Excess Noise in a Focused-Gain Amplifier

P. R. Battle, J. G. Wessel, and J. L. Carlsten
Physics Department, Montana State University, Bozeman, Montana 59717
(Received 12 November 1992)

The gain-guided nature of an amplifier can lead to more noise in the output than would be predicted based on the quantum noise limit of one photon per mode. We report the results of an experiment which demonstrate that the gain-guided nature of a focused-gain Raman amplifier can lead to excess noise in the amplified field. The results are compared with a theoretical analysis of the output of an amplifier which incorporates the effects of gain guiding as well as the focused nature of the gain profile.

PACS numbers: 42.60.Da

Optical systems in which the amount of noise effectively seeding the mode is larger than the usual quantum limit are said to have excess spontaneous emission or excess noise [1–3]. Theories based on nonorthogonal mode expansions have predicted, and subsequent experiments have verified, that excess spontaneous emission influences both the spectral width and the beam pointing fluctuations of the output field in an optical system [4,5]. Examples of optical systems, all described by non-Hermitian wave equations or boundary conditions, that may have excess noise include laser resonators with high output coupling, unstable resonators, and gain-guided amplifiers [3].

In this case of a gain-guided amplifier it is the presence of the gain term in the wave equation which makes the system non-Hermitian. Past studies of excess noise properties of gain-guided amplifiers have considered gain profiles which are functions of the transverse coordinates only. However, that approach is unrealistic for systems in which the gain region varies along the direction of propagation, as in the case of a focused-gain profile.

Previous investigations of growth from spontaneous emission in a Raman amplifier have shown that the output can be described well by a fully quantum mechanical plane wave theory in the high gain regime [6]. However, in the low gain regime where spontaneous Raman scattering is dominant the plane wave theory failed to describe the increased Stokes output. Though the effects on gain guiding were not manifest in those experiments, their results indicate that higher-order Stokes modes are coupled to the pump field which, as pointed out by LaSala, Deacon, and Madey [7], is a necessary condition if gain-guiding effects are to be seen in higher gain regimes. Gain-guiding effects did not occur in this earlier work [6] because the pump intensities used were too small to get significant coupling with the higher-order modes.

In this work we report the results of an experiment which show that gain guiding can affect the noise properties of a Raman amplifier. Theoretically, to model these results, the output field is found by solving Maxwell's wave equation; to account for spontaneous emission a quantum Langevin operator, representing the polarization fluctuations in the medium, is included. In a previous study of amplification in a focused-gain system, Perry, Rabinowitz, and Newstein solved for the output intensity by expanding the field into a set of orthogonal Gauss-Laguerre modes [8]. They found that, through amplification, the modes couple, leading to an enhancement of the gain in the amplified field. In contrast to the work of Perry, Rabinowitz, and Newstein, we begin by expanding the field into a set of nonorthogonal modes. By forcing the modes to satisfy a non-Hermitian equation we are led to a general expression for the growth of the amplified field from spontaneous emission. The expression predicts both gain enhancement and the amount of excess spontaneous emission in the field. Our analysis shows that as the gain of the nonorthogonal mode becomes larger than the usual scaled plane wave gain, the amount of noise effectively seeding that mode also increases. Thus we conclude that a measurement of enhanced gain is also an indication that excess spontaneous emission is present.

In this Letter we first give a brief summary of a general nonorthogonal mode theory incorporating excess noise for an amplifier with focused gain. A complete formulation of the theory will be presented in a future publication. The theoretical framework presented can be used to study noise properties of a variety of amplifiers, from a single pass x-ray laser to the Raman amplifier in a multipass cell [8,9]. We then report what we believe is the first direct measurement of gain enhancement from a Raman amplifier in a multipass cell. Finally, we show that, associated with the enhanced gain, there is an increase in the amount of spontaneous emission effectively seeding the amplified field.

Our starting point for describing the growth from spontaneous emission in an amplifier with focused gain is Maxwell's wave equation. In the steady state paraxial limit the wave equation for a slowly varying field traveling in the positive z direction is

$$[\nabla_T^2 - 2ik\partial_z + ikg(z,\mathbf{r}_T)]\hat{E}^{(-)}(z,\mathbf{r}_T) = -k^2 4\pi \hat{P}_{sp}^{\dagger}(z,\mathbf{r}_T)\,, \tag{1}$$

where $\nabla_T^2 = \partial_x^2 + \partial_y^2$ and $k=\omega/c$. $g(z,r)$ represents the gain profile which we take to be proportional to a focused Gaussian, i.e.,

$$g(z,r) = g_0 2e^{-2r^2/w_g^2(z)}/\pi w_g^2(z)\,, \tag{2}$$

where $w_g^2(z) = (2z_0/k_g)[1+(z/z_0)^2]$ is the transverse radius (squared), r is the magnitude of the transverse vector $\mathbf{r}_T$, and z_0, the Rayleigh range, is a constant which corresponds to the length of the focused region along the z axis. To account for spontaneous emission we have included a quantum Langevin operator $\hat{P}_{sp}$ which represents the quantum fluctuations in the polarization of the medium. The presence of the gain (or loss if g is negative) term in Eq. (1) makes the wave equation non-Hermitian.

To solve Eq. (1), the amplified field is expanded into a set of nonorthogonal modes,

$$\hat{E}^{(-)}(z,\mathbf{r}_T)=\sum_{n,l}\hat{a}_n^{l\dagger}(z)\Phi_n^l(z,\mathbf{r}_T)\,, \tag{3}$$

where $\hat{a}_n^{l\dagger}$ is a generalized creation operator for the photons in the mode [10] $\Phi_n^l(z,\mathbf{r}_T)$. We require the modes $\Phi_n^l(z,\mathbf{r}_T)$ to satisfy the eigenvalue equation

$$[\nabla_T^2-2ik\partial_z+ikg(z,r)]\Phi_n^l(z,\mathbf{r}_T)=\lambda_n^l\frac{4ik}{k_g w_g^2(z)}\Phi_n^l(z,\mathbf{r}_T)\,, \tag{4}$$

where λ_n^l is the eigenvalue associated with the mode $\Phi_n^l(z,\mathbf{r}_T)$. The indices n and l correspond to the radial and angular degrees of freedom, respectively. In contrast to previous work [1,2,5,9,10] in which the gain function was not a function of z, the modes defined above are necessarily a function of z because of the focused nature of the gain profile. In the limit of zero gain the modes defined by Eq. (4) are related to the free space Gauss-Laguerre modes. As we will see, the modes which solve this equation provide a natural basis in which to describe the amplified field. Because the modes $\Phi_n^l(z,\mathbf{r}_T)$ are solutions to a non-Hermitian equation they are not guaranteed to be complete nor are they orthogonal to each other. Completeness of these modes depends on the gain profile, and therefore must be proved on a case-by-case basis [11]. The standard power orthogonality relation is supplanted by

$$B_{n,n'}^l=\int d^2r_T\,\Phi_n^{l*}\Phi_{n'}^l\neq\delta_{n,n'}\,, \tag{5}$$

which is a measure of the overlap between the nonorthogonal modes. Even though the modes are functions of z, the overlap between modes does not change as a function of z. Because the overlap between different modes is nonzero, the noise in different modes is correlated [2,3]. The quantity $(B_{n,n}^l)^2$ is referred to as the excess spontaneous emission or Petermann factor for the mode $\Phi_n^l(z,\mathbf{r}_T)$ and is greater than or equal to unity.

While the modes of the amplifier are obtained by solving Eq. (4), to determine the population in those modes, the evolution of the generalized creation operators $\hat{a}_n^{l\dagger}$ must also be obtained. Substituting the field expansion, Eq. (3), into Eq. (1) and using Eq. (4) yields an equation of motion for the operators

$$\sum_n -2ik\Phi_n^l\left[\partial_z\hat{a}_n^{l\dagger}+\lambda_n^l\frac{2}{k_g w_g^2(z)}\hat{a}_n^{l\dagger}\right]=-k^2 4\pi\hat{P}_{sp}^\dagger(z,\mathbf{r}_T)\,. \tag{6}$$

If we assume that the spontaneous polarization correlation function $\langle\hat{P}_{sp}^\dagger(z,\mathbf{r}_T)\hat{P}_{sp}(z,\mathbf{r}_T)\rangle$ is delta correlated in space, which is the case for an inverted two-level amplifier, an expression for $\hat{a}_n^{l\dagger}$ can be obtained.

The total power in the amplified field can be determined, at a given point z, from

$$\begin{aligned}P&=\frac{c}{2\pi}\int dr_T^2\langle\hat{E}^{(-)}(z,\mathbf{r}_T)\hat{E}^{(+)}(z,\mathbf{r}_T)\rangle\\&=\frac{c}{2\pi}\sum_{n',n,k,l}\langle\hat{a}_{n'}^{k\dagger}(z)\hat{a}_n^l(z)\rangle\int dr_T^2\,\Phi_n^{l*}\Phi_{n'}^k\,.\end{aligned} \tag{7}$$

Note that, because the modes are nonorthogonal, the total power is not just the sum of power in each mode but has contributions from cross terms as well. Substituting Eq. (5) and $\hat{a}_n^{l\dagger}$, obtained from Eq. (6), into the expression for the total power gives

$$P=\Delta\nu\hbar\omega\sum_l\sum_{n',n}(B_{n',n}^l)^2\left(e^{(\lambda_{n'}^l+\lambda_n^{l*})(\theta-\theta_0)}-1\right)\,, \tag{8}$$

where $\theta=\tan^{-1}(z/z_0)$ and $\Delta\nu$ is the Hertzian bandwidth of the amplified field. We emphasize that the factor $(B_{n',n}^l)^2$ is not a function of the propagation distance z; however, it does depend on the gain of the amplifier.

The transformation from z to the new propagation variable θ, introduced by Perry, Rabinowitz, and Newstein [8], has the effect of folding out the focused nature of the gain profile; thus the growth of the amplified field is a simple exponential in θ. When the Rayleigh range z_0 of the amplifying field is large compared to the medium length, the propagation parameter θ can be approximated as z/z_0. In this limit Eq. (8) reduces to a form similar to those found in theories which consider excess noise in systems where the gain profile is a function of transverse coordinates only [1,2,5,9].

When the output of the amplifier is dominated by the lowest-order mode, as is the case for a long amplifier, the total power is approximately

$$P\approx\Delta\nu\hbar\omega(B_{0,0}^0)^2 e^{2\mathrm{Re}(\lambda_0^0)(\theta-\theta_0)}\,. \tag{9}$$

This expression shows that the amount of noise effectively seeding the mode is greater than the usual quantum limit of one photon per mode by precisely the excess spontaneous emission or Petermann factor, $(B_{0,0}^0)^2$.

To demonstrate the effects of gain guiding we present results from two experiments. In both experiments the output Stokes energy from a Raman generator in a multipass cell (MPC) was measured. The only difference between experiments was the length of the Raman cell used. The experimental apparatus is diagramed in Fig. 1. The frequency doubled output of a pulsed, single mode Nd:YAG laser was used to pump the Raman generator. The spatial profile of the pump beam was near Gaussian, which ensured that the stimulated Raman scattering gain profile would have the same form as the gain profile used in the theory. The Raman generator consisted of a cell of H_2 filled to 95 atm pressure. The MPC design is the same as used in previous investigations [12,13] with the

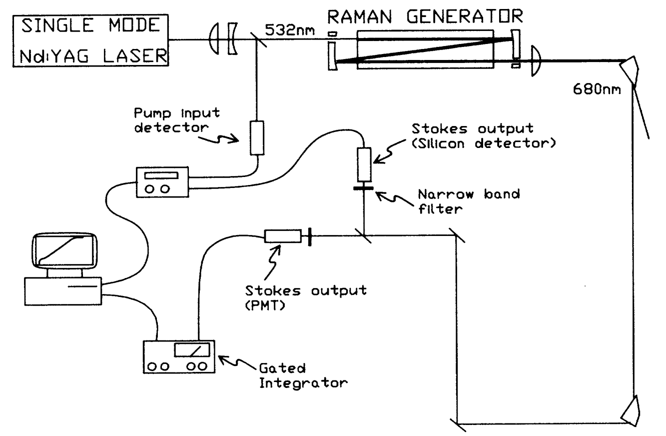

FIG. 1. Experimental apparatus used to measure enhanced gain in the amplified (Stokes) field.

following modifications: The mirror spacing was 186 cm and the number of passes was 11. A pyroelectric detector recorded the pump energy for each shot. The two Stokes detectors in Fig. 1, a photomultiplier tube (PMT) and a silicon detector, detected the Stokes energy over 12 orders of magnitude. To ensure that the experiments did not take place in the depleted pump region the transmission of the cell at the pump wavelength was monitored on each shot.

To compare the results of the experiments to the nonorthogonal mode theory, the instantaneous power given by Eq. (8) was integrated over the temporal profile of the pump pulse to give the energy in the Stokes pulse. Additionally, to account for the spectral gain narrowing [14] in the Stokes pulse we use [15] $\Delta\nu = \Gamma/\pi\sqrt{gz}$ for the Stokes bandwidth in the calculation; Γ is the HWHM of the Raman linewidth and gz is the scaled gain length product [12,13,16] used in earlier calculations of Stokes amplification in a multipass cell [12,13]. The plane wave gain coefficient and the Raman linewidth are calculated from Bischel and Dyer [17]. The multiple passes were accounted for by increasing the propagation parameter θ by the effective number of passes [8].

In Fig. 2 the results of the experiment using the long cell ($l_{\text{cell}} = 141.5$ cm) are presented. The horizontal axis is the scaled gain length product gz while the vertical axis is the output of the generator in joules. The dot-dashed line represents a fully quantum mechanical transient plane wave theory [14], in which the plane wave gain coefficient has been scaled to account for focusing and the multiple passes [12,13]. Also plotted in Fig. 2 is the output of the generator predicted by the nonorthogonal mode theory (solid line). For $gz < 10$ there is a substantial deviation from the scaled plane wave theory. The increase in the Stokes energy is due to spontaneous emission in the higher-order spatial modes, which are not accounted for in the plane wave theory [12]. Because the gain in the higher-order modes is less than the loss due to diffraction only the lowest-order Gaussian mode will experience ex-

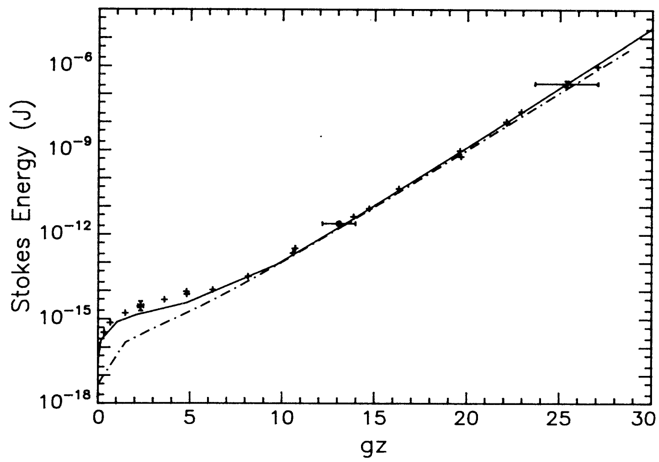

FIG. 2. Output of a long Raman generator in a MPC as a function of scaled plane wave gain coefficient, gz. The dot-dashed line is the transient scaled plane wave theory while the solid line is the nonorthogonal mode theory. Also plotted is the experimental output of a Raman generator (+'s). At low gain the deviation of the scaled plane wave theory from the nonorthogonal mode is due to the presence of higher-order nongaining spatial modes. In the high gain limit only the lowest-order Gaussian mode has significant gain and thus the Stokes output is well described by both the scaled plane wave theory and the nonorthogonal mode theory.

ponential growth. In this regime the Stokes output is well described by both the scaled plane wave theory (dot-dashed line) and the nonorthogonal mode theory (solid line).

In Fig. 3 we show the results of the experiment which used a short cell ($l_{\text{cell}} = 25.4$ cm). Again we find a substantial deviation from the transient plane wave theory

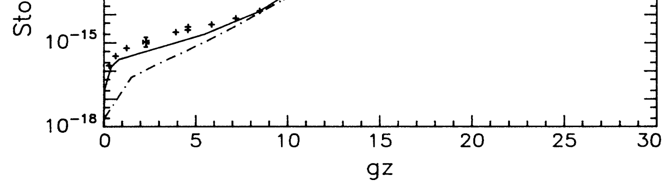

FIG. 3. Same as Fig. 2 except a short Raman cell is used. In the high gain limit transverse gain narrowing occurs leading to an enhancement in the gain, as indicated by the change in slope of the nonorthogonal mode theory. As can be seen, the data follow the nonorthogonal theory, indicating that gain enhancement is occurring in the experiment.

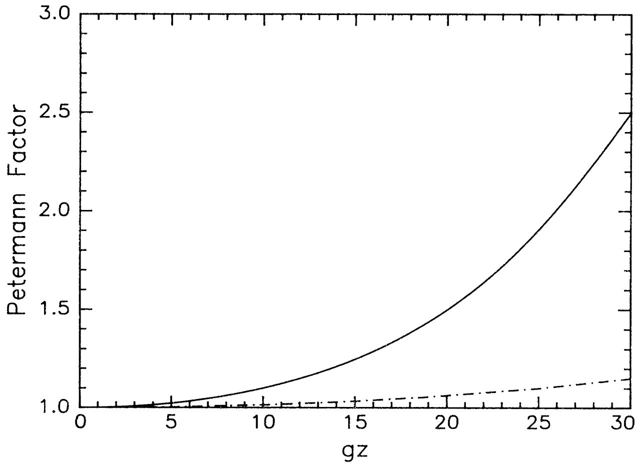

FIG. 4. The excess spontaneous emission or Petermann factor for the lowest-order nonorthogonal mode as a function of the scaled plane wave gain coefficient gz for both the short cell (solid line) and the long cell (dot-dashed line). In the region where enhanced gain occurs the Petermann factor increases. In this experiment an excess spontaneous emission factor as large as 2.5 is predicted.

(dot-dashed line) at low gz and as gz increases the Stokes growth initially follows the scaled plane wave theory. However, in contrast to the results of the long cell experiment, when $gz \geq 15$ the Stokes gain becomes larger than the scaled plane wave gain. In this experiment almost twice as much pump energy was needed to reach a given value of the gain length product gz than was needed for the long cell. The gain enhancement occurs when the Raman cell is pumped hard enough that the gain in the higher-order modes overcomes the loss due to diffraction, thus causing the Stokes field to narrow in the transverse direction. Even in the regime where gain enhancement is occurring the Stokes growth can still be described using a single nonorthogonal mode as in Eq. (9).

In both data sets presented, each experimental point represents approximately 100 shots, all collected at a particular pump input energy (within $\pm 2.5\%$). The theoretical plots in Fig. 2 (Fig. 3) were shifted up by a factor of approximately 5 (2). At this point it is not known if the needed offsets are an experimental artifact of some subtlety which has not been properly accounted for in the theory.

From the solution to the mode equation we are able to determine the Petermann factor for the dominant lowest-order nonorthogonal mode. In Fig. 4 this factor is plotted as a function of the scaled gain length parameter for both the short cell (solid line) and the long cell (dot-dashed line). The plot shows that the system begins with the usual one photon of noise per mode but, as the gain increases (recall that z is fixed in each experiment), the coupling between modes leads to an increase in the amount of spontaneous emission that effectively seeds the dominant nonorthogonal mode. Note that when gain enhancement occurs there is a corresponding increase in spontaneous emission seeding the mode. Thus in the region where enhanced gain was measured we expect excess spontaneous emission should also be present.

In conclusion, by combining the work of Perry, Rabinowitz, and Newstein [8] and techniques involving nonorthogonal mode expansions, a general theory incorporating both excess spontaneous emission and enhanced gain for systems with focused gain has been developed. Measurements of the output of a Raman generator in a multipass cell have confirmed the existence of enhanced gain and are in reasonable agreement with theory.

The authors gratefully acknowledge early discussions with M. G. Raymer on excess noise. This work was supported by National Science Foundation Grant No. PHY-9200104.

[1] K. Petermann, IEEE J. Quantum Electron. **15**, 566 (1979).
[2] H. A. Haus and S. Kawakami, IEEE J. Quantum Electron. **21**, 63 (1985).
[3] A. E. Siegman, Phys. Rev. A **39**, 1253 (1989).
[4] W. A. Hamel and J. P. Woerdman, Phys. Rev. Lett. **64**, 1506 (1990).
[5] S. J. Kuo, D. T. Smithey, and M. G. Raymer, Phys. Rev. Lett. **66**, 2605 (1991).
[6] P. R. Battle, J. G. Wessel, and J. L. Carlsten, Phys. Rev. A (to be published).
[7] J. E. LaSala, D. A. G. Deacon, and J. M. J. Madey, Phys. Rev. Lett. **59**, 2047 (1987).
[8] B. N. Perry, P. Rabinowitz, and M. Newstein, Phys. Rev. Lett. **49**, 1921 (1982).
[9] R. A. London, M. Stauss, and M. D. Rosen, Phys. Rev. Lett. **65**, 563 (1990).
[10] I. H. Deutsch, J. C. Garrison, and E. M. Wright, J. Opt. Soc. Am. B **8**, 1244 (1991).
[11] Completeness of the nonorthogonal modes for the case of a focused Gaussian profile, to our knowledge, has not been demonstrated. However, it appears that the nonorthogonal modes are complete enough to describe the growth of Stokes field from spontaneous emission.
[12] P. R. Battle, J. G. Wessel, and J. L. Carlsten (to be published).
[13] D. C. MacPherson, R. C. Swanson, and J. L. Carlsten, IEEE J. Quantum Electron. **25**, 1741 (1989).
[14] M. G. Raymer and J. Mostowksi, Phys. Rev. A **24**, 1980 (1981).
[15] This expression for the bandwidth is only valid at high gains [14]. At low gains the line shape is Lorentzian; however, for simplicity we approximate the bandwidth as $\Gamma/\pi\sqrt{gz+1}$. It is expected that this approximation leads to the discrepancy seen between the theory and experiment at low gain.
[16] G. D. Boyd, W. D. Johnston, Jr., and I. P. Kaminow, IEEE J. Quantum Electron. **5**, 203 (1969).
[17] W. K. Bischel and M. J. Dyer, J. Opt. Soc. Am. B **3**, 677 (1986).