Persistent Currents in an Interacting 1D Disordered Ring: Manifestations of the Mott-Hubbard Transition

Michael Abraham and Richard Berkovits
The Jack and Pearl Resnick Institute for Advanced Technology, Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel
(Received 4 December 1992)

We present an exact calculation of the persistent current in interacting disordered 1D rings. This system exhibits a variety of interesting behaviors depending on the disorder and the strength of electron-electron interactions. We find that even for the disordered case the current at high values of interactions is strongly suppressed due to the Mott-Hubbard metal-insulator transition. We observe that no dramatic increase of the averaged current or its rms value as a result of interactions is possible for these systems.

PACS numbers: 72.10.Bg, 05.30.Fk, 71.30.+h

Noninteracting electrons in an ordered 1D ring threaded by a magnetic flux are known to carry persistent currents periodic in the magnetic flux of amplitude ev_F/L (where v_F is the Fermi velocity, and L is the ring's circumference) [1–5]. For a disordered ring it is theoretically predicted that the averaged current amplitude will drop as a function of disorder. These results are also correct for a multichanneled quasi-1D ring. The rms value of the current is calculated to be of the same order as the amplitude of the current [2, 4, 5]. An experiment performed on 10^7 quasi-1D disordered rings measured the averaged amplitude of the current to be of order $10^{-2}ev_F/L$ [6], in good agreement with theory. On the other hand, a recent experiment on three single rings measured the current to be of order ev_F/L [7].

The large difference between the measured current amplitude in those two experiments has recently led to many theoretical attempts to explain this discrepancy. Most of the published works try to explain this discrepancy as the result of the fact that previous theoretical work on persistent currents ignored the role of electron-electron interactions in these systems [8–12]. Some of these works concentrate on the influence of interactions on the averaged amplitude [8, 9, 11], while others deal with the influence of interactions on the rms value of the amplitude [10, 12]. Nevertheless, no consensus on the exact influence of interactions in both cases has yet been reached.

The main motivation for considering the role of interactions in the disordered case is the hope that electron-electron interactions will tend to homogenize the system, thus offsetting the reduction in current due to disorder. In this Letter we present an exact study of the influence of interactions on a 1D single band spinless fermion model on a small lattice. Our results for the behavior of the current for different values of interactions and disorder are summed up in a schematic way in Fig. 1. It can be seen that interactions play a different role for different regimes of parameter space. There is a line of maximum enhancement for strong disorders and medium strength of interaction. Even for maximum enhancement the value of current is very far from the ordered noninteracting value of ev_F/L. For strong interactions the current is suppressed by the Mott-Hubbard transition. Maximum suppression of the current by interactions is achieved along a line corresponding to weak disorder and strong interactions. Returning to the general question of the influence of interactions on the averaged current, it can be seen that, although a complex behavior of the current as a function of interaction is observed, none of the changes will increase the averaged current to the value ev_F/L. The reason is that for values of interaction which are big enough to cancel out effects of disorder the current will decrease because of the Mott-Hubbard transition.

The disordered 1D ring system may be represented by the following Hamiltonian:

$$H = \sum_i \epsilon_i a_i^\dagger a_i + \left(V(\theta) \sum_i a_{i+1}^\dagger a_i + \text{H.c.} \right) + H_{\text{int}}, \tag{1}$$

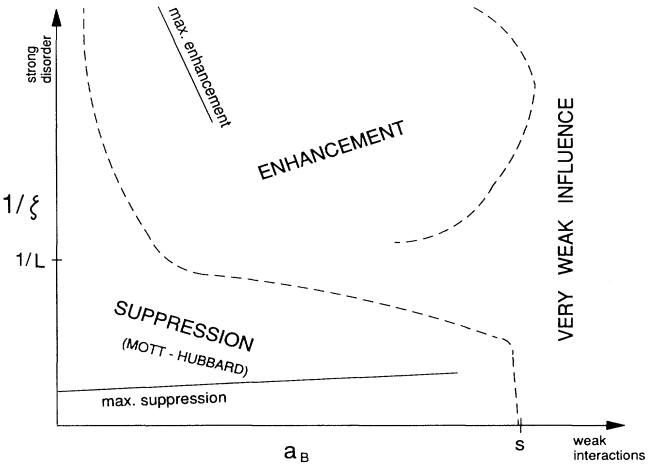

FIG. 1. A schematic drawing of the main features of the current dependence on disorder and interactions. The localization length ξ and Bohr radius a_B are defined in the text.

where ϵ_i is the on-site energy which is chosen randomly between $-W/2$ and $W/2$. $V(\theta) = \exp(i\theta)V$, where V is a constant hopping matrix element. Thus we are considering on-site diagonal disorder. The phase $\theta = 2\pi s\phi/L\phi_0$, where s is the lattice constant, L is the length of the ring, ϕ is the magnetic flux, and ϕ_0 is the quantum flux unit. In the remainder of the Letter we shall take $\phi_0 = 1$. $a_i^\dagger$ (a_i) is the fermionic creation (destruction) operation.

The interactions between the electrons are represented by H_{int}. We have chosen to consider a long range Coulomb interaction of the form

$$H_{\text{int}} = \frac{e^2}{s}\sum_{i>j}\frac{(a_i^\dagger a_i - K)(a_j^\dagger a_j - K)}{|i-j|}, \tag{2}$$

where K represents the average positive jellium which equals the average electron density. We shall define $\epsilon_c = e^2/s$ in order to simplify further discussions.

In order to obtain a reliable guide to the role played by interactions in determining the persistent currents in disordered rings, we have chosen to solve exactly the many-body Hamiltonian given in Eq. (1). This is achieved by an exact calculation of the ground state energy level, as a function of the magnetic flux ϕ, for a finite size system of m sites and n electrons. The number of eigenvectors spanning the many-body Hilbert space is $M = \binom{m}{n}$. The many-body Hamiltonian may be represented by an $M \times M$ matrix which can be numerically diagonalized. Once the lowest eigenvalue is obtained for different ϕ, the persistent current may be calculated using the usual definition

$$I(\phi) = -c\frac{\partial}{\partial\phi}E(\phi), \tag{3}$$

where $E(\phi)$ is the lowest eigenvalue, and c is the speed of light.

We have chosen to consider systems of 6 sites and 3 electrons, and systems of 10 sites and 5 electrons. The solution of these systems involves the diagonalization of 20×20 and 252×252 matrices. The persistent current for different values of W, V, and ϵ_c was calculated. Since we are studying the averaged persistent current we must average over many realizations of disorder. For the 6 site models we have averaged over 10 000 realizations, while for the 10 site models we have averaged over 1000 realizations.

A representative example of the persistent current for different values of V, W, and ϵ_c are plotted in Fig. 2. As one can expect the current is periodic in ϕ for all the cases. For the ordered sample plotted in Fig. 2(a), it is clear that interactions reduce the amplitude of the persistent current. For different values of disorder represented in Figs. 2(b)–2(d), it is seen that interactions play a more complicated role. In some they cases increase the current, while in other cases they decrease it. Generally it is seen that interactions do not dramatically change the basic shape of the curves, although their relative amplitude may change. Therefore, the interesting property of the persistent current is its amplitude, which may be characterized by the absolute current at $\phi/\phi_0 = 0.25$ [2], denoted as I. From the complicated behavior seen in Fig. 2 it is clear that one should investigate the whole parameter space using two relevant energy scales which will be defined latter.

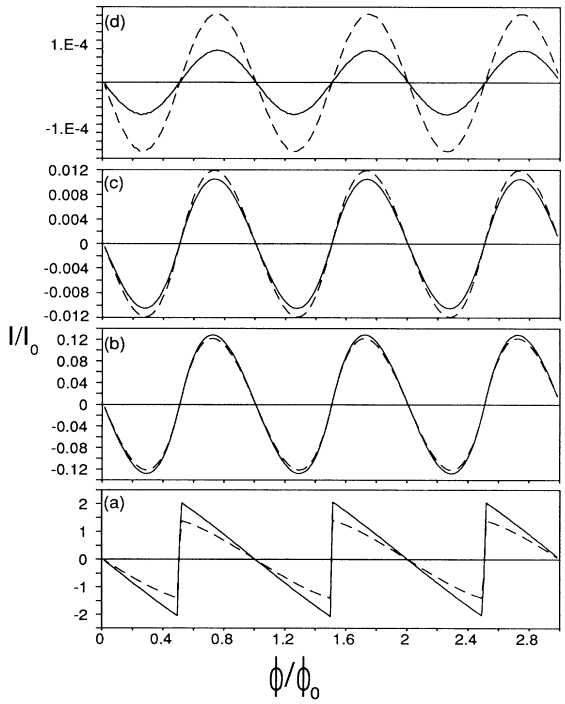

FIG. 2. The persistent current as a function of magnetic flux for different values of interactions and disorder. The full line represents the noninteracting case $\epsilon_c = 0$, while the dashed line represents the interacting case. (a) $W/V = 0$, $\epsilon_c/V = 10$; (b) $W/V = 10$, $\epsilon_c/V = 10$; (c) $W/V = 20$, $\epsilon_c/V = 20$; (d) $W/V = 100$, $\epsilon_c/V = 100$. The scale of the current is strongly dependent on disorder. The maximum influence of interactions is seen in (d) for which the current increases by a factor of 2.5.

First we shall study the two limiting cases, i.e., the behavior of the persistent current as a function of disorder with no interactions ($\epsilon_c = 0$), and for the interacting case with no disorder ($W = 0$). For the first case the relevant energy parameter is V/W, which is the usual measure of disorder in such models. It is well known that without interactions the current amplitude behaves in the following way [2]:

$$I = \frac{1}{2}I_0\exp(-L/\xi), \tag{4}$$

where $I_0 = 4\pi Vs/L$, and ξ is the localization length. For long samples ($L > 20s$) ξ was shown to follow two asymptotical forms: $\xi \sim s/\ln(W/V)$ for strong disorder ($W \gg 2\pi V$), and $\xi \sim s(V/W)^2$ for weak disorder ($W \ll 2\pi V$) [13]. In Fig. 3(a) we plot ξ using Eq. (4)

and our numerical calculations of I. We can see that the numerical results generally follow the above asymptotic behavior, although some deviations appear due to the short sample lengths.

For the ordered sample with interactions the relevant energy parameter is ϵ_c/V, which corresponds in terms of length units to the Bohr radius $a_B = \hbar^2/me^2$. Using the mass definition for tight binding models the Bohr radius is $a_B = (2V/\epsilon_c)s$. In Fig. 3(b) we present our numerical results for the persistent current in this case. One can clearly see the Mott-Hubbard metal-insulator transition expected for the current in 1D spinless fermionic models [14]. The current in the ordered interacting case may be written as $I(a_B) = ev_F^*/L$, where v_F^* is the effective Fermi velocity. For $a_B > s$ the effective Fermi velocity changes moderately as a function of a_B. For nearest neighbor interactions the Fermi velocity is given by $v_F^* = v_F\pi^2 \sin\mu/4\mu(\pi-\mu)$ [14–16], where $\mu = \arccos(1/a_B)$. As can be seen in Fig. 3(b), although our results are for long range interactions and short samples they fit quite well the above analytical formula. For $a_B < s$ a sharp decrease in the current is observed. For an infinite sample and nearest neighbor interactions a discontinuity in the Fermi velocity at $a_B = s$ is expected. For a finite sample one expects an exponential decrease to replace the discontinuity as can be seen in Fig. 3(b) [14].

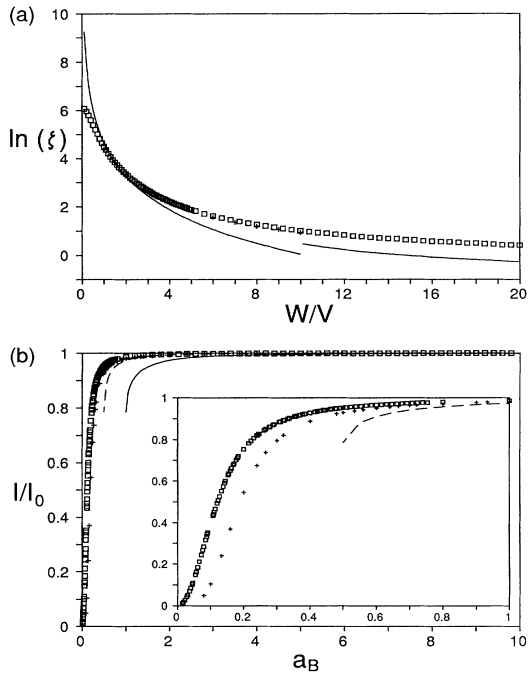

FIG. 3. (a) $\ln(\xi)$ as a function of W/V for noninteracting systems. Squares represent numerical results for the 6 site, 3 electron samples, while plusses correspond to the 10 site, 5 electron samples. The full line represents the asymptotical forms given in the text. (b) The current as a function of interactions a_B for ordered samples. The full line represents the results of Refs. [14–16], while the dashed line represents a correction due to long range interactions. In the inset we enlarge the transition region.

We now turn to the general case in which both length scales (ξ and a_B) determine the behavior of the persistent current. Quite surprisingly the exponential behavior of the current $I \sim \exp(-L/\xi)$ is still valid for most values of ξ even for the interacting case. In Fig. 4 we plot our numerical results of $I\exp(L/\xi)$ as a function of ξ for different values of a_B, and for different number of sites. As one can see the interactions modify the prefactor of the exponential which increases monotonically with a_B, except for the region of small localization length which will be discussed later. Therefore, one concludes that for not too strong interactions ($a_B > s$) and disorder ($\xi > L$) the current can be approximated as

$$I(a_B,\xi) = I(a_B)\exp(-L/\xi), \tag{5}$$

where $I(a_B)$ is the current in the ordered case, which is plotted in Fig. 3(b), and ξ is the noninteracting localization length. Hence, quite surprisingly the influence of interactions and disorder on the current may be treated independently in this regime.

As we have seen in Fig. 4 the simple relation of Eq. (5) does not hold for $a_B < s$ or $\xi < L$. In order to gain a better understanding of the current amplitude behavior we present in Fig. 5 all regions of parameter values. Since we are mainly interested in the influence of interactions on the averaged current we plot $I\exp(L/\xi)$, which enables one to immediately see whether the presence of interactions increased or decreased the averaged current. It can be clearly seen that there are different regions of influences which are described schematically in Fig. 1.

One can gain a physical insight into the role played by interactions using the following argument: Disorder tends to localize the electrons in the lowest energy sites in

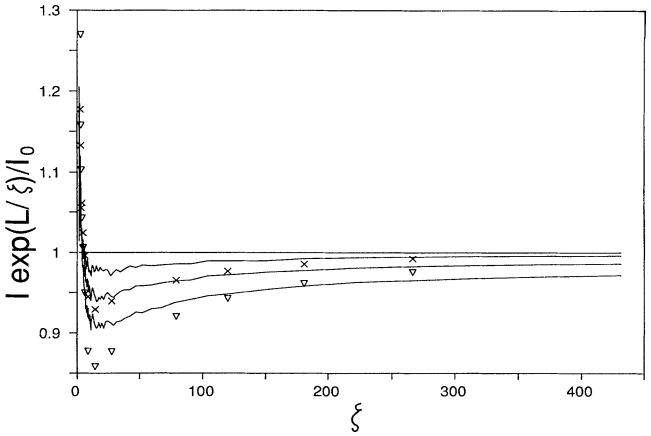

FIG. 4. The current as a function ξ for different values of a_B. The full lines represent the 6 site samples, while the symbols represent the 10 site samples. The higher curves correspond to higher values of a_B. We see that for $\xi > L$ and high values of a_B an almost exponential behavior is followed.

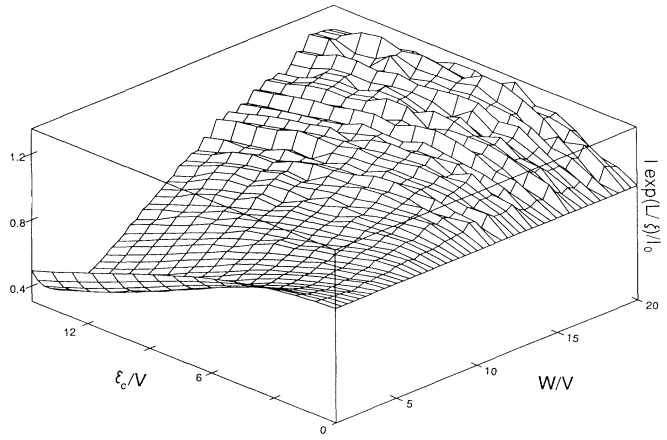

FIG. 5. The normalized current for the relevant regime of parameter space.

the sample, thus reducing the persistent current. Turning on electron-electron interactions will initially kick out the electrons from their preferred sites, thus delocalizing them, obviously increasing the persistent current. The above argument may be illustrated by the following consideration: The ground state wave function of the strong disordered case is denoted by Ψ_0 with energy ϵ_0. For small interactions the first excitation will be the movement of one electron to a position which has a higher on-site energy but more favorable from the interaction point of view. We shall denote this state as Ψ_1 which has an energy of order $\epsilon_0 + W - \epsilon_c$. It might be advantageous for the system to form a superposition of those two states in order to gain kinetic energy (from the hopping element V). We may then write the wave function of the system as $\Psi = \sqrt{C}\Psi_0 + \sqrt{1-C}\Psi_1$. By minimizing the energy $\langle\Psi|H|\Psi\rangle = \epsilon_0 + (1-C)(W-\epsilon_c) + 2\sqrt{C(1-C)}\,V'$ [where $V' = \mathrm{Re}V(\theta)$] one obtains C. Using the definition of Eq. (3), and the fact that only V' depends on ϕ, we get

$$I = \frac{\frac{2V'}{W-\epsilon_c}}{1+\left(\frac{2V'}{W-\epsilon_c}\right)^2}\left(\frac{\partial V'}{\partial\phi}\right). \tag{6}$$

One can see that for small ϵ_c the current is increasing as stated above. Another interesting feature is a maximum as a function of ϵ_c at $\epsilon_c = W - 2V'$. This maximum is seen in Fig. 5, and has the same qualitative characteristics as Eq. (6).

For much stronger interactions $a_B \sim s$ one can construct a similar argument for the influence of disorder. The ground state in the presence of small enough disorder is a pinned Wigner lattice. The first excitation will be a single electron shifting into a lower on-site energy which, of course, raises the interaction energy. This excitation will cost energy of order $\epsilon_c - W$. Following the line of our previous argument one obtains for the persistent current the same result as Eq. (6), where obviously $W - \epsilon_c$ is replaced by $\epsilon_c - W$. This current has a minimum as a function of W at $W = 2V' - \epsilon_c$. That minimum in current is clearly seen in Fig. 5. For larger interactions $a_B \ll s$ the system will mimic the Mott-Hubbard transition of an ordered system (see Fig. 1).

We have also calculated the influence of the interactions on the rms value of the current $\langle I^2\rangle$. The detailed results will be described elsewhere, but it can definitely be stated that interactions play a similar role in this case to the role played for the averaged current, i.e., $\langle I^2(\xi, a_B)\rangle/\langle I(\xi, a_B)\rangle^2 \sim \langle I^2(\xi, a_B = \infty)\rangle/\langle I(\xi, a_B = \infty)\rangle^2$.

In conclusion, we calculated exactly the persistent current of a small interacting disordered 1D ring. We found that depending on the disorder the interactions can increase or decrease the current. The main features in the parameter space were characterized. No dramatic increase in the averaged current or its rms value is possible. Therefore, for half filled 1D spinless models it is clear that interactions cannot explain the results of the single ring experiment [7]. Very recent perturbative calculations of Smith and Ambegaokar seem to support our results even for quasi-1D interacting systems [12].

We would like to thank B.L. Altshuler, Y. Avishai, and H. Widenmuller for useful discussions. R.B. would like to thank the Alon Foundation for financial support.

[1] M. Buttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).
[2] H.F. Cheung, Y. Gefen, E.K. Riedel, and W.H. Shih, Phys. Rev. B **37**, 6050 (1988); H.F Cheung, E.K. Riedel, and Y. Gefen, Phys. Rev. Lett. **62**, 587 (1989).
[3] H. Bouchiat and G. Montambaux, J. Phys. (Paris) **50**, 2695 (1989); G. Montambaux, H. Bouchiat, D. Sigeti, and R. Friesner, Phys. Rev. B **42**, 7647 (1990).
[4] F. von Oppen and E.K. Riedel, Phys. Rev. Lett. **66**, 80 (1991).
[5] B.L. Altshuler, Y. Gefen, and Y. Imry, Phys. Rev. Lett. **66**, 88 (1991).
[6] L.P. Levy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).
[7] V. Chandrasekhar, R.A. Webb, M.J. Brady, M.B. Ketchen, W.J. Galager, and A. Kleinsasser, Phys. Rev. Lett. **67**, 3578 (1991).
[8] V. Ambagokar and U. Eckern, Phys. Rev. Lett. **65**, 381 (1990).
[9] A. Schmid, Phys. Rev. Lett. **66**, 80 (1991).
[10] U. Eckern and A. Schmid, Europhys. Lett. **18**, 457 (1992).
[11] D. Loss, Phys. Rev. Lett. **69**, 343 (1992).
[12] R.A. Smith and V. Ambegaokar, Europhys. Lett. **20**, 161 (1992).
[13] P.D. Kirkman and J.B. Pendry, J. Phys. C **17**, 4327 (1984).
[14] B.S. Shastry and B. Sutherland, Phys. Rev. Lett. **65**, 243 (1990).
[15] R. Shankar, Int. J. Mod. Phys. B **4**, 2371 (1990).
[16] C.N. Yang and C.P. Yang, Phys. Rev. **150**, 321 (1966); **150**, 327 (1966).