goldstone boson, Π_{χ^3}, arise from the class of one-particle irreducible graphs in Fig. 2 and are $O(1/N_F)$. The contribution of a fermion multiplet to Π_{χ^3} may be computed as [11]

$$\Pi_{\chi^3}(p^2)=2y^2(s_0)\int\frac{d^4k}{(2\pi)^4}\frac{A_R((k+p)^2)k(k+p)+\hat{m}_U^2}{[A_R(k^2)k^2+\hat{m}_U^2][A_R((k+p)^2)(k+p)^2+\hat{m}_U^2]}\,, \tag{8}$$

where $\hat{m}_U^2\equiv y^2(s_0)v^2/2$ and $A_R(s)\equiv 1-y^2(s_0)N_F/(32\pi^2)\ln s/s_0$. The wave function renormalization factor Z_3 in (1) is related to this contribution as

$$Z_3(p^2)=1-\frac{d}{dp^2}\Pi_{\chi^3}(p^2)\,. \tag{9}$$

Using this relation and the definition of $\tilde{S}$ in (2), we obtain the following expression (in Euclidean space) after some algebra:

$$\tilde{S}=\frac{16\pi\hat{m}_U^4}{3}\int_{k^2<\Lambda^2}\frac{d^4k}{(2\pi)^4}\frac{\mathcal{N}}{k^2[A_R(k^2)k^2+\hat{m}_U^2]^5} \tag{10}$$

with

$$\mathcal{N}\equiv a_y(k^2)^2[A_R^2(k^2)-3a_yA_R(k^2)+3a_y^2]+\hat{m}_U^2k^2[3A_R^2(k^2)-7a_yA_R(k^2)+6a_y^2]+a_y\hat{m}_U^4\,, \tag{11}$$

where we used the shorthand notation $a_y\equiv y^2(s_0)N_F/32\pi^2$. In the above expression, there is a pole in the integrand above the triviality scale so that the integral is ill-defined unless we restrict the integration region. The pole is always larger than the triviality scale so that we cut off the integral at a scale Λ^2 below $s_{\rm triv}$, which is consistent with the existence of the intrinsic cutoff scale $s_{\rm triv}$ in the theory. This is how the physical cutoff comes to play the active role that one naturally expects and that is always missed in any perturbative treatment. The integral (10) may be computed after some work to be

$$\tilde{S}=\frac{1}{12\pi}\left[1+\frac{x_\Lambda^2a_y[-2A_R(\Lambda^2)+3a_y]+4x_\Lambda[-A_R(\Lambda^2)+a_y]-1}{[A_R(\Lambda^2)x_\Lambda+1]^4}\right], \tag{12}$$

where $x_\Lambda\equiv\Lambda^2/\hat{m}_U^2$.

If we expand this expression for $\tilde{S}$ in powers of the coupling constant as we would in perturbation theory, the need to restrict the integration region disappears. The truly remarkable fact regarding this parameter in this case is that to *all* orders in perturbation theory, this parameter $\tilde{S}$ is $1/(12\pi)$ and is independent of the Yukawa coupling, or equivalently the fermion mass, to leading order in the $1/N_F$ expansion; in other words, all the higher-order terms in the expansion for $\tilde{S}$ in (10) surprisingly cancel. In fact, it is clear from (12) that $\tilde{S}$ reduces to its constant value in the limit cutoff goes to infinity. The above expressions for $\tilde{S}$ in (10) or (12) include contributions from one-particle irreducible graphs of arbitrary high order (cf. Fig. 2) and these contributions are ultimately crucial, so that this is not a trivial fact. The dependence of $\tilde{S}$ on the mass of the fermion, then, comes solely from the necessity of imposing the cutoff in the theory, which makes this parameter an ideal setting for investigating the physical effects of the triviality cutoff.

We may compute the parameter numerically and our results are plotted in Figs. 3 and 4 against the renormalized coupling constant $y^2(s_0)$ and the mass of the

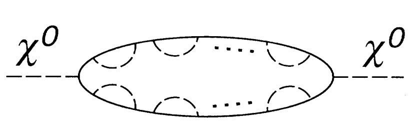

FIG. 2. The class of one-particle irreducible graphs contributing to the propagator of the neutral Nambu-Goldstone boson. Dashed and solid lines represent Nambu-Goldstone bosons and fermions, respectively.

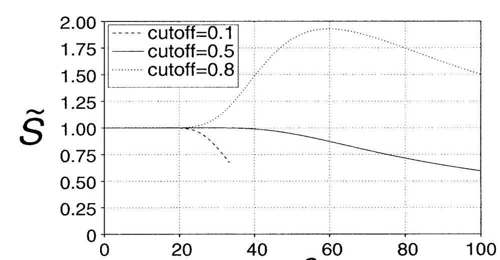

FIG. 3. $12\pi\tilde{S}$ plotted against the renormalized coupling constant $y^2(s_0)$ for the values of $\Lambda/s_{\rm triv}^{1/2}=0.1$, 0.5, and 0.8, which we call "cutoff" in the plot.

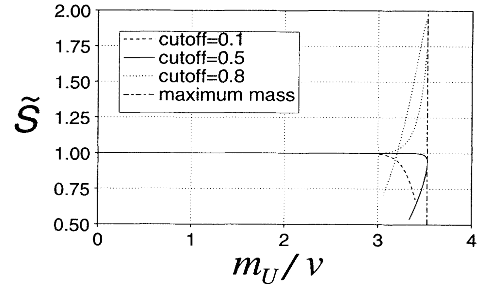

FIG. 4. $12\pi\tilde{S}$ plotted against m_U/v for the values of $\Lambda/s_{\text{triv}}^{1/2}=0.1$, 0.5, and 0.8, which we call "cutoff" in the plot.

fermion m_U, respectively, for a few cutoff values, $\Lambda/s_{\text{triv}}^{1/2}=0.1$, 0.5, and 0.8. As the Yukawa coupling grows, the cutoff Λ decreases and eventually the physical fermion mass would be larger than the cutoff. We have plotted only the region where the fermion mass is smaller than the corresponding cutoff Λ. All calculations agree in the perturbative regime. As the mass approaches the cutoff scale, the results depend on the cutoff scale and deviate from the perturbative result. As the mass increases to $m_U=3.12v$, the $\tilde{S}$ parameter computed with $\Lambda/s_{\text{triv}}=0.8$ differs 1% from the perturbative result, at which point, $s_{\text{triv}}^{1/2}=90v$, $\Gamma_U/m_U=0.51$, and $y^2(s_0)=23.5$. The maximum mass of the theory in the large-N_F limit is $3.52v$ so that the deviations from the perturbative result are appreciable only when the mass is close to its maximum value. As we can see, the contribution to $\tilde{S}$ does not vanish within the physical region defined by $m_U<\Lambda$, although there is an apparent decreasing trend at large couplings that is stronger for low values of the cutoff. If there is a way to make sense of the region $m_U>\Lambda$ in some framework, whether $\tilde{S}$ can vanish in this region might deserve some further investigation.

In closing, we point out that this contribution to $\tilde{S}$ can be understood as the effect of operators of dimension eight or higher in the effective scalar theory. At dimension eight, there is effectively only one operator, $\mathcal{O}\sim\phi^\dagger D_\mu D_\nu\phi\phi^\dagger D^\mu D^\nu\phi$, that contributes to $\tilde{S}$. The first constant term in (12) is generated by an operator like $\mathcal{O}/v^4$ and the cutoff-dependent terms are generated by $\mathcal{O}/\Lambda^4$, in both cases, up to higher dimension operators. The former does not fall off with the cutoff and is a perturbatively relevant, but a cutoff-*independent* contribution. The latter is a cutoff-dependent but a perturbatively irrelevant contribution. The sole reason that this term is not negligible is because the cutoff scale cannot be taken to infinity since it needs to be smaller than the triviality scale.

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