Two-Pole Structure of the $\frac{3}{2}^+$ Resonance of ^{5}He in a Dynamical Microscopic Model

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By a realistic dynamical microscopic reaction approach to ^{5}He we reproduce the empirical positions of the two S-matrix poles associated with the $\frac{3}{2}^+$ resonance, and unambiguously prove that it arises from the $t+d$ channel. The picture is not coherent though, unless the presently adopted $\alpha+n$ phase shift, extracted partly from $\alpha+p$ scattering data, is assumed to be incorrect. It is pointed out that the analog resonance in ^{5}Li behaves very differently because the shadow pole is located on another Riemann sheet.

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The $J^\pi = \frac{3}{2}^+$ resonance of ^{5}He at an excitation energy of 16.8 MeV is famous for its role in primordial nucleosynthesis and in the production of thermonuclear energy and of 14-MeV neutrons. The transition $t+d \to \alpha+n$ at this resonance is extremely strong although the coupling arises essentially from the relatively weak tensor force. A phenomenological analysis by Hale, Brown, and Jarmie [1] has recently revealed that this strong transition is accounted for by the presence of a so-called shadow pole, which is a pole of the S matrix, as a function of the complex energy, located on a Riemann sheet not adjacent, at that energy, to the physical sheet [2]. Although shadow poles currently attract much interest both in particle physics [3] and in atomic physics [4], in nuclear physics this is their first appearance hitherto.

While shedding light on one matter, the resort to the shadow pole arose controversy in another: the origin of the resonance. The aim of this Letter is to give a comprehensive description of the resonance, as fundamental as it can be, and thereby settle this dispute once and for all.

Common wisdom [5] associates this resonance with the $t+d$ channel, whose threshold lies some 60 keV below the resonance, and this assumption is in quantitative agreement with ^{6}Li$(e,e'p)^5$He$(\frac{3}{2}^+)$ experiments [6]. From the identification of the Riemann sheet on which the shadow pole resides, it was inferred that, on the contrary, the resonance originates from the $\alpha+n$ channel [1], almost 18 MeV over the $\alpha+n$ threshold. However, the sheet of the shadow pole carries such information only in the limit of zero coupling [2], and Pearce and Gibson [7] demonstrated in a schematic model that the pole may move from one sheet to another when the coupling strength is varied. Later Bogdanova, Hale, and Markushin gave another model example for such a pole migration [8]. These models, however, cannot be considered models of the $\{\alpha+n, t+d\}$ channels. For example, in the model of Bogdanova, Hale, and Markushin [8] the only interaction in the channels dubbed "$t+d$" and "$\alpha+n$" is the t-d Coulomb repulsion, thus, it is *a priori* certain that none of these channels can accommodate a resonance. In fact, the resonance originates from the third channel, a bound-state one, dubbed "^{5}He*" [9].

The origin of the resonance can only be pinned down by a realistic dynamical description, which reproduces the relevant experiments, and in which coupling strengths can be artificially turned down. In this mass range the interactions and exchanges between individual nucleons have strong effects, which calls for a microscopic approach. In this Letter we report on microscopic reaction calulations for the $\{\alpha+n, t+d\}$ system and a *polological* analysis of this coupled-channels problem. To our knowledge, this is the first such analysis performed on a realistic multichannel multiparticle problem. While settling the dispute on the nature of the resonance, we point out a hitherto unexposed aspect: a qualitative difference between the behaviors of systems $\{\alpha+n, t+d\}$ and $\{\alpha+p, {}^3\text{He}+d\}$.

Our approach is of the resonating-group type. We describe the internal motions of α and t by single $0s$ translation-invariant oscillator shell-model configurations, and that of d by a combination of three such functions of different sizes, which implies three model states [6] of the deuteron. The relative motions are represented by sets of angular-momentum projected shifted Gaussians, matched with S-matrix asymptotics [10]. The coefficients are determined variationally, and the S matrix is calculated with the Kohn-Kato formula [10].

The quantum numbers $J^\pi = \frac{3}{2}^+$ allow four combinations of channels, relative orbital momenta, and summed cluster spins. In spectrosopic notation and with the abbreviations $n \equiv \alpha+n$, $d \equiv t+d$, these are $^2D(n)$, $^4S(d)$, $^4D(d)$, and $^2D(d)$. The present model goes beyond that of Ref. [6] in that the force now contains noncentral terms, which give rise to new couplings. One of them, that between channels $^2D(n)$ and $^4S(d)$, is responsible for most of the $t+d \to \alpha+n$ transition. Together with the excited-deuteron pseudochannels, which allow for deuteron distortion, we thus have 10 channels coupled.

The 2^{10} sheets of the Riemann surface are distinguished by the signs of all Imk_i, where k_i is the wave number in channel i. The resonance being below the thresholds of the pseudochannels, all signs corresponding to these channels must be positive and will be omitted. Since the thresholds of channels ${}^4S(d)$, ${}^4D(d)$, and ${}^2D(d)$ coincide, on the sheets accessible from the physical sheet the corresponding three k values are the same. The sheets entering into consideration are thus fully distinguished by $[\mathrm{sgn}(\mathrm{Im}k_n), \mathrm{sgn}(\mathrm{Im}k_d)]$, where k_d belongs to the ground-state deuteron. The physical sheet is $(++)$, the "conventional" resonance pole above the $t+d$ threshold is on sheet $(--)$, and the shadow pole was found [1] on $(-+)$. In the limit of zero coupling the poles j belonging to the same resonance are at the same energy but on different sheets, and the channel i responsible for the resonance is distinguished by $\mathrm{Im}k_i^{(j)}$ being negative for all j [2, 7]. Thus, if the shadow pole did not change sheet between zero and realistic coupling, the resonance would be confirmed to belong to channel $\alpha + n$.

To localize the poles of the S matrix, we generalized the scattering formalism to complex energy. The pole positions were determined with an iterative procedure, which uses a set of scattering solutions at complex energies [11].

In choosing the parameters we rely on Blüge and Langanke [12, 13], who calculated astrophysical S factors in a similar model. So, for the central term of the interaction we employed the Minnesota force given in Ref. [14], and the corresponding size-correct oscillator constants [14]. Since the resulting distance between the $t+d$ and $\alpha+n$ thresholds, 19.6 MeV, somewhat differs from experiment (17.6 MeV), we fixed our energy scale to the $t+d$ threshold, and cared about the quality of description only in this region. We set the free space-exchange parameter [14] to $u = 0.835$ [12, 13], and used the spin-orbit force of Reichstein and Tang [15], with strength and width of -224.8 MeV and 0.707 fm, respectively. We followed Ref. [13] also in adopting a slightly modified version of the tensor interaction of Heiss and Hackenbroich [16].

We concentrate primarily on the processes with entrance channel $t+d$. The elastic phase shift and the strength $|S_{dn}|^2$ of the transition ${}^4S(d) \to {}^2D(n)$ produced by the force specified above are the dashed curves in Fig. 1. To improve the fit to the phase shift, we varied u and the strength of the short-range tensor term, and the best result, obtained with 0.811 and -112.94 MeV, is the solid curve. The corresponding pole positions are given in Table I. The results of Hale, Brown, and Jarmie [1], for both energy and Riemann sheet, are fully reproduced.

The pole trajectories obtained by varying the strength of the tensor force, i.e., essentially the coupling between channels ${}^2D(n)$ and ${}^4S(d)$, are shown in Fig. 2. When the coupling is zero, the two pole energies coincide, and the shadow pole is on sheet $(+-)$, which proves with full certainty that the resonance arises from the $t+d$ channel. In Fig. 1(a) we see that a pure $t+d$ model does produce a resonance. With the coupling increased, the conventional and the shadow poles move parallel and perpendicular to the real axis, respectively. The shadow pole at $\mathcal{E}'(k_n', k_d')$ does reach the axis, at that point coinciding with its conjugate pair at $\mathcal{E}'^*(-k_n'^*, -k_d'^*)$ [2, 7], and, with exchanged identities, both poles walk over to sheet $(-+)$. This is in accord with the scenario envisaged in Refs. [7, 8], and we seem to understand our resonance perfectly.

Nevertheless, it can still cause a bit of a surprise. Figure 3 shows a marked disagreement in the $\alpha+n$ phase shift between the model and the only published data

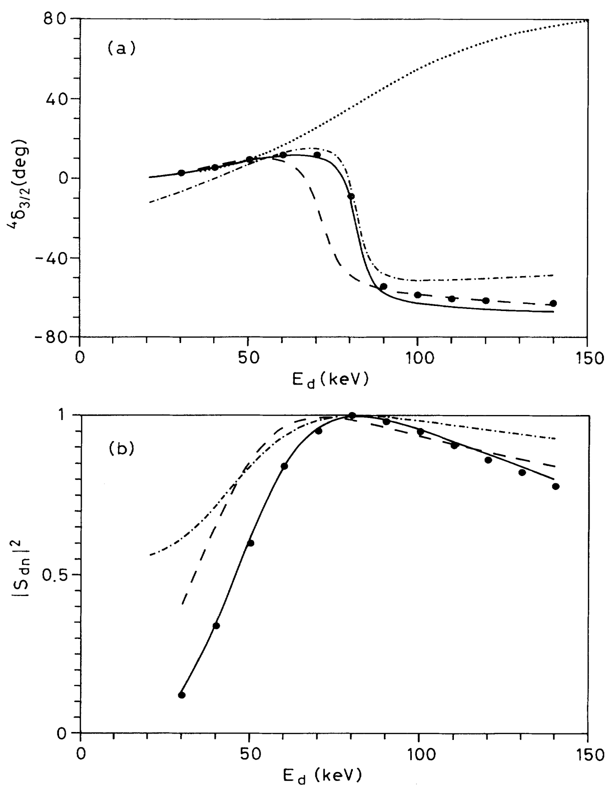

FIG. 1. (a) The ${}^4S_{3/2}$ $t+d$ phase shift and (b) squared modulus S-matrix element of transition ${}^4S(d) \to {}^2D(n)$ in the 10-channel model with the original potential (dashed line) and with the readjusted potential (solid line); in a pure ${}^4S_{3/2}(t+d)$ model (dotted line), and with the modified Breit-Wigner formula (2) (dash-dotted line). The heavy dots are the R-matrix fits of Ref. [1], taken from Refs. [1, 8], which represent the experimental data accurately [1, 17].

TABLE I. Pole positions in keV.

Riemann sheet	$(--)$	$(-+)$
Phenomenological[a]	$46.97 - i37.10$	$81.57 - i3.64$
10-channel, original	$41.88 - i31.78$	$71.33 - i5.20$
10-channel, fitted	$43.51 - i37.40$	$81.70 - i3.38$

[a]Reference [1].

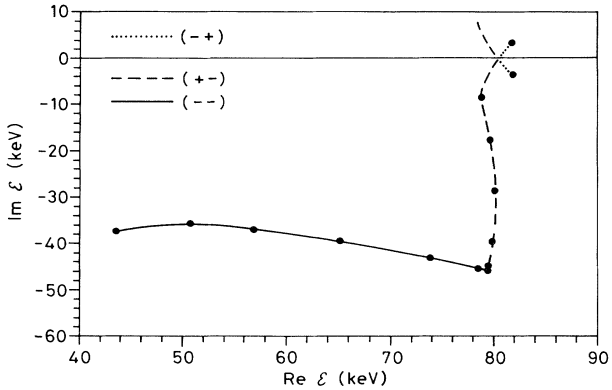

FIG. 2. Trajectories of the conventional pole on sheet (−−) (solid curve) and of the shadow pole on sheets (−+) (dotted) and (+−) (dashed curve) when the coupling is varied between 0 and its physical value with the readjusted potential.

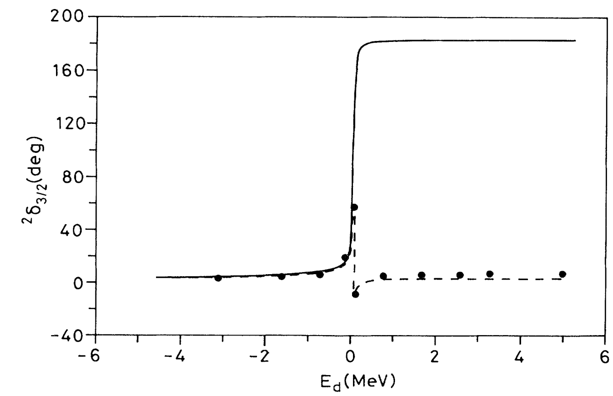

FIG. 3. Theoretical $^2S_{3/2}$ $\alpha + n$ phase shift calculated with the potential best fitting $t + d$ (full line) and with a slight arbitrary change (dashed line). Dots: extracted from experiment [18].

analysis we are aware of, that due to Hoop and Barschall [18]. These data have been reproduced in Ref. [13], and we can also reproduce the very similar phase shifts of $\alpha + p$ scattering [19]. Indeed, with a slight reduction of the coupling strength, we can also produce a similar $\alpha+n$ curve (dashed line), but, at the same time, the agreement in $t + d$ breaks down.

The root of this discontinuous behavior is again to be found in the position of the shadow pole. When reducing the coupling strength or increasing the charge of the system, the shadow pole is pulled back to sheet (+−). According to the taxonomy of two-channel resonances given by Pearce and Gibson [7], the position of the shadow pole strictly determines the shapes of both elastic scattering phase shifts and when the pole crosses the real energy axis, these phase shifts undergo violent changes; in fact, they exchange their shapes. The shape that the shadow pole on sheet (−+) implies for the $\alpha + n$ curve is at variance with the curve of Hoop and Barschall [18]. This, however, does not destroy the theory because this phase shift was in fact extracted partly from $\alpha + p$ data corrected for the charge difference. As we have seen, the difference in charge is enough for the shadow poles to settle on two different Riemann sheets, and thus to yield qualitatively different shapes for the two α+nucleon phase shifts. The predicted shape of the $\alpha + n$ curve is yet to be confirmed experimentally, but the corresponding difference between the $t + d$ and ^{3}He+d phase shifts is observable [20].

The sensitivity of the model to details is extraordinary. As a result of an omission of any of the minor channels $^4D(d)$, $^2D(d)$, or of deuteron distortion, the shadow pole moves to sheet (+−). Thus the observed behavior of the resonance results from an interplay of all channels. Nevertheless, our conclusions do not hinge on the adopted version of the model because, if any of these simplifications is accompanied by a readjustment of the force to the $t+d$ phase shift, the original behavior is qualitatively restored. To obtain essentially the same effect with refitted parameters, it is enough to keep the two coupled channels $^2D(n)$ and $^4S(d)$, without distortion.

The single-channel $t+d$ resonance in Fig. 1 looks obviously like a Breit-Wigner resonance, while the deviating shape of the double-pole resonance indicates a breakdown of the Breit-Wigner formula. It brings us closer to understanding the effect of the shadow pole if we derive a modified formula for this case.

When a single-channel S matrix has a pole at $\mathcal{E}(k) = E_0 - \frac{i}{2}\Gamma$, the extended unitarity relation $S[\mathcal{E}^*(k^*)]S^*[\mathcal{E}(k)] = 1$ implies a zero on the physical sheet at $\mathcal{E}^*(k^*) = E_0 + \frac{i}{2}\Gamma$, so that the single-pole approximation yields the Breit-Wigner formula

$$S(E) \approx e^{2i\phi}\frac{E - E_0 - \frac{i}{2}\Gamma}{E - E_0 + \frac{i}{2}\Gamma}, \tag{1}$$

with ϕ a constant phase. In a multichannel case the zeros of S_{ii} are scattered at various locations. For example, the zero of S_{dd}, of the two-channel problem $\{^2D(n), ^4S(d)\}$, that is near the physical axis can be found by applying the unitarity relation $S_{dd}[\mathcal{E}^*(-k_n^*, k_d^*)]S_{dd}^*[\mathcal{E}(k_n, k_d)] = 1$ [2] to the conjugate shadow pole at $\mathcal{E} = \mathcal{E}'^*(-k_n'^*, -k_d'^*) \in (-+)$. This shows that at $\mathcal{E}'(k_n', -k_d') \in (--)$, adjacent to the physical sheet, $S_{dd} = 0$. Thus (1) is modified to

$$S_{dd}(E) \approx \frac{-ie^{2i\phi'}\Gamma_d}{E_0 - E_0' - \frac{i}{2}(\Gamma - \Gamma')}\frac{E - E_0' + \frac{i}{2}\Gamma'}{E - E_0 + \frac{i}{2}\Gamma}, \tag{2}$$

where the E-independent factor is chosen to set the residue of the conventional pole to $-ie^{2i\phi'}\Gamma_d$ [cf. (1)]; then the modulus of the residue Γ_d is the d-channel partial width belonging to the conventional pole. Partial widths Γ_i' can be assigned to the shadow pole similarly.

Table II shows that these partial widths agree well with

TABLE II. Partial widths for channels $^2D(n)$ and $^4S(d)$ in keV. Experiment: Ref. [1].

Channel	$^2D(n)$		$^4S(d)$	
Pole	Theory	Experiment	Theory	Experiment
Conv.	39.84	39.83	24.22	25.10
Shadow	72.26	68.77	3163.9	2861.6

those extracted from experiment [1], thus bearing out the relations $\Gamma \neq \Gamma_n + \Gamma_d$ and $\Gamma' \neq \Gamma'_n + \Gamma'_d$, and hence the breakdown of the probability meaning of the partial widths. The dash-dotted curves in Fig. 1 show that Eq. (2) gives an acceptable qualitative fit.

Equation (2) expresses most succinctly the role of the shadow pole in bringing about the strong $t + d \to \alpha + n$ transition. It reveals that, for $|S_{dn}|^2 \equiv 1 - |S_{dd}|^2$ to be close to 1 at $E = E'_0$, the closeness of the zero of the S matrix to the real axis (i.e., $\Gamma' \approx 0$) [1] is not enough; if it were so, all narrow Breit-Wigner resonances would cause strong transitions. It is also necessary that the shadow pole be displaced well enough from the conventional pole [i.e., $(E_0 - E'_0)^2 \gg 0$ and/or $\frac{1}{4}(\Gamma - \Gamma')^2 \approx \frac{1}{4}\Gamma^2 \gg 0$].

The results can be summarized as follows. In a dynamical microscopic model that describes the $t+d$ elastic scattering and $t + d \to \alpha + n$ transition in the resonance region we have found the two poles very close to the positions known from the phenomenological analysis [1]. The shadow pole has been confirmed to be on Riemann sheet $(-+)$. However, when the coupling is set to zero, the shadow pole gets transferred to sheet $(+-)$. This implies that the resonance originates from partial wave $^4S_{3/2}$ of channel $t+d$. The strong $t+d \to \alpha+n$ transition is caused by the shadow pole being close to the real axis as well as far enough from the ordinary pole.

The observed constellation of the poles implies a characteristic shape of the $\alpha + n$ phase shift as well, which sharply contradicts the phase shift deduced with the use of $\alpha + p$ data [18]. An experimental check of the $\alpha + n$ phase shift would be a very stringent test of whether we really understand the effect of shadow poles. The model predicts that the analogous shadow pole for the ^{5}Li system is on sheet $(+-)$, which gives the resonance an utterly dissimilar appearance. While in electron-atom collisions the shadow poles can be shifted from one Riemann sheet to the other with a perturbation exerted by strong radiation field [4], the mirror five-nucleon systems provide a natural example for such a phenomenon. The strong dependence, on the charge of the system, that is caused by the changed location of the shadow pole may give an opportunity to pinpoint charge asymmetry effects in nuclear forces.

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