# Implications of $b \to s\gamma$ Decay Measurements in Testing the Higgs Sector of the Minimum Supersymmetric Standard Model

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The observation that the $b \to s\gamma$ decay rate is close to the standard model value implies a large mass for the charged Higgs boson in the minimum supersymmetric standard model (MSSM), which nearly closes the $t \to bH^+$ decay channel. For $m_t = 150$ GeV, the parameter region $m_A \lesssim 130$ GeV is excluded; this largely preempts searches at the CERN $e^+e^-$ collider LEP II and also partially excludes a region that would be inaccessible to MSSM Higgs boson searches at LEP II and at the Superconducting Super Collider and the CERN Large Hadron Collider.

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In the minimum supersymmetric standard model (MSSM), the inclusive $b \to s\gamma$ branching fraction is very sensitive to charged Higgs boson loop contributions, since they interfere constructively with the standard model (SM) $W$-loop amplitude and both receive a strong QCD enhancement; this offers a constraint on charged Higgs parameters [1, 2]. The present experimental upper limit $B(b \to s\gamma) < 8.4 \times 10^{-4}$ from the CLEO collaboration [3] already implies severe constraints [4] on the charged Higgs boson mass $m_{H^\pm}$ [2] for given top quark mass $m_t$ and model parameter $\tan\beta = v_2/v_1$, the ratio of the two vacuum expectation values appearing in the MSSM. In the present Letter we point out that these constraints almost close the interesting decay channel $t \to bH^+$ (the basis of all viable $H^\pm$ studies at hadron colliders [5–9]) and exclude most of the $(m_A, \tan\beta)$ parameter region accessible to LEP II Higgs boson searches. They also partially exclude a parameter region believed to be inaccessible to combined LEP I, LEP II, and SSC/LHC searches [6–9]. These important constraints will become even more far reaching when more precise theoretical calculations are made and a more accurate determination of $B(b \to s\gamma)$ becomes possible, for example at future $B$ factories.

For calculating QCD enhancements of the $b \to s\gamma$ decay amplitudes, we use the prescription of Grinstein, Springer, and Wise [10]. The relevant operator arising from the dominant $tW^+$ and $tH^+$ loop contributions at scale $m_b$ has the form

$$c_7(m_b) = \left[\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right]^{16/23} \left\{ c_7(M_W) - \frac{8}{3}c_8(M_W)\left[1 - \left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{2/23}\right] + \frac{232}{513}\left[1 - \left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{19/23}\right]\right\}, \quad (1)$$

where for the MSSM

$$c_7(M_W) = -\frac{1}{2}A(x) - B(y) - \frac{1}{6\tan^2\beta}A(y), \quad (2)$$

$$c_8(M_W) = -\frac{1}{2}D(x) - E(y) - \frac{1}{6\tan^2\beta}D(y), \quad (3)$$

with $x = (m_t/M_W)^2$, $y = (m_t/m_{H^\pm})^2$. The functions $A$, $B$, $D$, and $E$ are defined in Ref. [10]. The ratio of $\Gamma(b \to s\gamma)$ to the inclusive semileptonic decay width is then given by

$$\frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\nu)} = \frac{6\alpha}{\pi\rho\lambda}|c_7(m_b)|^2, \quad (4)$$

where $\alpha$ is the electromagnetic coupling. The phase-space factor $\rho$ and the QCD correction factor $\lambda$ for the semileptonic process are given by $\rho = 1 - 8r^2 + 8r^6 - r^8 - 24r^4\ln(r)$ with $r = m_c/m_b$ and $\lambda = 1 - \frac{2}{3}f(r,0,0)\alpha_s(m_b)/\pi$ with $f(r,0,0) = 2.41$ [11]. Note that the $m_b^5$ dependence of the partial widths cancels out in Eq. (4), and also the Cabbibo-Kobayashi-Maskawa matrix elements cancel to a good approximation. We ignore charm quark contributions (~0.1% in the amplitude). We evaluate the $b \to s\gamma$ branching fraction from Eq. (4) using the accurately determined semileptonic branching fraction $B(b \to ce\nu) = 0.107$ and the estimate $\alpha_s(M_W)/\alpha_s(m_b) = 0.548$ based [12] on a three-loop formula with $m_b = 4.25$ GeV.

The $B(b \to s\gamma)$ results depend sensitively on both $m_{H^\pm}$ and $\tan\beta$. The MSSM sets a lower bound $m_{H^\pm}^2 = M_W^2 + m_A^2 > M_W^2$ at tree level; with one-loop radiative corrections for $M_{\rm SUSY} = 1$ TeV and experimental limits on $m_A > 40$ GeV this bound becomes approximately $m_{H^\pm} > 90$ GeV, well above the LEP detection limits for $H^\pm$. There are bounds $m_t/600 < \tan\beta < 600/m_b$ from requiring Yukawa couplings to remain perturbative [2] and $\tan\beta < 85$ from the proton lifetime [13]. There are also constraints from low-energy data (principally $B$-$\overline{B}$, $D$-$\overline{D}$, $K$-$\overline{K}$ mixing) that exclude low values of $\tan\beta$ [2, 14] but these are less stringent than the $b \to s\gamma$ constraint of present concern.

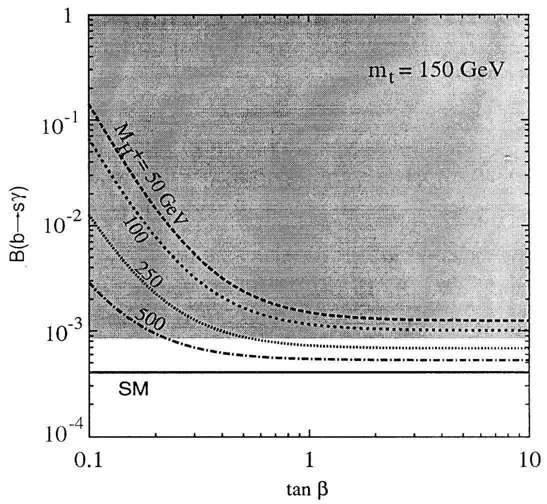


FIG. 1. Calculated dependence of the inclusive branching fraction $B(b \to s\gamma)$ on $\tan\beta$ in the MSSM, with various values of $m_{H^\pm}$ , for $m_t = 150$ GeV. The shaded area is excluded by the CLEO bound $B(b \to s\gamma) < 8.4 \times 10^{-4}$.

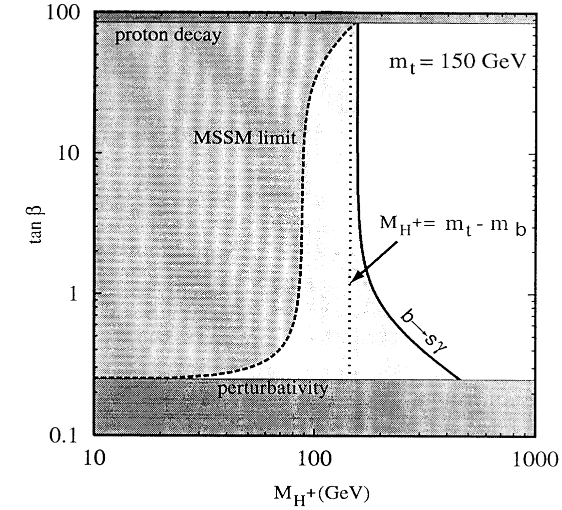


FIG. 2. Comparison of the $b \to s\gamma$ bound with other bounds from perturbativity, proton lifetime, and the MSSM mass formulas, in the $(m_{H^\pm}, \tan\beta)$ plane, for $m_t = 150$ GeV. The threshold for $t \to bH^+$ decay at $m_{H^\pm} = m_t - m_b$ is also shown. Shaded areas are excluded by one or more bounds.

Figure 1 shows the dependence of $B(b \to s\gamma)$ on $\tan\beta$ for $m_t = 150$ GeV, with various choices of $m_{H^\pm}$. We see that the CLEO bound $B < 8.4 \times 10^{-4}$ not only excludes small values of $\tan\beta$ for any $m_{H^\pm}$, but also excludes a range of lower $m_{H^\pm}$ values ($m_{H^\pm} \lesssim 155$ GeV in the $m_t = 150$ GeV case shown) for any $\tan\beta$. However, this lower limit on $m_{H^\pm}$ depends quite sensitively on the theoretical calculation as discussed below.

Figure 2 translates the $b \to s\gamma$ bound into the $(m_{H^\pm}, \tan\beta)$ plane for $m_t = 150$ GeV and compares it with the bounds from perturbativity, proton decay, and the MSSM $m_{H^\pm}$ formula. The bound is beyond the threshold $m_{H^\pm} = m_t - m_b$, so that the decay mode $t \to bH^+$ is closed. As $m_t$ is reduced, the $b \to s\gamma$ bound and the $t \to bH^+$ threshold move left toward the MSSM constraint on $m_{H^\pm}$ (that we have calculated from the one-loop mass formula [15] plus LEP limits on $m_A$ [16]). For $m_t \lesssim 130$ GeV the $b \to s\gamma$ bound overtakes the $t \to bH^+$ threshold and this decay mode becomes marginally open, in our present calculations. At $m_t \simeq 95$ GeV, the threshold crosses the MSSM bound and the $t \to bH^+$ decay becomes closed once more.

Figure 3 translates the $b \to s\gamma$ bound into the $(m_A, \tan\beta)$ plane, where coverage of the MSSM is usually discussed [6–9]. The area below and to the left of the $b \to s\gamma$ curve is excluded. The boundary of the region accessible to $e^+e^- \to Zh, Ah$ searches at LEP II is below and to the left of the dashed curve; we see that a large part of this LEP II range is preempted. Heavy shading shows the area of parameter space that appears to be inaccessible to MSSM Higgs boson searches at LEP I, LEP II, and SSC/LHC (reproduced here from Ref. [9]); we see that this inaccessible region is already partially covered by the $b \to s\gamma$ bound. With further improvements in measurements of $B(b \to s\gamma)$, the coverage of the MSSM may in fact be complete after all.

The $b \to s\gamma$ bound of Fig. 3 also leads to an interesting possible correlation between $m_t$ and the lighter $CP$-even Higgs boson mass $m_h$, if we inject an additional theoretical requirement on Yukawa couplings $\lambda_b(M_G) = \lambda_\tau(M_G)$ from SUSY-GUT unification following Refs. [12, 17]. Given $m_t < 175$ GeV, the authors of Ref. [12] find just

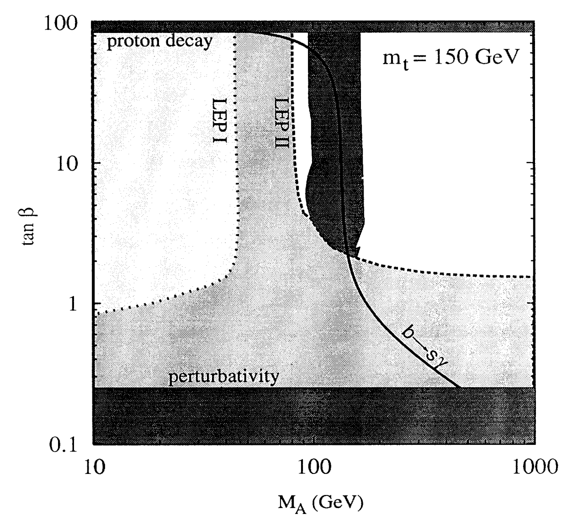


FIG. 3. Comparison of the $b \to s\gamma$ bound in the $(m_A, \tan\beta)$ plane, with the region accessible to LEP I and LEP II searches (lightly shaded) and the region apparently inaccessible to LEP I, LEP II, and SSC/LHC MSSM Higgs boson searches (heavily shaded vertical region) for $m_t = 150$ GeV.

two solutions for $\beta$, namely, $\sin\beta = 0.78(m_t/150 \text{ GeV})$ and $\tan\beta > m_t/m_b$. For $m_t = 150$ GeV, the first solution gives $\tan\beta = 1.25$ for which the $b \to s\gamma$ bound allows only the limited range $68 < m_h < 76$ GeV [assuming a squark mass scale of order 1 TeV as in Ref. [9]; this range shifts up (down) by approximately 10 GeV when the SUSY scale is increased (decreased) by a factor 2]; such an observed correlation of $m_h$ with $m_t$ would support this solution. Note that threshold corrections to the Yukawa coupling unification constraint could shift this predicted Higgs mass range somewhat. For the large-$\tan\beta$ solution, the $b \to s\gamma$ bound does not effectively constrain $m_h$.

The above analysis has neglected various theoretical uncertainties. The leading-log form in Eq. (1) is obtained by truncating the anomalous dimension matrix to three operators. This truncation was initially estimated to introduce a theoretical uncertainty of at most 15% to $c_7(m_b)$ in the standard model [10]. Subsequent calculations by Cella, Curci, Ricciardi, and Vicerè [18] and especially Misiak [19] of the previously unknown entries in the anomalous dimension matrix demonstrated that the effects were of order $-5\%$. This uncertainty should be reduced somewhat in the MSSM where the charged Higgs contribution increases the leading-order result as in Eqs. (2) and (3). Recently Misiak [20] has also calculated the next-to-leading log corrections and found a $-5\%$ to $-10\%$ change in $c_7(m_b)$ for $\alpha_s(M_Z)$ in the range 0.110 to 0.123; this contribution is not included in our analysis. The lower limit on the charged Higgs mass is sensitive to these uncertainties as can be seen in Fig. 1. The leading-log result in Eq. (1) is obtained by integrating out the top quark (and the charged Higgs boson) at the mass of the $W$. Recently the leading corrections have been obtained when the top quark is integrated out at a scale larger than $M_W$ [21]. These corrections enhance $c_7(m_b)$ by as much as 7%. This enhancement partially compensates for the reductions enumerated above.

We have neglected any other sources of FCNC in the supersymmetric model to the $b \to s\gamma$ decay rate. This was investigated thoroughly in Ref. [22] under the tight assumptions of minimal $N = 1$ supergravity with radiative breaking of the electroweak symmetry and more recently in Ref. [23] with more relaxed assumptions. Only for larger values of $\tan\beta$ ($\gtrsim 10$) and relatively light supersymmetric particles are the chargino and gluino contributions comparable to the standard model rate. These additional contributions can add constructively or destructively to the amplitudes we have considered here. Only if extra contributions conspire to reduce the decay rate can the bounds in this paper be evaded.

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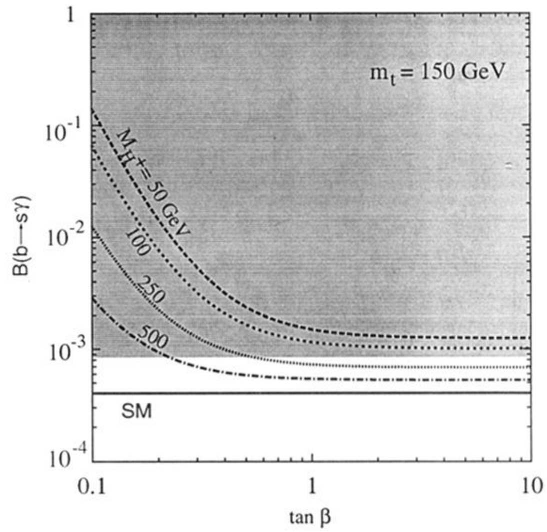


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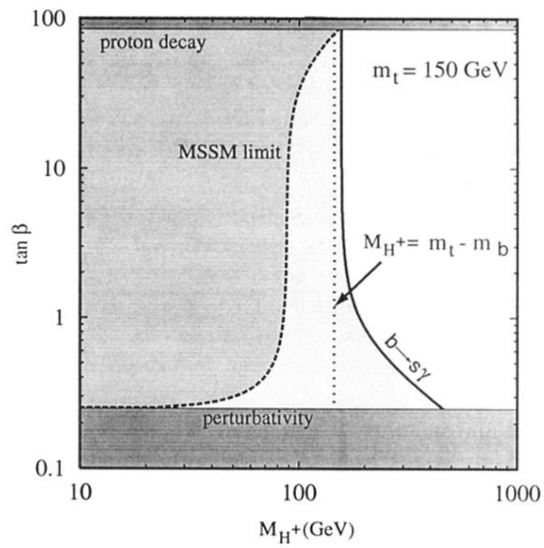


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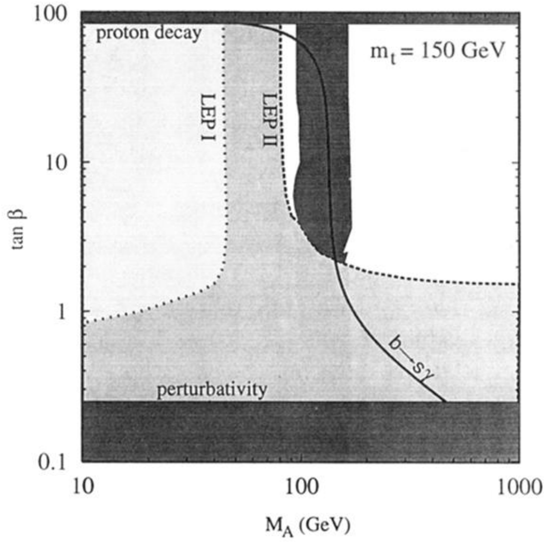


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