Search for $\tau^- \to \gamma\mu^-$: A Test of Lepton Number Conservation

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A search for the lepton number violating decay of the τ lepton to the $\gamma\mu$ final state has been performed with the CLEO II detector at the Cornell e^+e^- storage ring CESR. In a data sample that corresponds to an integrated luminosity of 1.55 fb^{-1}, we observe no candidates in the signal region. We thus determine an upper limit of $B(\tau^- \to \gamma\mu^-) < 4.2\times10^{-6}$ at 90% confidence level.

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The search for lepton number violation has long been an area of interest. Lepton number violating decays are predicted in several models which go beyond the standard model. Among them are models which require additional Higgs doublets [1], heavy leptons [2,3], supersymmetry [4], or a left-right symmetry [5,6]. Particularly interesting is the decay channel $\tau^- \to \gamma\mu^-$ [7], since new predictions were recently obtained in a superstring model by extrapolating physics at the Planck scale into the low-energy regime [8]. Even though firm predictions of absolute branching ratios are currently not possible because of unknown masses and couplings of hypothetical fermions, the model relates the branching ratios of $\tau^- \to \gamma\mu^-$ and $\mu^- \to \gamma e^-$ by $B(\tau^- \to \gamma\mu^-) \simeq 2\times10^5 B(\mu^- \to \gamma e^-)$. The large enhancement for $\tau^- \to \gamma\mu^-$ is primarily due to mass dependent couplings and behaves as $(m_\tau/m_\mu)^5$, thus making this channel a particularly attractive one to pursue experimentally. The related search for $\mu^- \to \gamma e^-$ has been the subject of dedicated experiments at PSI [9], at TRIUMF [10], and at LAMPF [11]. These experiments yield an upper limit for this branching fraction of 4.9×10^{-11} at 90% confidence level (C.L.), which corresponds to an upper limit for $\tau^- \to \gamma\mu^-$ of $\sim 1\times10^{-5}$. The current limit on $\tau^- \to \gamma\mu^-$ is 3.4×10^{-5} at 90% C.L. [12].

We present here a search for $\tau^- \to \gamma\mu^-$ using the CLEO II detector at the Cornell Electron Storage Ring (CESR) which operates at center-of-mass energies $E_{\text{c.m.}} \sim 10$ GeV. The τ's are produced in pairs by the continuum QED process $e^+e^- \to \gamma^* \to \tau^+\tau^-$. We use data obtained at center-of-mass energies at the $\Upsilon(4S)$ resonance and slightly above and below the $\Upsilon(4S)$. The total integrated luminosity of these data is 1.55 fb^{-1}. Events originating from the decay $\tau^- \to \gamma\mu^-$ have distinct characteristics: (i) the energy of the $\gamma\mu$ system equals the beam energy (E_b); (ii) the invariant mass of the $\gamma\mu$ system reconstructs to the τ mass (m_τ); and (iii) the invariant mass of the decay daughters of the other tau, combined with the unmeasured four-momentum in the event, must also sum to m_τ. Due to the boost of the τ's at CESR energies, the energy of the photon in the $\mu\gamma$ decay in the laboratory frame is distributed uniformly between 0.1 and 5.1 GeV, while the angle between the photon and the μ is typically less than 90° ($\cos\theta_{\gamma\mu} > 0$).

The features of the CLEO II detector important for this analysis are the excellent photon energy resolution of the CsI calorimeter, $\sigma_E/E = 0.35\%/E^{0.75} + 1.9\% - 0.1\% \times E$ in the barrel region and $\sigma_E/E = 0.26\%/E + 2.5\%$ in the end caps with E in GeV, the good momentum resolution of the drift chamber, $(\delta p_t/p_t)^2 = (0.15\% \times p_t)^2 + (0.50\%)^2$ with p_t in GeV/c, and the high efficiency for detecting muons, $\epsilon = 98\%$ for momenta greater than 0.6 GeV/c over 85% of the solid angle. Details of the detector performance are given elsewhere [13].

The analysis is based on the ability to "tag" τ-pair events by reconstructing one of the τ's in a number of exclusive decay topologies and then looking for a $\gamma\mu$ pair in the recoil system. As shown in Table I, we use seven different tag topologies [14], comprising 99.5% of all τ decays. This allows us both to optimize the detection efficiency and to reject non-τ backgrounds due to the use of tag-specific selection criteria. The tags are reconstructed as follows: A charged particle with $p > 0.5$ GeV/c and more than 85% of its energy deposited in the electromagnetic calorimeter is designated an e^+ tag, and one which penetrates more than three absorption lengths of iron is designated a μ^+ tag. The remaining tags are selected according to their composition of charged hadrons with no restriction on momentum and π^0's made of distinct photons, where any two-photon combination with an invariant mass satisfying $|m_{\gamma\gamma} - m_{\pi^0}| \le 25$ MeV/c^2 is considered a π^0 candidate. If one of the photons is used in the reconstruction of more than one π^0 candidate, we choose that π^0 candidate with a mass closest to the nominal π^0 mass. If several π^0-π^0 combinations exist in the $\pi^+(\ge 2)\pi^0$ tag, we choose the one minimizing $\chi^2 = \sum_{i=1,2}(m_{\gamma\gamma_i} - m_{\pi^0})^2/\sigma_i^2$. The photon candidate in the $\gamma\mu$ final state is selected as the highest energy shower ($E_\gamma > 0.1$ GeV) which is not associated with a π^0.

Candidate events for this analysis are selected from a sample of 8.6×10^6 low-multiplicity triggers. Events with two or four charged particles which have zero net charge, originate from the interaction region, and contain at least one identified muon and at least one photon with an energy greater than 30 (50) MeV in the barrel (end cap), reduce the initial sample to 233000 events. For both the μ^+ and π^+ tags, the minimum photon energy is further increased to 80 MeV to reduce background from doubly radiative $\mu\mu$ events. The muon energy deposited in the calorimeter is limited to < 500 MeV in order to reject QED background where a radiative photon overlaps a muon. Several kinematic conditions are then imposed to reduce backgrounds from ordinary τ pairs, radiative τ pairs, and radiative μ pairs: the cosine of the angle θ between the direction of the missing momentum in the event P_{miss} and the beam directions must satisfy $|\cos\theta| < 0.95$; $|P_{\text{miss}}| < E_b/c$; and the visible energy must be

TABLE I. Detection efficiencies and observed final states for τ tag modes.

τ tag[a]	Detection efficiency (%) [15]	Observed final state
$e^+\nu\bar\nu$	26.0 ± 0.8	$\mu^-e^+\gamma$
$\mu^+\nu\bar\nu$	14.5 ± 0.6	$\mu^-\mu^+\gamma$
$\pi^+\bar\nu$	23.0 ± 0.8	$\mu^-\pi^+\gamma$
$\pi^+\pi^0\bar\nu$	21.0 ± 0.5	$\mu^-\pi^+3\gamma$
$\pi^+\pi^+\pi^-\bar\nu$	28.3 ± 1.2	$\mu^-\pi^+\pi^+\pi^-\gamma$
$\pi^+\pi^+\pi^-(\ge 1)\pi^0\bar\nu$	13.9 ± 0.8	$\mu^-\pi^+\pi^+\pi^-(\ge 3)\gamma$
$\pi^+(\ge 2)\pi^0\bar\nu$	15.9 ± 0.8	$\mu^-\pi^+(\ge 5)\gamma$

[a]We do not distinguish between charged π's and K's in this analysis.

less than $0.98E_{\mathrm{c.m.}}$ For the e^+, π^+, and μ^+ tag samples, the fraction of visible energy is lowered to 95%, 90%, and 85%, respectively. For the μ^+ tag both charged particles must be identified as muons and the mass determined from the tag muon and the missing particles has to be consistent with that of a τ ($|m-m_\tau|<3\sigma$ with $\sigma=150$ MeV/c^2). The π^+ tag must be inconsistent with being a muon, i.e., it has to lie within the fiducial volume of the muon detectors and should be stopped within the first three absorption lengths of iron. These selection criteria reduce the data sample to 52 900 events.

The final sample is then obtained by requiring $|E_{\gamma\mu}-E_b|<150$ MeV and $\cos\theta_{\gamma\mu}>0$. The first constraint eliminates about 9% of all $\tau^- \to \gamma\mu^-$ Monte Carlo signal events, but reduces the remaining data by 98%, leaving 930 events. The loss in signal is due to initial state radiation producing a non-Gaussian $\gamma\mu$ energy distribution [see Fig. 1(a)]. The second constraint leads to an additional loss of $\sim 1\%$ in signal, but yields an additional factor of ~ 20 in background reduction. Candidates are further required to satisfy $|m_{\gamma\mu}-m_\tau|<3\sigma_m$, where σ_m is the mass resolution of the $\gamma\mu$ system. The mass resolution of the $\gamma\mu$ system as well as the individual tag efficiencies are determined from a sample of 27 000 Monte Carlo events generated at different beam energies. One τ is decayed to $\gamma\mu$ according to a phase-space distribution, and the other τ is decayed generically using the KORALB Monte Carlo generator [16], which includes corrections for radiative effects [17].

Figure 1(a) shows the energy distribution $E_{\gamma\mu}-E_b$ obtained from the Monte Carlo sample before applying the final selection criteria. The $\gamma\mu$ invariant mass spectrum obtained from the Monte Carlo sample after applying all selection criteria is shown in Fig. 1(b). The $\gamma\mu$ mass distribution is modeled with a fit by a Gaussian ($\chi^2/N_{\mathrm{DF}}=1.4$ for 57 degrees of freedom). The $\gamma\mu$ mass spectrum peaks slightly below the nominal τ mass due to non-Gaussian tails in the photon energy response of the electromagnetic calorimeter and has a resolution of $\sigma_m=22.5\pm0.4$ MeV/c^2. The detection efficiencies (ϵ_i) obtained for each tag topology including trigger efficiencies are listed in Table I. Using the branching ratios compiled by the Particle Data Group [18], we determine the total detection efficiency to be $\epsilon_{\mathrm{tot}}=\sum_i B_i\epsilon_i=(20.5\pm0.4)\%$.

Figure 2(a) shows a scatter plot of the $\gamma\mu$ invariant mass spectrum versus $\cos\theta_{\gamma\mu}$ for the final data sample summed over all seven tag modes. The plot contains 48 events, which cluster in the low mass range (<1.0 GeV/c^2) and around 3 GeV/c^2. The shape of the $\gamma\mu$ mass spectrum is well modeled with a fit by a fourth-order polynomial ($\chi^2/N_{\mathrm{DF}}=1.03$ for 8 degrees of freedom). Using this parametrization we estimate a background of 0.36 ± 0.04 event in the τ mass region. Figure 2(b) shows the $\gamma\mu$ invariant mass spectrum in the 1.5–2.0 GeV/c^2 mass region. The solid curve indicates the expected signal region as obtained from the Monte Carlo simulation; the arrows show the 3σ boundaries. Two events are visible between 1.5 and 2.0 GeV/c^2, neither of which falls into the τ mass region.

The sensitivity of our detector to the $\gamma\mu$ final state has been tested studying radiative μ pairs. Using data accumulated both at the $\Upsilon(4S)$ and in the continuum and calculating the efficiency from a Monte Carlo sample, we determine a μ-pair cross section of 0.84 ± 0.05 nb. This is consistent with the expected cross section of 0.92 nb.

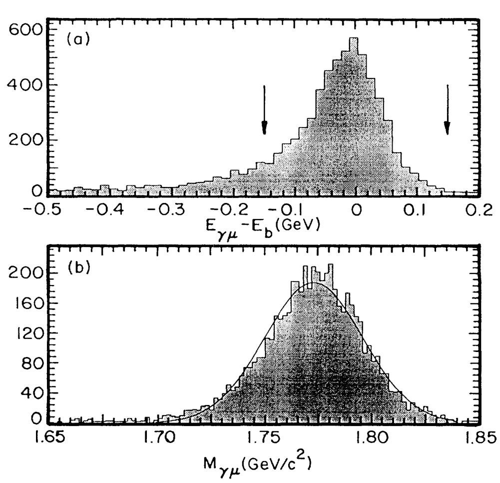

FIG. 1. (a) The energy distribution $E_{\gamma\mu}-E_b$ and (b) the $\gamma\mu$ invariant mass spectrum obtained from the Monte Carlo sample. The arrows show the accepted $\gamma\mu$ energy range. The solid line represents a Gaussian fit (see text).

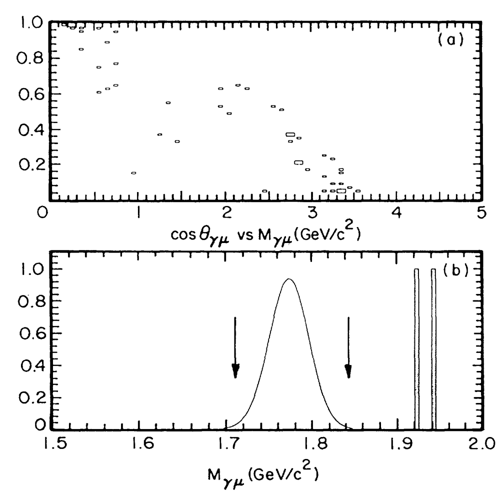

FIG. 2. (a) A scatter plot of the projection of the γ and μ directions ($\cos\theta_{\gamma\mu}$) vs the $\gamma\mu$ invariant mass of the final data sample summed over all tag modes. The size of the boxes is proportional to the number of events in that bin. (b) The $\gamma\mu$ invariant mass distribution of the final sample in the 1.5–2.0 GeV/c^2 mass range. The solid line indicates the τ mass resolution obtained from the Monte Carlo simulation; the arrows indicate the 3σ window.

Finally, we have checked the stability of our result by loosening the selection criteria on both the $\gamma\mu$ energy and invariant mass to $|E_{\gamma\mu}-E_b|<250$ MeV and $|m_{\gamma\mu}-m_\tau|<5\sigma_m$. No candidates are found with these looser selection criteria.

A Monte Carlo study of generic τ decays shows that the main background contribution to the $\tau^-\to\gamma\mu^-$ channel comes from ordinary τ decays, where the muon is produced in the decay $\tau^-\to\mu^-\nu\bar{\nu}$ and the photon originates from either initial-state radiation, final-state radiation of the muon, a π^0 decay with one photon undetected, or a spurious signal in the calorimeter. From a sample of 5.3×10^5 such τ pairs, eighteen events satisfy the selection criteria but none of them falls in the τ mass region. This indicates that the background in our final data sample can be entirely accounted for by generic τ decays. In addition, the $\gamma\mu$ mass spectrum of the generic τ decays also shows a low-mass enhancement and a clustering at 3 GeV.

The low-mass enhancement results from combining a muon with a background low-energy photon, for example, one which may have originated from initial-state radiation. Except for the e^+ tag mode which gets an additional contribution from final-state radiation of the electron, we expect all other tag modes to contribute equally to the low-mass enhancement. The clustering at higher masses comes mainly from τ events where the muon of the τ^- decay is combined with an energetic photon (typically $E_\gamma>1.5$ GeV) originating in the τ^+ decay. For example, the photon can come either from a π^0 decay or from final-state radiation of an electron in $\tau\to e^+\nu\bar{\nu}$. Since the photon and the μ come from different τ's, the angle between them is typically larger than 90°. Therefore, most of these events, which would yield high $\gamma\mu$ masses, are rejected by $\cos\theta_{\gamma\mu}>0$ and only a small fraction of events, for which $\theta_{\gamma\mu}$ is slightly below 90°, are accepted.

Though our results are consistent with the background arising from generic τ decays, small contributions from QED processes like $\gamma\mu^+\mu^-$ or $\gamma\gamma\mu^+\mu^-$ in the μ^+ tag, and low-multiplicity $q\bar{q}$ events in the hadronic tag modes, are also possible. We expect that backgrounds from these sources contribute mainly to the low $\gamma\mu$ mass region. Contributions from two-photon events, on the other hand, are expected to be negligible, because it is rather unlikely that such events contain a muon and a photon which together add up to the beam energy.

In order to set an upper limit on the branching ratio for $\tau^-\to\gamma\mu^-$, we need the number of $\gamma\mu$ candidates in the τ mass region, the total detection efficiency, and the total number of τ pairs. Since we observe no $\gamma\mu$ candidates in our final data sample in the τ mass region, the upper limit for $\gamma\mu$ candidates is $N_{\gamma\mu}=2.3$ events at 90% C.L. The total detection efficiency is $\epsilon_{\rm tot}=0.205\pm0.004$. The total number of τ pairs in our current data sample is calculated from the known τ cross sections and luminosities determined at the individual beam energies, yielding $N_{\tau\tau}=(1.44\pm0.03)\times10^6$. Thus, if we neglect all systematic effects, we obtain an upper limit of $B(\tau\to\gamma\mu)<3.9\times10^{-6}$ at 90% C.L. We estimate the following systematic uncertainties on the upper limit: 2.1% due to the determination of $N_{\tau\tau}$, which reflects equal contributions from uncertainties in the luminosity determination and in the cross section added in quadrature, 1.4% due to statistical errors in our Monte Carlo samples, 1.2% due to measurement errors on the τ branching ratios, and 3% due to our data selection. Adding all systematic errors linearly the upper limit for $\tau^-\to\gamma\mu^-$ increases to

$$B(\tau^-\to\gamma\mu^-)<4.2\times10^{-6}\quad\text{at 90\% C.L.}$$

In conclusion, we have performed a search for the lepton number violating decay $\tau^-\to\gamma\mu^-$ using 1.55 fb^{-1} of CLEO II data. We observe no candidates in the τ mass region, yielding an upper limit on the branching ratio of $B(\tau\to\gamma\mu)<4.2\times10^{-6}$ at 90% C.L. This limit is a factor of 8 lower than a limit from a previous experiment [12]. We note that the current experimental limit for $\mu^-\to\gamma e^-$ is 4.9×10^{-11} at 90% C.L. Using the enhancement factor predicted in the superstring model [8], our limit corresponds to an upper limit for $\mu^-\to\gamma e^-$ of $<2.1\times10^{-11}$ at 90% C.L. We also note that independent of this particular model our limit is the most stringent test of τ lepton number conservation to date.

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[15] We include feed-down in hadronic tags from channels with multiple π^0's, except for the π^+ tag which is more susceptible to background from radiative μ pairs. This is reflected in the varying detection efficiencies. For example, due to feed-down from the $\pi^+(\geq 2)\pi^0\bar{\nu}$ channel, the efficiency $\epsilon_{\pi^+\pi^0}$ is quite close to ϵ_{π^+}, which is reduced due to tight selection criteria. Similarly, $\epsilon_{\pi^+\pi^+\pi^-}$ is increased by feed-down from $\pi^+\pi^+\pi^-(\geq 1)\pi^0\bar{\nu}$, which in turn shows a reduced efficiency.

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