Critical Constraints on Chiral Hierarchies

R. Sekhar Chivukula,[1],[a] Mitchell Golden,[2],[b] and Elizabeth H. Simmons[2],[c]

[1]*Department of Physics, Boston University, 590 Commonwealth Avenue, Boston, Massachusetts 02215*
[2]*Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138*

Critical dynamics constrains models of dynamical electroweak symmetry breaking (EWSB) in which the scale of high-energy physics is far above 1 TeV. A big hierarchy requires the high-energy theory to have a second-order chiral phase transition, near which the theory is described by a low-energy effective Lagrangian with composite "Higgs" scalars. As scalar theories with more than one Φ^4 coupling can have a Coleman-Weinberg instability and a *first-order* transition, such dynamical EWSB models cannot always support a large hierarchy. If the large-N_c Nambu–Jona-Lasinio model is a good approximation to the top-condensate and strong extended technicolor models, they will not produce acceptable EWSB.

PACS numbers: 12.50.Lr, 11.30.Rd

(1) Introduction.— Much recent work has focused on top-quark-condensate (and related) models [1–5] as well as models with strong extended technicolor interactions [6]. In these theories chiral symmetry breaking driven by dynamics at a high scale ($\Lambda \gg 1$ TeV) plays a significant role in electroweak symmetry breaking. Typically, the high-energy dynamics is assumed to be a broken gauge theory—either extended technicolor (ETC) dynamics in strong ETC models or the dynamics of some grand unified theory in top-condensate models. The high-energy dynamics is usually modeled by a local Nambu–Jona-Lasinio (NJL) four-fermion interaction [7] that is attractive in the chiral symmetry breaking channel. When the strength of the four-fermion interaction is tuned close to the critical value for chiral symmetry breaking, it would appear possible for the high-energy dynamics to play a role in electroweak symmetry breaking without driving the electroweak scale to be of order Λ.

The argument that high-energy dynamics can play a role in electroweak symmetry breaking is independent of the NJL approximation [8]: If the coupling constants of the high-energy theory are small, only strong low-energy dynamics (such as technicolor) can contribute to electroweak symmetry breaking. On the other hand, if the coupling constants of the high-energy theory are large and the interactions are attractive in the appropriate channels, chiral symmetry will be broken by the high-energy interactions and the scale of electroweak symmetry breaking will be of order Λ. If the transition between these two extremes is continuous, i.e., if the chiral symmetry breaking phase transition is *second order* in the high-energy couplings, then it is possible to adjust the high-energy parameters so that the dynamics at scale Λ can contribute to electroweak symmetry breaking. Moreover, if the transition is second order, then close to the transition the theory may be described in terms of a low-energy effective Lagrangian with composite "Higgs" scalars—the Ginzburg-Landau theory of the chiral phase transition.

It is crucial that the transition be second order in the high-energy couplings. If the transition is first order, then as one adjusts the high-energy couplings the scale of chiral symmetry breaking will jump discontinuously from approximately zero at weak coupling to approximately Λ at strong coupling. In general it will not be possible to maintain a hierarchy between the scale of electroweak symmetry breaking and the scale of the high-energy dynamics Λ.

In this Letter we show that there are cases in which the transition cannot be self-consistently second order. A scalar theory in which there is more than one Φ^4 coupling can have a *first-order* phase transition instead, due to the Coleman-Weinberg instability [9]. Therefore, top-condensate or strong ETC theories in which the composite scalars have more than one Φ^4 coupling cannot always support a large hierarchy.

(2) $U(N_f) \times U(N_f)$ models.— For simplicity, we first consider a theory of N_f left- and right-handed fermions Ψ with a chiral $U(N_f) \times U(N_f)$ symmetry. As usual, we assume that the high-energy dynamics is attractive in the $\bar{\Psi}\Psi$ channel. Therefore, the order parameter Φ of chiral symmetry breaking transforms as an $(\bar{N}_f, N_f)$ under the chiral symmetry. If it is possible to arrange for a large hierarchy, then at energies below Λ the dynamics can be described in terms of a Ginzburg-Landau theory for the order parameter Φ coupled to the fermions:

$$\mathcal{L} = \bar{\Psi} i \not{D} \Psi + \frac{\pi y}{N_f^{1/2}} (\bar{\Psi}_L \Phi \Psi_R + \text{H.c.}) + \text{tr}(\partial^\mu \Phi^\dagger \partial_\mu \Phi)$$
$$- M^2 \text{tr}(\Phi^\dagger \Phi) - \frac{\pi^2}{3} \frac{\lambda_1}{N_f^2} (\text{tr}\Phi^\dagger \Phi)^2$$
$$- \frac{\pi^2}{3} \frac{\lambda_2}{N_f} \text{tr}(\Phi^\dagger \Phi)^2 + O\left[\frac{\Phi^\dagger \Phi}{\Lambda^2}, \frac{\partial^2}{\Lambda^2}\right]. \quad (2.1)$$

The quantities y, M^2, λ_1, and λ_2 are functions of the couplings of the fundamental high-energy theory. This effective Lagrangian can be considered the theory of a composite $U(N_f) \times U(N_f)$ Higgs boson Φ.

At tree level, if the high-energy couplings can be

chosen so that $M^2 \ll \Lambda^2$, then it is possible to establish a large hierarchy. This prediction can be affected by quantum corrections: As shown by Coleman and Weinberg [9], if M^2 is adjusted to be close to zero, then quantum corrections can destablize the minimum at $\Phi = 0$. More precisely, if one computes the renormalization-group–improved effective potential and requires that the second derivative of the potential at $\Phi = 0$ be small, one finds that the potential is minimized far away from the origin. Consequently, if one adjusts the couplings in the high-energy theory so that M^2 goes through zero, one finds that the location of the effective potential's absolute minimum jumps discontinuously from $\Phi = 0$ to some large nonzero value of Φ. In other words, the transition which at tree level was second order is driven first order by quantum fluctuations. [The stability of the $U(N_f) \times U(N_f)$ linear sigma model, without fermions, was considered in [10]].

Yamagishi [11] has shown that the condition that the effective potential be minimized away from the origin can be stated purely in terms of the couplings $\lambda_1(\mu)$ and $\lambda_2(\mu)$, by following their flows from $\mu \approx \Lambda$ as the scale μ is decreased. We apply the results of [11] to the Lagrangian (2.1). The effective potential is minimized away from the origin if the couplings cross the line

$$4(\lambda_1 + \lambda_2) + \beta_1 + \beta_2 = 0 \tag{2.2}$$

in a region where $\lambda_2 > 0$, $\lambda_1 + \lambda_2 < 0$, and

$$4(\beta_1 + \beta_2) + \sum_{i,j=1,2} \beta_i \frac{\partial \beta_j}{\partial \lambda_i} > 0. \tag{2.3}$$

Here β_1 and β_2 are the beta functions for the couplings λ_1 and λ_2, respectively. We will refer to the line (2.2) as the "stability line."

If the couplings never cross the stability line, quantum corrections do not drive the transition first order and the high-energy theory may self-consistently have a second-order transition. However, if the couplings do cross the stability line, the low-energy effective theory has a first-order transition and therefore the high-energy theory cannot self-consistently have a second-order transition [12].

In practice, of course, one can only compute the beta functions in perturbation theory. At one-loop order the beta functions are

$$\beta_1 = \frac{1}{12} \left(1 + \frac{4}{N_f^2}\right) \lambda_1^2 + \frac{1}{3}\lambda_1\lambda_2 + \frac{1}{4}\lambda_2^2 + \frac{y^2 N_c}{4N_f}\lambda_1 \tag{2.4}$$

and

$$\beta_2 = \frac{1}{2N_f^2}\lambda_1\lambda_2 + \frac{1}{6}\lambda_2^2 + \frac{y^2 N_c}{4N_f}\lambda_2 - \frac{3N_c}{8N_f}y^4. \tag{2.5}$$

(The contributions from λ_1 and λ_2 differ from those given in [13] and [10] by a factor of $\frac{1}{4}$. We note that in [10] the complex scalar field is incorrectly normalized and this explains the discrepancy.) Here N_c is the number of

colors or technicolors of fermions Ψ. Note that if y is constant, the one-loop β functions for the quantities λ_1/y^2 and λ_2/y^2 are independent of y. The β functions (2.4) and (2.5) have a fixed point which is the analog of the fixed point for the Higgs self-coupling noted in [2].

In these theories, typically the Yukawa coupling is drawn quickly to a low-energy "fixed point" [14,15], where its value runs very slowly due to the running of a relatively weak gauge coupling (color or technicolor). For the purposes of illustration, therefore, we ignore the running of the Yukawa coupling y.

In Fig. 1 we plot the renormalization-group trajectories of λ_1/y^2 and λ_2/y^2 for the model with $N_f = 2$ and $N_c = 3$. These figures show that the couplings λ_1 and λ_2 run toward the fixed point of (2.4) and (2.5) discussed above. If $\lambda_2 > \lambda_1$ and if both are sufficiently strong at $\mu = \Lambda$, the couplings run in such a way as to intersect the stability line. In fact, these trajectories intersect the line *twice*. One can check that as one scales to the infrared (toward the fixed point), condition (2.3) is satisfied only at the *first* intersection and this intersection corresponds to the minimum of the effective potential. We have numerically checked that the picture does not qualitatively change with a running Yukawa coupling or for different values of N_f and N_c.

In the cases where the two λ's start at reasonably large values, they run quickly and intersect the stability line after a small change in μ. At one-loop order, the value of Φ at the minimum of the potential is equal to the value of
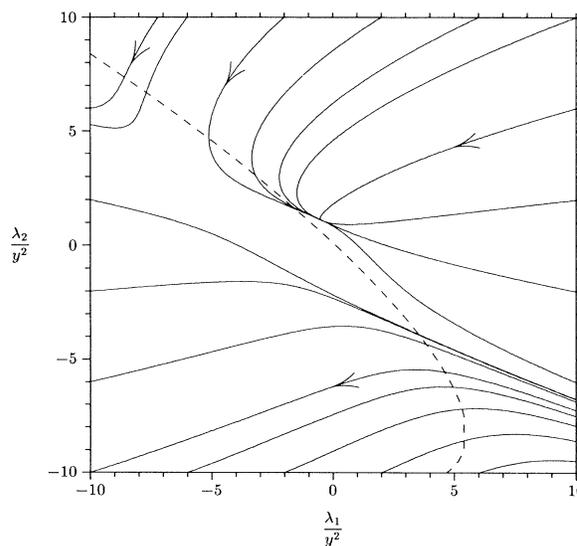
FIG. 1. The trajectories for the quantities λ_1/y^2 and λ_2/y^2 in the $U(N_f) \times U(N_f)$ model. The arrows indicate the behavior as one scales toward the infrared. Here we have taken $N_f = 2$ and $N_c = 3$. Because of the form of Eqs. (2.4) and (2.5), this plot is independent of y. The "stability line" is shown in dashes. Note that the curves that start at large λ_2 and small λ_1 cross the stability line twice, and thus have a Coleman-Weinberg instability.

μ at which the stability line is crossed. Therefore, if the couplings cross the stability line quickly, then $\langle \Phi \rangle$ is of order Λ and there can be no large hierarchy.

Of crucial importance, then, is what values the couplings $\lambda_1(\mu)$ and $\lambda_2(\mu)$ take when $\mu = \Lambda$. This is a nonperturbative problem. In the NJL model one may show [2] that to leading order in $1/N_c$, $\lambda_1(\mu) \to 0$ and $\lambda_2(\mu) \to \infty$ as $\mu \to \Lambda$. This boundary condition puts the $U(N_f) \times U(N_f)$ model in the region which flows rapidly toward the stability line and therefore suggests that it is not possible to obtain a large hierarchy. [These predictions will be modified in a generalized NJL model [16]. However, even in generalized models $\lambda_1(\mu) \to 0$ as $\mu \to \Lambda$ to leading order in $1/N_c$. Therefore, we expect that the transition will still be first order if $\lambda_2(\Lambda)$ is not small.]

One may be concerned that we are investigating the Coleman-Weinberg phenomenon in perturbation theory, but have been forced to consider potentially large values of the couplings λ. However, since the phenomenon depends only on the qualitative features of the renormalization-group flows, we do not expect that higher-order effects will qualitatively change the conclusions. This issue may be tested by simulating the model (2.1) nonperturbatively using lattice techniques. While this has not been done in four dimensions, numerical simulations in three dimensions without fermions (where the lowest-order renormalization-group analysis also predicts a first-order transition [17]) confirm that the transition is first order [18].

The point is that it is not sufficient to adjust the couplings of the high-energy theory so that the second derivatives of the scalar potential at the origin are small. One will also have to adjust the theory so that, at $\mu \approx \Lambda$, one is in a region of coupling constant space which does not quickly flow toward the stability line. In a spontaneously broken gauge theory with a simple gauge group, however, having fixed the scale of symmetry breaking one can only adjust *one* parameter: the value of the gauge coupling at the symmetry breaking scale. One cannot, therefore, simply assume that a large hierarchy of scales is possible. One must check that the effective low-energy theory does not suffer from a Coleman-Weinberg instability. As we have seen, the large-N_c limit of the high-energy NJL model places the $U(N_f) \times U(N_f)$ low-energy model in a region which has this instability.

(3) Other models.— We now consider some other examples. Consider first a generic theory without fermions. As before, we can introduce a field Φ to represent the order parameter of chiral symmetry breaking. If the symmetry of the high-energy theory is such that the Ginzburg-Landau theory for Φ has more than one coupling of dimension four, then, at least in the ϵ expansion, the only fixed point is the infrared-unstable Gaussian fixed point. One therefore expects that the couplings generally flow toward the unstable region, i.e., most trajectories are pushed away from the origin and flow toward large negative values of the couplings.

Now consider the theory with fermions. As we have seen, there will in general be infrared-stable fixed points. However, if the scalar self-couplings are large compared to the Yukawa couplings, the coupling constant flows will (at least initially) look the same as they did without fermions and should, therefore, still cross the stability line.

Accordingly, in a model of composite scalars in which there is more than one Φ^4 coupling and in which the scalar self-interactions become strong at the compositeness scale Λ, the chiral phase transition may not be second order. Such a model will not always sustain a large hierarchy between the compositeness scale Λ and the weak scale.

In top-condensate-inspired models with two composite Higgs bosons [19,20], for example, one has five Φ^4 couplings and three mass terms. It can be argued that one has enough freedom to adjust the three mass terms to be close to zero, but for the reasons discussed above the theory can still have a fluctuation-induced first-order phase transition. Again, if large-N_c arguments apply, the model will not sustain a large hierarchy. (The instability was noted in [19], but its implications were not discussed.)

By contrast, the standard O(4) model [1] has only one quartic coupling. In this case, the "stability line" is a point, and it is at a *lower* value of λ than the fixed point. Therefore, if, as in [2], the value of $\lambda(\Lambda)$ is large, then the trajectory hits the fixed point without crossing the stability point and it may be possible to sustain a large hierarchy.

Note that our results apply only in cases in which the scalar self-interactions become strong at the compositeness scale. In composite-Higgs models in which *all* of the scalars are Goldstone bosons of some chiral symmetry breaking transition at a higher-energy scale [21], the nonderivative self-couplings of the scalars are related to small symmetry breaking effects can naturally be small at $\mu = \Lambda$. Although the transition may in principle be first order, it may take a very large change of scale before the couplings cross the stability line since the couplings are weak. In this case the hierarchy can be large.

Conclusions.— In conclusion, theories of composite Higgs scalars may have a first-order chiral symmetry breaking phase transition if there is more than one Φ^4 coupling and if the scalar self-interactions become strong at the compositeness scale. One must check that the theory does not suffer from the Coleman-Weinberg instability. In particular, in strong ETC models or generalized top-condensate models with more than one Φ^4 coupling in the low-energy theory, one may not be able to adjust the high-energy theory to obtain a large hierarchy between the scale of the high-energy dynamics and the weak scale. If the NJL model solved in the large-N_c limit is a good approximation to the high-energy dynamics, then these models will not produce acceptable electroweak symmetry breaking.

[a] Electronic address: sekhar@weyl.bu.edu

[b] Electronic address: golden@physics.harvard.edu

[c] Electronic address: simmons@physics.harvard.edu

[1] Y. Nambu, Enrico Fermi Institute Report No. EFI 88-39 (unpublished); V. A. Miransky, M. Tanabashi, and K. Yamawaki, Phys. Lett. B **221**, 177 (1989); Mod. Phys. Lett. A **4**, 1043 (1989).

[2] W. A. Bardeen, C. T. Hill, and M. Lindner, Phys. Rev. D **41**, 1647 (1990).

[3] C. T. Hill, M. Luty, and E. A. Paschos, Phys. Rev. D **43**, 3011 (1991); T. Elliot and S. F. King, Phys. Lett. B **183**, 371 (1992).

[4] C. T. Hill, D. C. Kennedy, T. Onogi, and H-L. Yu, Fermilab Report No. FERMI-PUB-92/218-T (to be published).

[5] C. T. Hill, Phys. Lett. B **266**, 419 (1991); S. Martin, Phys. Rev. D **45**, 4283 (1992); **46**, 2197 (1992); N. Evans, S. King, and D. Ross, Southampton University Report No. SHEP-91-92-11 (to be published).

[6] T. Appelquist, T. Takeuchi, M. Einhorn, and L. C. R. Wijewardhana, Phys. Lett. B **220**, 223 (1989); T. Takeuchi, Phys. Rev. D **40**, 2697 (1989); V. A. Miransky and K. Yamawaki, Mod. Phys. Lett. A **4**, 129 (1989).

[7] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).

[8] T. D. Chivukula, A. G. Cohen, and K. Lane, Nucl. Phys. **B343**, 554 (1990).

[9] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).

[10] A. J. Paterson, Nucl. Phys. **B190 [FS3]**, 188 (1981).

[11] H. Yamagishi, Phys. Rev. D **23**, 1880 (1981).

[12] See, for example, D. J. Amit, *Field Theory, the Renormalization Group, and Critical Phenomena* (World Scientific, Singapore, 1984), 2nd ed.

[13] R. D. Pisarski and D. L. Stein, Phys. Rev. B **23**, 3549 (1981); J. Phys. A **14**, 3341 (1981).

[14] B. Pendleton and G. Ross, Phys. Lett. **98B**, 291 (1981).

[15] C. Hill, Phys. Rev. D **24**, 691 (1981); M. Fischler and C. Hill, Nucl. Phys. **B193**, 53 (1981).

[16] A. Hasenfratz, P. Hasenfratz, K. Jansen, J. Kuti, and Y. Shen, Nucl. Phys. **B365**, 79 (1991).

[17] R. D. Pisarski and F. Wilczek, Phys. Rev. D **29**, 338 (1984).

[18] H. Gausterer and S. Sanielevici, Phys. Lett. B **209**, 533 (1988).

[19] M. A. Luty, Phys. Rev. D **41**, 2893 (1990).

[20] M. Suzuki, Phys. Rev. D **41**, 3457 (1990); C. D. Froggatt, I. G. Knowles, and R. G. Moorhouse, Phys. Lett. B **249**, 273 (1990).

[21] D. B. Kaplan and H. Georgi, Phys. Lett. **136B**, 183 (1984).