CP Violation in Semileptonic Top Quark Decays in the Weinberg Model

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We show that the charged Higgs sector in Weinberg's model for *CP* violation can cause a large *CP* violation effect in $t \to b\tau\nu_\tau$. The asymmetry in the *partial* rate is dominated by contributions such as Z(and γ)-H^+ box diagrams interfering with a (near-resonance) W tree diagram. But far more significant are the asymmetries in the *partially integrated rates* and in the energy between the τ^+ and the τ^-. These stem from (near-resonance) W-tree–H^+-tree interference.

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Any clue from experiments in new systems, especially those involving leptons, will be more than welcome to help unveil the source of *CP* nonconservation. In the standard model (SM) *CP* violation emanates from a *CP*-odd phase [1] in the Cabibbo-Kobayashi-Maskawa (CKM) matrix which influences directly only the quark sector. Thus *CP* violation in the leptonic sector is in general extremely small in the SM, resulting indirectly from the CKM phase. For example, the electric dipole moment (EDM) of the electron [2] in the SM is at least a three-loop effect [3], predicted to be $<10^{-38}e\,\mathrm{cm}$. In the Weinberg model of *CP* violation (WM) [4], which has recently gained renewed interest [5], there are *CP* violation phases within the enlarged Higgs sector. These phases may therefore lead to large effects in the lepton sector. Indeed, Barr and Zee [6,7] have shown that one may gain about 10 orders of magnitude over the EDM of the electron in the SM, by considering two-loop contributions in the WM involving the top loop. It is therefore interesting to investigate whether the same model that gives a large EDM of the electron via t quark loops and Higgs exchange predicts a measurable *CP* violation asymmetry for semileptonic top decays, thus establishing a kind of "low-scale–high-scale connection."

CP violation in the production and decay of top quarks has recently been discussed in the literature, both within and beyond the SM [8,9]. In particular, it was shown [10] that a *CP*-even phase from the W width opens new possibilities for observables like a partial rate asymmetry (PRA), which require such a phase in addition to a *CP*-odd phase. Let us consider this *CP* violating PRA, i.e.,

$$\alpha = \frac{\Gamma(t \to b\tau^+\nu_\tau) - \Gamma(\bar{t} \to \bar{b}\tau^-\bar{\nu}_\tau)}{\Gamma(t \to b\tau^+\nu_\tau) + \Gamma(\bar{t} \to \bar{b}\tau^-\bar{\nu}_\tau)} \tag{1}$$

within the WM. As we show here, the main contributions to the above asymmetry come from terms such as interference of the W tree diagram $t \to bW \to b\tau\nu_\tau$ [see Fig. 1(a)] with the Z-H^+ and γ-H^+ box contributions [see, for example, Fig. 1(b)], or H tree [see Fig. 1(c)] interfering with γ-W boxes [see, for example, Fig. 1(d)]. In all these diagrams, the W resonates, as denoted by the bubble on its line. Therefore, although strictly speaking, diagram 1(a) is not a tree graph, for brevity, we will refer to it as such. The presence of this resonance is crucial in enhancing the contribution to the PRA of the class of graphs with it, compared to those without it, at a given order in perturbation theory, here that being g^8. Note that in order not to violate the combined symmetry of *CPT* [11], all fermion pairs, except $\tau\nu_\tau$, run in that bubble (i.e., no rescattering). We find that in the WM this *CP* violation effect is larger by more than 9 orders of magnitude than in the SM [8]. As such the effect we report may well be within the reach of currently envisioned top quark production rates at the supercolliders, although precise predictions cannot be made due to the lack of knowledge of the relevant parameters of the WM.

Before presenting our results we note that it was previously suggested [12,13] that a large *CP* violating PRA can result from the interference of diagrams 1(a) and 1(c), where the *CP* even phase is provided by the W propagator and the *CP*-odd phase lies in the couplings of the

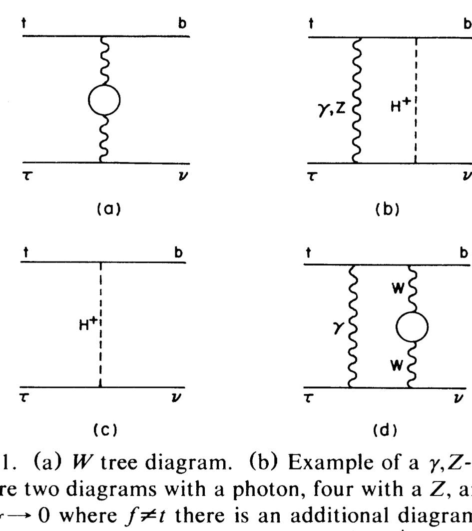

FIG. 1. (a) W tree diagram. (b) Example of a γ,Z-H^+ box. There are two diagrams with a photon, four with a Z, and in the limit $m_f \to 0$ where $f \neq t$ there is an additional diagram with χ, χ being the unphysical neutral scalar. (c) H^+ tree diagram. (d) γ-W box (there are two diagrams of this type).

charged Higgs bosons. This is incorrect due to the following facts: (1) The contribution to α from the W-tree–Higgs-tree interference is proportional to $\mathrm{Im}G_L$, the longitudinal part of the W propagator, which (in the unitary gauge, for example) is

$$\rho_T^{\mu\nu}G_T+\rho_L^{\mu\nu}G_L\equiv\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)G_T+\left(\frac{q^\mu q^\nu}{q^2}\right)G_L\,. \quad (2)$$

This results from a helicity mismatch, and can be explicitly seen from the calculation of the interference. (2) Unlike the case for

$$G_T=\frac{1}{q^2-M_W^2+i\Gamma_W M_W}\,, \quad (3)$$

$\mathrm{Im}G_L$ is proportional to squares of fermion masses in the W loop [14], the largest one being, in our case, the charm quark mass, as the b quark contribution is Cabibbo suppressed and the $\tau\nu_\tau$ loop does not contribute to the asymmetry. The new effect is therefore forcing an extremely small PRA from W-tree–H-tree interference, contrary to previous claims [12,13]. The error originated from the use of the naive Breit-Wigner form for the unitary gauge W propagator

$$\frac{-g^{\mu\nu}+q^\mu q^\nu/M_W^2}{q^2-M_W^2+i\Gamma_W M_W}\,, \quad (4)$$

rather than the correct form [14]

$$\frac{-g^{\mu\nu}+q^\mu q^\nu[q^2+i(\Gamma_W M_W+\epsilon_L)]/q^2(M_W^2+i\epsilon_L)}{q^2-M_W^2+i\Gamma_W M_W}\,, \quad (5)$$

where ϵ_L is, again, proportional to squares of the masses of those fermions to which the W decays to. (The explicit form of ϵ_L is given in Ref. [14].) It is important to note that the above form of the W propagator implies that its transverse piece is exactly as given in Eq. (3) and that is the same as commonly used in the literature. The rest of this paper will make use of this transverse piece only.

In this work we discuss two ways to obtain CP violating asymmetries that are proportional to $\mathrm{Im}\hat{G}_T$ (where a caret on G_T indicates the absence of $\tau\nu$ in the numerator of $\mathrm{Im}G_T$) and to $\mathrm{Im}G_T$, respectively, rather than to the vanishingly small $\mathrm{Im}G_L$. In one method the necessary theoretical calculation is harder while in the other it may be slightly more difficult to implement experimentally as it requires dealing with an asymmetry in *partially integrated rates.* We first deal with the former method. However, since the second method leads to significantly larger results we are only interested in an order of magnitude estimate for the first one.

Because the lowest order (tree-tree) contribution to the PRA is vanishingly small it is expected to be very sensitive to radiative corrections. One is thus motivated to go to a higher order to obtain an asymmetry proportional to $\mathrm{Im}\hat{G}_T$. We first study vertex corrections to diagrams 1(a) and 1(c). As can be easily demonstrated, once again the PRA is proportional to $\mathrm{Im}\hat{G}_L$, and thus is negligibly small irrespective of whether the vertex correction is done to the W or H vertices. As a specific lowest order example of a significant contribution to the PRA, we consider the box-tree interference between diagrams 1(a) and 1(b) [and between 1(c) and 1(d)] as it takes advantage of the transverse part of the resonance propagator, and therefore peaks at $q^2\simeq M_W^2$, where $q^2=(p_\nu+p_\tau)^2$. Fortunately, the CP violation effect is still quite large, considering its high order, because of resonance enhancement [15].

Let us now give a brief outline of the CP violation from the charged Higgs sector in the WM. Rather than working with complex couplings among the Higgs sector, as originally suggested by Weinberg [4], we refer here to the mass eigenstates of the charged Higgs particles H_1 and H_2 (a third one is absorbed by the W) related to the three original charged scalar fields by a unitary transformation. Writing this transformation as a CKM-like matrix [16], pushing the mass of one of the charged Higgs particles to very high values, and denoting the remaining one by H, we are left with the following Lagrangian for the Yukawa couplings [17] (where only Htb and $H\tau\nu$ terms are displayed):

$$\mathcal{L}=\frac{gm_b}{\sqrt{2}M_W}\bar{t}_L b_R\frac{s_1 s_3}{c_1}H^+ +\frac{gm_t}{\sqrt{2}M_W}\bar{t}_R b_L\frac{c_1c_2s_3-s_2c_3e^{i\delta}}{s_1c_2}H^+ -\frac{gm_\tau}{\sqrt{2}M_W}\bar{\nu}_L\tau_R\frac{c_1s_2s_3+c_2c_3e^{i\delta}}{s_1s_2}H^+ +\mathrm{H.c.} \quad (6)$$

Note that s_i, c_i, and δ appear in the CKM-like matrix operating in the charged Higgs sector, and are not elements of the usual CKM matrix.

Calculation of the tree-box interference and the resulting partial rate asymmetry is a straightforward, though lengthy, process. We first emphasize that on resonance, the interference of diagram 1(a) with the diagrams represented by Fig. 1(b), is effectively of order g^4, since the W resonates and we are only interested in absorptive (tree) $\times$ dispersive (box) contribution [i.e., on resonance the absorptive part of Fig. 1(a) goes as $g^2\Gamma_W/\Gamma_W^2\simeq O(g^0)$]. Thus contributions of effective $O(g^6)$ or higher have been ignored. Both γ and Z contribute, giving seven diagrams for Fig. 1(b) in the 't Hooft-Feynman gauge. This counting includes one graph, resulting from exchange of the unphysical scalar χ, that survives in the limit of $m_f\rightarrow 0$, for fermions different from the top quark. The leading contribution for the interference between the H tree [Fig. 1(c)] and γ-W box [Fig. 1(d), representing two diagrams] is again, effectively, of order g^4, this time only for those photons that allow the W to resonate since otherwise the effective order increases. This also explains why there is no interference of H tree with Z-W box of

TABLE I. Results for the *CP* violating asymmetry α in the partial rates [see Eq. (1)]. Contribution to α from each tree-loop interference is separately listed as well as the total. Notation: T is for "tree" and B is for "box." Thus, for example, T_W-$B_{H\gamma}$ is the interference between Fig. 1(a) (i.e., T_W) and Fig. 1(b) with γ (i.e., $B_{H\gamma}$), etc. The CKM-like angles s_1,s_2,s_3 given in Table II, and $\delta=\pi/2$, maximize the Higgs coupling ImU [see Eq. (8)].

M_{H^+}	T_W-$B_{H\gamma}$	T_W-B_{HZ}	T_H-$B_{W\gamma}$	α	$(\alpha^2 B)^{-1}$
200	3.4×10^{-6}	-2.5×10^{-5}	-1.7×10^{-5}	-3.9×10^{-5}	5.9×10^{9}
300	1.5×10^{-6}	-1.8×10^{-5}	-8.3×10^{-6}	-2.4×10^{-5}	1.6×10^{10}

order g^4. Note that in all these cases we are calculating the dispersive (tree) $\times$ absorptive (box) contribution to the PRA. We also note here that the four photon diagrams (two each from Figs. 1(b) and 1(d)] are infrared divergent. We have therefore included in our calculation the four corresponding bremsstrahlung interferences: External γ attached to the τ in Fig. 1(a) interfering with a γ on a t line, or with a γ on a b line in Fig. 1(c), and two more interferences as described above, but with the W and H^+ interchanged.

For the interference of Figs. 1(a) and 1(b) we can write for the *CP* violation asymmetry defined in Eq. (1)

$$\alpha \simeq \frac{(\mathrm{Im}U/512\pi^3 m_t^3)\int dq^2 du\, \mathrm{Im}\hat{G}_T\, \mathrm{Re}(\mathrm{tree}\times\mathrm{box})}{\Gamma(t\to bW\to b\tau\nu)}, \quad (7)$$

where the interference in the denominator has been neglected, and

$$U=\frac{c_1c_2s_3-s_2c_3e^{-i\delta}}{s_1c_2}\,\frac{c_1s_2s_3+c_2c_3e^{i\delta}}{s_1s_2},$$
$$u=(p_\tau+p_b)^2,\quad q^2=(p_\nu+p_\tau)^2 \quad (8)$$

and

$$\mathrm{Im}\hat{G}_T=-\frac{M_W\hat{\Gamma}_W}{(q^2-M_W^2)^2+(\Gamma_W M_W)^2},$$
$$\hat{\Gamma}_W=\Gamma_W-\Gamma_{W\to\tau\nu}. \quad (9)$$

For the interference of Figs. 1(c) and 1(d) we use a similar formalism, here the Im part comes from the box with the resonating W propagator. For the bremsstrahlung processes we assume soft external photons with an energy cutoff taken to be 1 GeV. (The dependence of the PRA on this cutoff is very weak.)

We now obtain numerical values for the asymmetry α, and for the relevant quantity $(\alpha^2 B)^{-1}$ which is a measure of the number of events required to observe any effect, where B is the branching ratio for $t\to bW$ followed by $W\to\tau\nu$. We assume that $\delta=\pi/2$ and search for the maximal value of ImU [see Eq. (8)] by varying s_1,s_2,s_3 subjecting the charged Higgs couplings to all experimental constraints. A further enhancement of the effect is possible when $M_{H^+}<m_t$ and the absorptive part of the Higgs tree interferes with the box. We do not include this contribution and present numerical results only for $M_{H^+}>m_t$ in Table I. Note that about 10^9 to 10^{10} $t\bar{t}$ pairs are needed to see such a rate asymmetry.

Now, we discuss the other method alluded to above which allows an asymmetry proportional to ImG_T (and not to Im$\hat{G}_T$ as the CPT restriction is not operational here) to result even from the much simpler tree-tree interference, i.e., the product of Fig. 1(a) and Fig. 1(c) [18]. For this purpose we consider an asymmetry in the partially integrated rate replacing Eq. (1)

$$\Delta\alpha=\frac{\Gamma_{pi}(t\to b\tau^+\nu_\tau)-\Gamma_{pi}(\bar{t}\to\bar{b}\tau^-\bar{\nu}_\tau)}{\Gamma_{pi}(t\to b\tau^+\nu_\tau)+\Gamma_{pi}(\bar{t}\to\bar{b}\tau^-\bar{\nu}_\tau)}, \quad (10)$$

where Γ_{pi} is the partially integrated width for $t\to b\tau\nu$ obtained by integrating over only a part of the full kinematic range of u for a fixed q^2 [see Eq. (7)].

The point is that the integration over the full range of u necessarily results in the replacement of the p_τ^μ (from the τ-ν_τ loop sandwiched between the W and the H) with q^μ. This emergence of q^μ forces the contribution proportional to G_T to vanish since $q^\mu\rho_T^{\mu\nu}=0$, by definition. Thus to enable the transverse part of the W propagator to contribute, instead of considering PRA, we now consider *CP* violating *partially integrated rate asymmetry* (PIRA), *à la* Eq. (10). For clarity we can think of the integration over u (for a fixed q^2) in the rest frame of the W boson, i.e., $\mathbf{q}=0$. The integration over u is now equivalent to that over the angle (θ) between ($-\mathbf{p}_\tau$) and $\mathbf{p}_b$. Thus for positive values of $\cos\theta$ the PIRA is defined to be α_+. Calculation of α_+ from the tree-tree interference [i.e., Figs. 1(a) and 1(c)] leads to

$$\alpha_+=\frac{\sqrt{2}}{4\pi}\frac{G_F m_\tau^2 r_{WH}\,\mathrm{Im}U}{(2+r_{Wt})(1-r_{WH})[B(W\to\tau\nu)]}, \quad (11)$$

where $r_{WH}=M_W^2/m_H^2$ and $r_{Wt}=M_W^2/m_t^2$. The resulting numbers are given in Table II. We thus see that there is as much as 3 orders of magnitude increase over the previous case (Table I) in the sensitivity depending on the value of M_{H^+}. The number of t quarks required to see these effects (i.e., 10^6–10^7 pairs) may well therefore be within the reach of the supercolliders.

TABLE II. Results for the *CP* violating PIRA (α_+) and the minimal number of events required, i.e., $(\alpha_+^2 B_+)^{-1}$ for two values of M_{H^+} and $m_t=150$ GeV. B_+ is the appropriate branching fraction $\simeq0.04$. See also caption to Table I.

M_{H^+}	s_1	s_2	s_3	α_+	$(\alpha_+^2 B_+)^{-1}$
200	0.252	8.29×10^{-3}	0.707	2.8×10^{-3}	3.2×10^{6}
300	0.210	9.99×10^{-3}	0.707	1.4×10^{-3}	1.3×10^{7}

Another (related) method to obtain a significant CP violating contribution from the tree-tree interference is to focus on the energy asymmetry (α_E) between the τ_+ and the τ_- resulting from decays of $t\bar{t}$ pair at rest. We define α_E to be the ratio of the difference in the averages of the τ energies to their sum. Unlike the preceding case, a cut on the phase space is no longer required here. A calculation similar to the above leads to

$$\alpha_E = \frac{\sqrt{2}}{12\pi} \frac{G_F m_\tau^2 r_{WH}(1-r_{Wt})\mathrm{Im}U}{(1+3r_{Wt}^2+2r_{Wt})(1-r_{WH})[B(W\to\tau\nu)]} . \tag{12}$$

Thus for $m_t = 150$ GeV, α_E is approximately equal to $\alpha_+/3$.

The main conclusion is that in the WM there can be a large CP violation effect for a leading decay $t\to b\tau\nu_\tau$ of the top quark with an enhancement of more than 9 orders of magnitude insofar as the required number of $t\bar{t}$ pairs is concerned over the most optimistic semileptonic mode (i.e., $t\to d\tau\nu_\tau$) in the SM [8]. It will be interesting to investigate the CP violation effect for top quark decays in other extensions such as supersymmetry and leptoquark models.

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Note added.—After completion of this work we received a paper [19] from Liu in which he also addresses Refs. [12] and [13]. The approximate form for the W propagator, given in his Eq. (14), is inconsistent with the optical theorem at $q^2 = M_W^2$. In any case, we would like to stress that his statement about the longitudinal component of the resonant W propagator, whether compatible or not with the correct form given in our Eq. (5) (see Ref. [14] for further details), is not relevant for the present work, since we are using only the *transverse* part of the W-boson propagator. There is universal agreement on this transverse piece. We maintain that it gives rise to large CP violating PRA, PIRA, and energy asymmetry without any conflict with CPT.

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[14] The form of G_L, the longitudinal part of the W propagator, is discussed by the present authors in Report No. TECHNION-PH-92-39 (to be published).
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