Persistent Spin and Mass Currents and Aharonov-Casher Effect

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Spin-orbit interaction produces persistent spin and mass currents in a ring via the Aharonov-Casher effect. An experiment in the ^{3}He-A_1 phase in which this effect leads to the excitation of mass and spin supercurrent is proposed.

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The discovery in 1983 of the Berry phase [1] brings a new understanding of different topological effects in quantum mechanics. The simplest realization of the Berry phase is the Aharonov-Bohm (AB) effect of a charged particle in an external electromagnetic field. A transparent demonstration of the AB effect is a persistent current in mesoscopic rings threaded by magnetic field [2], as well as many other experiments, which proved the relevance of the AB effect on transport in multiconnected geometry. More recently it has been pointed out that there is an analog of the AB effect in the presence of the spin-orbit (SO) interactions, which leads to nontrivial phase shifts and to a topological interference effect of the wave function of a particle with spin, which was called the Aharonov-Casher (AC) effect [3].

There is a major difference between AB and AC effects: The AB effect comes from the true gauge invariant coupling $j_\mu A_\mu$ between the current j_μ and the electromagnetic vector potential A_μ and for that reason can be observed even in the absence of magnetic field $\mathbf{B}=\mathrm{curl}\mathbf{A}$ in the region where particles are propagating. In the AC effect, on the other hand, the phase of the wave function is changed as a result of SO interaction, i.e., as a coupling of the spin current $j_\mu^{\sigma_i}$ to an effective tensor gauge potential $E_\nu \epsilon_{\mu\nu i}$, where $\epsilon_{\mu\nu\lambda}$ is the antisymmetric tensor and $\mathbf{E}$ is the electric field. Therefore nonzero electric field on the path of the particle is necessary in order to produce the AC phase shift [4]. In the case of electrons moving in the atomic electric field, SO coupling can be written in a more familiar form as $\boldsymbol{\sigma}\cdot\boldsymbol{l}$ with $\boldsymbol{l}$ being the orbital momentum of the electron [5].

In spite of this important difference the unifying point of view on both effects is that they are a consequence of the Berry phase acquired by the wave function of the particle under the transport from some initial state through the set of intermediate states in the Hilbert space back to its original configuration.

Here we will implement this point of view and report on the observation that the AC effect can result in persistent spin and mass currents. The wave function of a particle in external magnetic and electric fields and in the presence of SO interaction will acquire the spin dependent Berry phase: (i) The additional phase of the wave function is given by $\phi=\Phi_{AB}+\sigma_z\Phi_{AC}$, where Φ_{AB} is the AB flux piercing the ring (in case particles are charged) and Φ_{AC} is the AC flux due to the AC effect; $\sigma_z=\pm$ is the spin projection (we assume spin $\frac{1}{2}$ particles). The formerly doubly degenerate eigenstates and eigenvalues will acquire spin dependent shifts, $E_n(\Phi_{AB}+\sigma_z\Phi_{AC})$, $\Psi_n(\Phi_{AB}+\sigma_z\Phi_{AC})$; see also [6]. (ii) This spin dependent shift will lead to a persistent spin current:

$$\begin{aligned} j_\varphi^{\sigma_z} &= -\frac{c}{4\pi R}\mathrm{Tr}\frac{\partial E(\Phi_{AB}+\sigma_z\Phi_{AC})}{\partial\Phi_{AC}}\sigma_z \\ &= -\frac{c}{2\pi R}\frac{\partial E}{\partial\Phi_{AB}} \end{aligned} \quad (1)$$

with $j_\varphi^{\sigma_z}$ being the z component of the spin current along the ring; σ_i are the Pauli matrices. In the presence of the net spin polarization the AC effect also leads to mass current proportional to $n_\uparrow - n_\downarrow$.

Another realization of the Berry phase leading to persistent spin currents was discussed in [7]. Note that the state with persistent spin current does not violate P (parity) and T (time reversal) invariance, in contrast to the persistent mass current. As a result of this, persistent spin current can be excited in the absence of external magnetic flux. Actually for an electron state with nonzero angular momentum l (say in an atom) persistent spin current means nothing but the existence of the SO interaction. (iii) The AC effect and persistent spin currents are independent of the charge of the particle and can be observed for neutral particles, as was done in the original observation of the AC effect for neutrons [8]. (iv) The time-dependent AC flux generates an effective spin dependent "electric" field $\mathbf{E}_{AC}$ via the Faraday law $(1/c)\partial_t\Phi_{AC}=-\oint\mathbf{E}_{AC}\cdot d\boldsymbol{l}$ acting on neutral as well as on charged particles with spin. This is a local effect, independent on the macroscopic phase coherence [9].

The closely related, but different, problem on the effect of random SO interactions on the electron transport properties in a mesoscopic ring was considered by Meir, Gefen, and Entin-Wohlman [6]. They showed that in the presence of SO scattering the flux dependence of energy and eigenfunctions acquires the spin dependent shift $\Phi_{AB}\pm\delta$ where shift δ is governed by an average over the random SO interactions of the ring. Thus the overall effect of the SO interaction in this approach is given by the particular impurity configuration and is independent of external fields. In our case we will neglect the random SO scattering and consider the AC effect in external electric field, which can be varied in the direction and magnitude. This will lead to spin current excitation due to the Faraday law for time dependent AC flux. An interesting realization of this effect will be shown below to take place in superfluid ^{3}He, where external crossed electric and magnetic fields should cause a supercurrent.

Consider first a one-dimensional ring of radius R. We will describe it using a tight binding model on a closed chain of N sites separated by a distance $a=R/N$. An external magnetic field perpendicular to the plane, $\mathbf{B}\|\mathbf{e}_z$ (we will use cylindrical coordinates given by unit vectors $\mathbf{e}_\phi,\mathbf{e}_\rho,\mathbf{e}_z$), results in the twisted boundary conditions for a charged particle wave function $\Psi(N)=\exp(i2\pi\Phi_{AB}/\Phi_0)\Psi(0)$ with the AB flux $\Phi_{AB}=B_z\pi R^2$ and $\Phi_0=hc/e$. The Hamiltonian of this chain, taking into account the SO interaction and Zeeman splitting $g\mu_B\hat{\boldsymbol\sigma}\cdot\mathbf{B}$ (g being the gyromagnetic ratio), can be written as

$$H=-t\sum_{n,\sigma,\sigma'}\hat\Lambda(n)|n,\sigma\rangle\langle n+1,\sigma'|+\text{H.c.}+\sum_{n,\sigma,\sigma'}(\epsilon_n+g\mu_B\hat{\boldsymbol\sigma}\cdot\mathbf{B})|n,\sigma\rangle\langle n,\sigma'|\,. \tag{2}$$

Here n labels sites, ϵ_n denotes the on-site energies, and

$$\hat\Lambda_{\sigma\sigma'}(n)=\exp[i\hat A_{\rm AC}(n,n+1)]_{\sigma\sigma'}=\exp\left(i\frac{g\mu_B}{\hbar c}\hat{\boldsymbol\sigma}\int_n^{n+1}d\mathbf{r}\times\mathbf{E}(\mathbf{r})\right)_{\sigma\sigma'},$$

where $\hat A_{\rm AC}$ is the AC analog of the vector potential $\Phi_{\rm AC}=\sum_{n=1}^{n=N-1}A_{\rm AC}(n,n+1)$.

Following the approach of Ref. [6] we use the transfer matrix T_N defined as

$$\begin{pmatrix}\psi_N\\ \psi_{N-1}\end{pmatrix}=T_N\begin{pmatrix}\psi_1\\ \psi_0\end{pmatrix}, \tag{3}$$

where ψ_n is the spinor wave function of the particle in coordinate representation, and we drop out the spin indexes for simplicity. Using the structure of the Hamiltonian Eq. (2) we can write T_N as a direct product:

$$T_N=\hat S\otimes T'_N,\ T'_N=t^N\begin{pmatrix}1 & 0\\ \sum_{i=1}^N(\epsilon_i+g\mu_B\sigma_z) & 1\end{pmatrix}, \tag{4}$$

$$\hat S=\prod_{i=1}^N\hat\Lambda(i)\,. \tag{5}$$

To derive this equation we had to neglect the noncommutativity of $[\hat\Lambda,\sigma_z]\sim(g\mu_B/\hbar c)E_z a$, which leads to an $O(a/R)$ contribution. As we will see, the Berry phase will be proportional to the circulation of E_ρ on the ring; thus taking into account the commutator will lead to higher order terms. Strictly speaking, Eqs. (4),(5) are valid in the $N\to\infty$ limit when $\hat\Lambda(0)=\hat\Lambda(N)$.

For the slowly varying electric field $\mathbf{E}$ ($|\partial_\phi\mathbf{E}|/|\mathbf{E}|a\ll1$, where a is a distance between sites) the spin transfer matrix $\hat S$ equals

$$\hat S=\prod_{i=1}^N\exp\left(i\frac{g\mu_B}{\hbar c}\hat{\boldsymbol\sigma}\int_i^{i+1}d\mathbf{r}\times\mathbf{E}(\mathbf{r})\right)\simeq\exp\left(i\frac{g\mu_B}{\hbar c}\hat{\boldsymbol\sigma}\oint d\mathbf{r}\times\mathbf{E}(\mathbf{r})\right). \tag{6}$$

We neglected the commutator in the exponent in Eq. (6) coming from the Hausdorff formula

$$e^Ae^B=e^{A+B+(1/2)[A,B]},$$

since it is proportional to $\partial_\phi\mathbf{E}$. Only the radial component of the electric field E_ρ contributes to the contour integral in Eq. (6). As a result

$$\frac{g\mu_B}{\hbar c}\oint d\mathbf{r}\times\mathbf{E}(\mathbf{r})=-\mathbf{e}_z2\pi\frac{\Phi_{\rm AC}}{\Phi_0}, \tag{7}$$

and using Eqs. (6),(7) the eigenfunction equation with twisted boundary conditions can be written in the compact form

$$T'_N\begin{pmatrix}\psi_1\\ \psi_0\end{pmatrix}=\exp\left(2\pi i\frac{\Phi_{\rm AB}}{\Phi_0}+2\pi i\sigma_z\frac{\Phi_{\rm AC}}{\Phi_0}\right)\begin{pmatrix}\psi_1\\ \psi_0\end{pmatrix}. \tag{8}$$

It is obvious that in this geometry the spin dependent AC flux coming from SO interactions enters the transfer matrix as a spin dependent phase. This equation allows one to find energy eigenvalues and eigenfunctions of the tight binding Hamiltonian equation (2) if they are known for the bare problem without SO interactions. Namely, the energy spectrum and the wave functions will depend on the effective flux which is a sum of spin independent (AB) and spin dependent (AC) parts,

$$E_n=E_n(\Phi_{\rm AB}+\sigma_z\Phi_{\rm AC}),\quad \Psi_n=\Psi_n(\Phi_{\rm AB}+\sigma_z\Phi_{\rm AC}),$$

as was mentioned. The energy dependence on the $\Phi_{\rm AC}$ leads to the persistent spin current $j_\varphi^{\sigma_z}$ [see Eq. (1)]. Note again that the spin dependent Berry phase $\Phi_{\rm AC}$ is nonzero even for neutral particles. This equivalence of the AC and AB effect holds only if the spin relaxation is neglected.

It is instructive to estimate the magnitude of the AC flux for realistic mesoscopic systems. From Eq. (7) it follows that

$$\frac{\Phi_{\rm AC}}{\Phi_0}=g\mu_B\frac{RE}{c\hbar}\,. \tag{9}$$

The relativistic nature of the AC effect is reflected in the ratio of electric potential on the scale of the size of the ring eRE to the rest energy of the particle mc^2.

If the electric field is time dependent the variation of the AC flux results in the appearance of a spin dependent driving force due to the Faraday law. This causes the spin current to be in accord with Ohm's law. Applying the Faraday law to spin $\uparrow\downarrow$ liquids (assuming Φ_{AB} =const) we find that particles experience the spin dependent force $-\sigma_z(1/c)\partial_t\Phi_{AC}=\sigma_z\oint\mathbf{E}_{AC}\cdot d\mathbf{l}$. Again, in the presence of net spin polarization this force will cause a mass current. This effect is a consequence of the Lorentz invariance and electrodynamics and is a local phenomenon, as has been pointed out previously [9]. In order to excite spin and mass current due to local force the global phase coherence along the ring *is not required.*

For a ring of the radius $R\sim10^{-3}$ cm, in external electric field $E\sim10^5$ V/cm, Eq. (9) gives for particles with gyromagnetic ratio $g\sim1$ that $\Phi_{AC}/\Phi_0\sim10^{-3}$ [see Eq. (9)], that is, a tiny effect. On the other hand, in semiconductors, the effective g factor can be 2 orders of magnitude larger [10]. For these samples the effective flux will be of the order of $10^{-1}\Phi_0$, which makes the interference effects associated with the AC effect in external fields experimentally observable. Still the experimental observation of the effect in a real mesoscopic ring does not look like a simple problem. The main difficulties are due to the screening of the electric field and the necessity to work with considerably strong magnetic fields which prevent the usage of a SQUID.

Consider another class of systems with phase coherence established on the macroscopic scale—superfluid ^{3}He. In strong enough magnetic field ($B\sim10$ kG) the superfluid transition is known to split into two phase transitions [11]: (1) First, atoms with spins along the field are paired—this is called the ^{3}He-A_1 phase. (2) After a second phase transition, atoms of both spin directions are paired in the ^{3}He-A_2 phase.

The ^{3}He-A_1 phase is a superfluid with $S_z=+1$ Cooper pair condensate. Atoms with spins opposite to the magnetic field are not condensed. In a certain range of temperatures, $\Delta T\sim(6\times10^{-3}$ mK/kG$)B$, the ^{3}He-A_1 is the only superfluid phase. Thus the ^{3}He-A_1 phase has important features: (1) phase coherence of the $S_z=+1$ condensate over macroscopic distances and (2) total spin polarization of the coherent subsystem. This implies that in external electric field the condensate will exhibit the AC effect.

We propose the following experiment. Consider the ^{3}He-A_1 phase within a capacitor formed by two coaxial cylinders. Let x and y be the coordinates on the surface of the cylinder perpendicular and along the axis correspondingly, so that electric field has only a z component. If the spins are polarized along the y axis, the phase of the condensate wave function acquires the position dependent shift $\phi(x)=\phi(0)+2\pi\Phi_{AC}(x)/\Phi_0$, with $\Phi_{AC}(x)$ given by Eq. (7) with the integral taken along the x direction, i.e., around the cylinder. This generates a condensate flow around the ring with the velocity $v_s=(\hbar/2m_3)\partial_x\phi(x)$, i.e., a supercurrent. For $E\sim10^5$ V/cm this current will be $v_s\simeq2\times10^{-7}$ cm/sec (compare with the critical velocity in ^{3}He-A, $v_c\simeq0.02$ cm/sec [11,12]). The phase difference accumulated by the Cooper pair upon transport through the cylinder capacitor of circular perimeter L will be $\Delta\phi\simeq6\times10^{-4}L$[cm]. In order to have a phase difference $\Delta\phi\simeq0.1(2\pi)$ we need to have a channel of length $L\sim1$ m. Presently most experiments on ^{3}He are done in containers with characteristic size of a few cm.

On the other hand, in the singly connected geometry, with a plane capacitor instead of the cylindrical one, there will be no persistent current in the stationary state. The boundary conditions for the wave function are not periodic and the AC gauge potential does not produce a net flux through the system and no persistent supercurrent will be induced.

The ac modification of the same idea looks more interesting. We propose the experiment in the singly connected capacitor with electric field oscillating with some frequency ω: $E=E_0\cos\omega t$. Such a field will cause oscillating super and therefore normal currents which apparently can be detected. The power loss of the viscous flow is known [13] to be equal to $P\simeq8\pi L\eta v_s^2$, where L is the length of the channel and $\eta\sim3\times10^{-2}$ g/cm sec is the normal ^{3}He viscosity. The estimation for the field $E\sim10^5$ V/cm gives $P\simeq10^{-14}$ erg/sec. The power loss in this case is frequency independent since $v_s^2\sim E_0^2\cos^2\omega t=\frac{1}{2}E_0^2$. For example, the temperature dependence of the impedance of the capacitor can be measured with high accuracy. As a result of viscous flow of the normal component the impedance in ^{3}He-A_1 will be different from both the normal and any other superfluid phase: Only in A_1 phase does the AC effect lead to the mass supercurrent and therefore to the normal component flow.

To conclude, we proposed a new realization of the AC effect, which leads to persistent spin and mass currents in mesoscopic rings and in the superfluid ^{3}He-A_1 phase. We argue that a time dependent electric field in a particular geometry will excite *locally* spin current even in macroscopic samples, via the Faraday law.

After we submitted this paper we became aware of the recent preprint by A. Aronov and Y. B. Lyanda-Geller (to be published), in which the closely related effect of the spin-orbit Berry phase on the transport in semiconductors was considered.

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