## High-Energy Pion-Induced Double Charge Exchange and Isovector Renormalization

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The  $(\pi^+,\pi^-)$  double-charge-exchange reaction for pion energies of 200 to 1400 MeV is calculated with a zero-parameter Glauber theory that includes spin flip and pion absorption. Differential cross sections and excitation functions are obtained for <sup>14</sup>C and <sup>18</sup>O targets. Results for  $T_{\pi}$ =300-525 MeV are compared with recent experimental results and suggest a new quenching process. We have calculated effects of medium polarization on the isovector pion operator and find a strong reduction in good agreement with data. A pronounced dip in the cross section near  $T_{\pi}$ =1300 MeV offers a unique testing ground for unconventional mechanisms.

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The pion-induced double-charge-exchange (DCX) reaction,  $(\pi^+,\pi^-)$ , is unique among reactions using elementary particles to probe nuclei as it requires a scattering of the incident particle from at least two nucleons. In the usual sequential picture, DCX results from two successive single-charge-exchange reactions: The  $\pi^+$  scatters from a neutron resulting in a  $\pi^0$  and a proton; the  $\pi^0$ subsequently undergoes another charge exchange, resulting in a  $\pi^-$  and another proton. Thus, this reaction has been suggested as a probe both of short-range nucleonnucleon correlations in the nucleus and of the pion propagation in the nuclear medium.

DCX reactions have generated considerable excitement and theoretical and experimental work [1,2] since the pion factories began operation. Most of the work to date has concentrated in the region of the  $\pi N(3,3)$  resonance and in the lower-energy region around 50 MeV incident pion energy [3]. The DCX reaction at these energies is extremely complicated; a plethora of possible mechanisms—successive deltas [4], exchange currents [5], short-range correlations, six-quark bags [6]—have been found to contribute to DCX reactions at and below resonance energies. However, at higher energies most of these mechanisms are not expected to be important and a theoretical analysis should in principle be simpler.

With the advent [7] of DCX at LAMPF at energies up to 550 MeV pion kinetic energy and the possibility of DCX at greater energies at KEK, it is of interest to explore the nature of the reaction at these higher energies. Although there have been attempts to describe DCX reactions for incident pion energies between 200 and 700 MeV [8], the results reported here are the first to use modern  $\pi N$  phase shifts at these energies and are the first systematic DCX calculations for incident energies up to 1.4 GeV which include pion absorption and higher partial waves. In the calculations reported here, we confine ourselves to a discussion of the sequential mechanism. This is the simplest and—away from the (3,3) resonance—the most likely mechanism for DCX. We consider only the  $\pi^0$  as the intermediate meson although other neutral mesons, such as the  $\eta$ ,  $\rho^0$ , or  $A_1$ , could be included. Recent work by Chiang, Oset, and Liu [9] suggests the width of the  $\eta$ is much broader in nuclei than previously thought [10] and should have little impact on DCX.

As one moves into this new energy regime, one should employ a model that has a minimum number of free parameters, in which the effects of higher partial waves may be explicitly included, and which is readily open to physical interpretation. We use the Glauber model which has been found to provide reliable estimates of cross sections at the lower energies. With increasing pion energies the model should be more accurate as the  $\pi N$  cross section becomes increasingly forward peaked. The pions can travel further without scattering and short-range correlations or off-shell effects are less relevant. The DCX reaction is sensitive to details of nuclear structure [11]. Our version of the Glauber model [12] explicitly uses shellmodel wave functions and thus incorporates microscopic nuclear structure effects.

The amplitude for  $(\pi^+, \pi^-)$  on a nucleus of A nucleons in the Glauber model is

$$F(q) = \frac{ik}{2\pi} \int d^2 b \, e^{i\mathbf{q}\cdot\mathbf{b}} \langle \Psi_{\text{fin}} | 1 - \prod_j^A (1 - \Gamma_j) | \Psi_{\text{in}} \rangle \,, \qquad (1)$$

where **b** is the impact parameter, **k** the incident pion momentum, and  $\mathbf{q} = \mathbf{k} - \mathbf{k}'$  the momentum transfer. In Eq. (1),  $\Gamma$  is the single-particle profile function,

$$\Gamma(\mathbf{b}-\mathbf{s}) = \frac{1}{2\pi i k} \int d^2 q \, h(q) e^{-i\mathbf{q}\cdot(\mathbf{b}-\mathbf{s})}, \qquad (2)$$

in which s is the projection of the bound nucleon on the

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impact-parameter plane. The variables k and q in Eq. (1) are laboratory variables while those in Eq. (2) refer to the  $\pi N$  c.m. system. The profile function is obtained from the  $\pi N$  amplitude:

$$h(q) = h^{(s)}(q) + \Theta \cdot \tau h^{(v)}(q), \qquad (3)$$

where  $h^{(\alpha)}(q) = f^{(\alpha)} + ig^{(\alpha)}(q)\sigma \cdot \hat{\mathbf{n}}$  with  $\alpha = s.v.$  In Eq. (3), the superscripts s and v refer to isoscalar and isovector amplitudes and the operators  $\boldsymbol{\Theta}$  and  $\boldsymbol{\tau}$  are isospin operators for the pion and nucleon, respectively, with  $\hat{\mathbf{n}} = (\mathbf{k} \times \mathbf{k}')/|\mathbf{k} \times \mathbf{k}'|$ . The  $\pi N$  amplitudes h(q) are calculated using a partial-wave expansion rather than the commonly used approximation using a Gaussian depending on  $q^2$  [13]. We can therefore use the  $\pi N$  phase shifts and inelasticity parameters of Arndt's 1987 analysis [14] without modification. The  $\pi N$  amplitudes were calculated including partial waves up to an l of 5. Effects of spin flip are included.

Since the operator in Eq. (1) is a product of A onebody operators, if the shell-model wave functions are determinants, then the evaluation of Eq. (1) reduces to

the evaluation of a sum of 
$$A \times A$$
 determinants. By using  
the Glasgow shell-model code [15], the wave functions  
are naturally expressed as a sum of Slater determinants,  
and hence, antisymmetry is explicitly included. The nu-  
clear wave functions for <sup>18</sup>O and <sup>18</sup>Ne were obtained us-  
ing matrix elements obtained by Kuo [16]. The wave  
functions for mass 14 were obtained using the matrix ele-  
ments of Cohen and Kurath [17]. Oscillator parameters  
obtained from electron scattering of  $\alpha^2 = 0.39$  fm<sup>-2</sup> for  
<sup>14</sup>C and 0.319 fm<sup>-2</sup> for <sup>18</sup>O were used. Thus, this calcu-  
lation has no free parameters.

A complication from time ordering arises when charge exchange is allowed to occur. Clearly, the intermediate  $\pi^0$  cannot scatter off a nucleon until it has been created through an earlier charge exchange. Mathematically, this is expressed by the noncommutativity of the pion isospin operator  $\Theta$  with itself. There is no difficulty in the leading term of DCX as in this case both  $\Theta$  operators are necessarily lowering operators which do commute. There are no problems in the time ordering if we neglect  $\Gamma^{(v)}$  in the remainder of the operators. These terms were estimated to be small in Ref. [12] and a recent evaluation [18] shows them to be negligible. Hence, we obtain

$$F(q) = \frac{ik}{2\pi} \int d^2 b \, e^{i\mathbf{q}\cdot\mathbf{b}} \langle \Psi_{\rm fin} | -2\sum_{i < j} \Gamma_i^{(v)} \tau_i^+ \Gamma_j^{(v)} \tau_j^+ \prod_{k \neq i,j} (1 - \Gamma_k^{(s)}) | \Psi_{\rm in} \rangle \,. \tag{4}$$

The  $\Gamma^{(s)}$  and  $\Gamma^{(v)}$  are obtained from Eq. (2) by replacing h(q) with the appropriate expression for  $h^{(s)}$  or  $h^{(v)}$ .

In addition we have in this work for the first time included in the Glauber formulation effects of pion absorption of DCX reactions. This has been done following the work of Ref. [19] in which the Glauber formalism for pion-nucleus single charge exchange was modified to account for pion absorption on two and three nucleons. The correction is applied by multiplying the distortion factor  $\Pi(1-\Gamma)$  in Eq. (4) by the additional distortion factor

$$\exp\left(-i\int_{-\infty}^{\infty}\frac{1}{2k}\mathrm{Im}\Pi^{(2)}(k,\mathbf{b},z)dz\right),$$
(5)

where  $\Pi^{(2)}(k,\mathbf{r})$  is the piece of the pion self-energy accounting for pion absorption. We find the effects of pion absorption in DCX are moderate. It reduces the forward cross section of the  ${}^{18}O(\pi^+,\pi^-){}^{18}Ne$  reaction by 15% around  $T_{\pi}$ =300 MeV and by only 10% around  $T_{\pi}$ =500 MeV with even smaller effects as the energy increases.

The effects of spin flip in the isoscalar and isovector amplitudes are also moderate except near the (3,3) resonance and around  $T_{\pi}$ =1300 MeV. In the latter case there are large cancellations among the isovector amplitudes resulting in an abnormally small cross section. Without contributions from spin flip the cross section at  $T_{\pi}$ =1300 MeV would be an order of magnitude smaller.

The results for  $d\sigma/d\Omega$  at zero degrees as a function of the energy including absorption and spin flip are shown in Fig. 1 for <sup>18</sup>O( $\pi^+, \pi^-$ )<sup>18</sup>Ne. The results obtained by including different partial waves are shown. The f wave



FIG. 1. Differential cross section at  $\theta = 0^{\circ}$  in the laboratory system for the  ${}^{18}O(\pi^+, \pi^-){}^{18}Ne$  reaction as a function of the pion kinetic energy, showing the contribution of several partial waves. The solid line is the result including all partial waves up to l=5. Effects of absorption and spin flip are included. The thick, solid line shows the result when the isovector polarization is included.

TABLE I. The center-of-mass differential cross section in  $\mu$ b/sr at 5° for the ( $\pi^+,\pi^-$ ) reaction leading to the ground state of the resultant nucleus. The experimental results are from Williams *et al.* [7]. The columns labeled TH are the results of theory without polarization and those labeled THP include the isovector polarization.

$T_{\pi}$ (MeV)	<sup>14</sup> C					
	Experiment	TH	THP	Experiment	TH	ТНР
300	$3.84 \pm 0.54$	11.9	3.0	$2.68 \pm 0.37$	8.9	3.0
350	$4.15 \pm 0.42$	14.8	3.6	$3.00 \pm 0.27$	9.4	3.0
400	$3.14 \pm 0.39$	15.1	3.8	$3.06 \pm 0.29$	8.9	2.8
450		14.2	3.7	$2.94 \pm 0.33$	8.1	2.6
500	$3.62 \pm 0.65$	13.7	3.8	$2.69 \pm 0.35$	7.7	2.6
525		13.5	3.7	$2.65\pm0.80$	7.5	2.6

becomes significant above 400 MeV, the g wave above 700 MeV, and the h wave above 1000 MeV. The effects of including the h wave are small up to 1400 MeV; additional partial waves were not included.

The zero-degree cross section has a peculiar energy dependence: It is flat around 300-500 MeV as observed in a recent experiment [7], has a dip around 700 MeV, rises again with a maximum around 850 MeV, and has a very pronounced dip around 1300 MeV with a cross section 3000 times smaller than around 400 MeV. Results for  ${}^{14}C(\pi^+,\pi^-){}^{14}O$  are similar. The phase shifts are poorly known around 1300 MeV and since the small cross section results from a cancellation among the several partial waves, the absolute magnitude of the cross section should not be considered reliable. This region may be an ideal place in which to search for exotic mechanisms that contribute to DCX but which would be otherwise unobservable because of their small contribution.

In Table I we give the values of  $d\sigma/d\Omega$  at 5° in the center-of-mass system for the  ${}^{18}O(\pi^+,\pi^-){}^{18}Ne$  and  ${}^{14}C(\pi^+,\pi^-){}^{14}O$  reactions and compare the results with the experimental ones of Ref. [7]. Theory reproduces the flat shape of the excitation function but is about a factor of 3 too large compared with experiment. This is similar to the results found in single charge exchange [19] in which the theoretical results exceeded experiment [20] by (20-50)% but reproduced the shape. We reiterate that our calculation employs no free parameters and is therefore devoid of the uncertainty engendered by varying parameters.

The approximate mass and energy independence of the theory-to-experiment ratio strongly suggests there is a renormalization of the isovector pion operator that has not been previously reported. Such a renormalization is well known for isoscalar electromagnetic transitions and has been investigated for pion-induced reactions [21,22]. It can arise from the processes shown in Fig. 2. These processes can be interpreted as a medium polarization in the t channel induced by the pion source or, analogously, as the coupling of the pion to the RPA components of the nuclear excitation. The effect of an isoscalar renormalization on the elastic scattering is small: We have ob-

tained a good reproduction [21] of pion-nuclear elastic cross sections at these energies using the present approach. More precisely, our results in Ref. [23] underestimate the elastic cross section at small angles by approximately 20% in <sup>12</sup>C and 10% in <sup>40</sup>Ca. The effects of the Coulomb interaction were not included. Intrinsic uncertainties of our approach to DCX of about (20-25)% must therefore be expected.

We estimate the isovector renormalization by using the effective interaction given by Speth, Werner, and Wild [24]. In coordinate space in the isovector channel the interaction is  $f'_0 \tau \cdot \tau \delta(\mathbf{r} - \mathbf{r}')$  where

$$f_0' = C_0 \left[ f'^{(\text{in})} \frac{\rho(r)}{\rho_0} + f'^{(\text{ex})} \frac{1 - \rho(r)}{\rho_0} \right], \tag{6}$$

with  $C_0=380$  MeV fm<sup>3</sup>,  $f'^{(in)}=0.33$ ,  $f'^{(ex)}=0.45$ , and  $\rho_0$ the central density. We implement the isovector renormalization implicit in Fig. 2 by means of a local density approximation. A local Fermi sea is assumed at each point of the nucleus having density  $\rho_0(\mathbf{r})$ ; the terms implicit in Fig. 2 lead in momentum space to a geometrical series which when summed is equivalent to multiplying the isovector  $\pi N$  amplitude  $h^{(v)}(q)$  in Eq. (2) by the quantity  $[1-U(q^0=0,q,\rho=\rho(\mathbf{r}))f'_0]^{-1}$ , where  $U(q^0,q,\rho)$  is the Lindhard function [25] for particle-hole excitations in nuclear matter of density  $\rho$ .

The inclusion of the renormalization results in a reduction of the calculated cross section approximately independent of energy and target mass. The results are



FIG. 2. Schematic representation of renormalization of the  $\pi N$  amplitude.

shown in Table I and are in good agreement with experiment. The differential cross sections of Ref. [7] are also reproduced, as are the single-charge-exchange results of Ref. [20]. Further details will be published elsewhere [23].

In summary, we have demonstrated that the isovector operator appearing in pion scattering is strongly reduced in nuclei and that this reduction can be understood within the context of core polarization. Further, there is a region above 1.2 GeV pion kinetic energy where the predicted pion-induced double-charge-exchange cross section is very small for the conventional sequential mechanism. Although additional work-including pion-nucleon experiments to better determine the elementary  $\pi N$  amplitudes-is required to validate this result, this work suggests that this energy region may be an excellent place in which to seek new, exotic processes that contribute to pion-induced charge exchange. The present zeroparameter calculation, as accurate as possible within the conventional multiple scattering formalism in its eikonal form, should serve as a guideline for future experiments and as a starting point to extract new information from them when they become available.

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- W. R. Gibbs and B. F. Gibson, Annu. Rev. Nucl. Part. Sci. 37, 207 (1987); D. Strottman, in *Fundamental Symmetries and Nuclear Structure*, edited by J. N. Ginocchio and S. P. Rosen (World Scientific, Singapore, 1989), p. 247.
- [2] Proceedings of LAMPF DCX Workshop, August 1989, edited by W. R. Gibbs and M. J. Leitch (World Scientific, Singapore, 1990).
- [3] N. Auerbach, W. R. Gibbs, and E. Piasetzky, Phys. Rev. Lett. 59, 1076 (1987); M. J. Leitch *et al.*, Phys. Rev. C 39, 2356 (1989).
- [4] M. B. Johnson, E. R. Siciliano, H. Toki, and A. Wirzba,

Phys. Rev. Lett. **52**, 593 (1984); E. Siciliano, M. B. Johnson, and H. Sarafian, Ann. Phys. (N.Y.) **203**, 1 (1990).

- [5] E. Oset, D. Strottman, M. J. Vicente-Vacas, and M. Wei-Hsing, Nucl. Phys. A408, 461 (1983).
- [6] G. A. Miller, Phys. Rev. Lett. 53, 2008 (1984).
- [7] A. L. Williams et al., Phys. Lett. B 216, 11 (1989).
- [8] L. C. Liu and V. Franco, Phys. Rev. C 11, 760 (1975); D. J. Ernst and G. E. Parnell, as reported in Ref. [7]; M. Arima and R. Seki, as reported in Ref. [2].
- [9] H. C. Chiang, E. Oset, and L. C. Liu, Phys. Rev. C 44, 738 (1991).
- [10] Q. Haider and L. C. Liu, Phys. Lett. B 172, 57 (1986); Phys. Rev. C 36, 1636 (1987).
- [11] E. Oset, D. Strottman, and G. E. Brown, Phys. Lett. 73B, 393 (1978).
- [12] E. Oset and D. Strottman, Nucl. Phys. A355, 437 (1981);
   A377, 297 (1982).
- [13] R. H. Bassel and C. Wilkin, Phys. Rev. 174, 1179 (1968);
   M. Mizoguchi and H. Toki, Nucl. Phys. A513, 685 (1990).
- [14] R. Arndt, computer code SAID, Phys. Rev. D 28, 97 (1983).
- [15] R. R. Whitehead, A. Watt, B. J. Cole, and I. Morrison, in Advances in Nuclear Physics, edited by M. Baranger and E. Vogt (Plenum, New York, 1978), Vol. 9.
- [16] T. T. S. Kuo (private communication).
- [17] S. Cohen and D. Kurath, Nucl. Phys. 73, 1 (1965).
- [18] V. Franco and H. G. Schlaile, Phys. Rev. C 41, 1075 (1990).
- [19] E. Oset and D. Strottman, Phys. Rev. C 42, 2454 (1990).
- [20] S. Rokni et al., Phys. Lett. B 202, 35 (1988).
- [21] E. Oset and D. Strottman, Phys. Rev. C 44, 468 (1991).
- [22] A. C. Hayes, P. J. Ellis, D. J. Millener, and J. J. Kelley (to be published).
- [23] E. Oset, D. Strottman, H. Toki, and J. Navarro (to be published).
- [24] J. Speth, E. Werner, and W. Wild, Phys. Rep. C 33, 127 (1977).
- [25] A. L. Fetter and J. D. Walecka, Quantum Theory of Many Particle Systems (McGraw-Hill, New York, 1971). One must multiply the relevant expressions found in this text by 2 to account for the isospin of the nucleons. We have also introduced a small gap of 5 MeV in the Fermi sea that serves to impose a minimum excitation energy for the particle-hole excitations.