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Quark Matter Droplets in Neutron Stars

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We show that, for physically reasonable bulk and surface properties, the lowest energy state of dense matter consists of quark matter coexisting with nuclear matter in the presence of an essentially uniform background of electrons. We estimate the size and nature of spatial structure in this phase, and show that at the lowest densities the quark matter forms droplets embedded in nuclear matter, whereas at higher densities it can exhibit a variety of different topologies. A finite fraction of the interior of neutron stars could consist of matter in this new phase, which would provide new mechanisms for glitches and cooling.

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Over the past two decades many authors have considered the properties of neutron stars with a core of quark matter [1]. (References to more recent work may be found in Ref. [2].) In the usual picture the transition between nuclear and quark matter occurs at a unique pressure. Consequently in neutron stars the density is expected to jump discontinuously at the boundary between the two phases. This is because, for electrically neutral matter in β equilibrium, there is only one independent variable, the baryon density. Recently Glendenning [3] considered the possibility of bulk quark and nuclear matter coexisting in a uniform electron gas. The density of electrons in quark matter was assumed to be the same as in nuclear matter, and its value was such as to ensure that the total charge vanished. He found that quark and nuclear matter could coexist for a finite range of pressures, and therefore over a finite region of the star. In this Letter we investigate Coulomb and surface effects, which were not considered in the earlier work. We address three important questions. When is it legitimate to regard the electron density as uniform, what is the spatial structure of the new phase, and is it energetically favorable?

At lower densities the new phase may be regarded as droplets of quark matter immersed in nuclear matter, and we shall refer to it as the *droplet phase*, even though at higher densities its structure is more complicated, as we shall show. If droplet sizes and separations are small compared with Debye screening lengths, the electron density will be uniform to a good approximation. The Debye screening length λ_D is given by

$$1/\lambda_D^2 = 4\pi \sum_i Q_i^2 \left(\frac{\partial n_i}{\partial \mu_i}\right)_{n_j, j\neq i}, \tag{1}$$

where n_i, μ_i, and Q_i are the number density, chemical potential, and charge of particle species i. Considering only electrons gives a screening length

$$\lambda_D^{(e)} = \frac{\sqrt{\pi/4\alpha}}{k_{F,e}}, \tag{2}$$

where $\alpha \simeq 1/137$ and the Fermi momentum $k_{F,e} = \mu_e$ since the electrons are always relativistic at these densities. For $\mu_e \lesssim 150$ MeV we thus obtain $\lambda_D^{(e)} \gtrsim 13$ fm. The screening length for protons alone $\lambda_D^{(p)}$, is given by $[\pi v_{F,p}/c4\alpha(1+F_0)]^{1/2}/k_{F,p}$, where F_0 is the Landau parameter which gives the energy for proton density variations. At the saturation density for symmetrical nuclear matter, $F_0 \simeq 0$, whereas at higher densities $F_0 \sim 1$ [4]. Since $\mu_p \sim m$, the nucleon mass, we find $\lambda_D^{(p)} \gtrsim 10$ fm, somewhat shorter than the electron screening length, and therefore in the nuclear matter phase protons are the particles most effective at screening. The screening length for quarks is $\lambda_D^{(q)} \simeq 7/k_{F,q}$, where q=u, d, and s refer to up, down, and strange quarks. It depends only slightly on whether or not s quarks are present, so for $\mu_q \simeq m/3$ we find $\lambda_D^{(q)} \simeq 5$ fm.

In a composite system, such as the one we consider, screening cannot be described using a single screening length, but it is clear from our estimates that if the characteristic spatial scales of structures are less than about 10 fm for the nuclear phase, and less than about 5 fm for the quark phase, screening effects will be unimportant, and the electron density will be essentially uniform. In the opposite case, when screening lengths are short compared with spatial scales, the total charge densities in bulk nuclear matter and quark matter will both vanish.

Consider now the case when screening lengths are much larger than the spatial scale of structures. This latter condition implies that the electron density is uniform everywhere, and all other particle densities are uniform within a given phase. The problem is essentially identical to that of matter at subnuclear densities [5], and the structure is determined by competition between Coulomb and interface energies. When quark matter occupies a small fraction f of the total volume, it will form spherical droplets immersed in nuclear matter. For higher filling fractions, the quark matter will adopt shapes more like rods ("spaghetti") and plates ("lasagna"), rather than spheres. For $f \geq 0.5$, the structures expected are the same as for a filling factor $1 - f$, but with the roles of nuclear matter and quark matter reversed. Thus one expects for increasing f that there will be regions with nuclear matter in rodlike structures, and roughly spherical droplets.

To estimate characteristic dimensions, we consider some special cases. When f is small or close to unity, the minority phase will form spherical droplets. The surface energy per droplet is given by

$$\mathcal{E}_S = \sigma 4\pi R^2 , \tag{3}$$

where σ is the surface tension, and the Coulomb energy is

$$\mathcal{E}_C = \frac{3}{5}\frac{Z^2e^2}{R} = \frac{16\pi^2}{15}\left(\rho_Q - \rho_N\right)^2 R^5. \tag{4}$$

Here Z is the excess charge of the droplet compared with the surrounding medium, $Ze = (\rho_Q - \rho_N)V_D$, where $V_D = (4\pi/3)R^3$ is the droplet volume and ρ_Q and ρ_N are the total charge densities in bulk quark and nuclear matter, respectively. Minimizing the energy density with respect to R we obtain the usual result that $\mathcal{E}_S = 2\mathcal{E}_C$ and find a droplet radius

$$R = \left(\frac{15}{8\pi}\frac{\sigma}{(\rho_Q - \rho_N)^2}\right)^{1/3} \simeq (5.0\,\text{fm})\left(\frac{\sigma}{\sigma_0}\right)^{1/3}\left(\frac{\rho_Q - \rho_N}{\rho_0}\right)^{-2/3} . \tag{5}$$

In the second formula we have introduced the quantities $\rho_0 = 0.4e$ fm^{-3} and $\sigma_0 = 50$ MeV$\cdot$fm^{-2} which, as we shall argue below, are typical scales for the quantities. [A droplet of symmetric nuclear matter in vacuum has a surface tension $\sigma = 1$ MeV$\cdot$fm^{-2} for which (5) gives $R \simeq 4$ fm, which agrees with the fact that nuclei like ^{56}Fe are the most stable form of matter for roughly symmetrical nuclear matter at low density.] The form of Eq. (5) reflects the fact that on dimensional grounds, the characteristic length scale is $[\sigma/(\rho_Q - \rho_N)^2]^{1/3}$ times a function of f. The total Coulomb and surface energy per unit volume is given for small f by

$$\epsilon_{S+C} = f\,9\left(\frac{\pi}{15}\sigma^2(\rho_Q - \rho_N)^2\right)^{1/3} \simeq (44\text{ MeV}\cdot\text{fm}^{-3})\,f\left(\frac{\sigma}{\sigma_0}\frac{\rho_Q - \rho_N}{\rho_0}\right)^{2/3} . \tag{6}$$

The result for f close to unity is given by replacing f by $1 - f$. Next we consider the case when the volumes of quark and nuclear matter are equal, $f = 1/2$. We approximate the structure as alternating layers, with thickness $2a$, of quark and nuclear matter. The surface energy per unit volume is $\sigma/(2a)$ and the Coulomb energy is $(2\pi/3)(\rho_Q - \rho_N)^2a^2$, and therefore the equilibrium value of a is

$$a = \left(\frac{3}{2\pi}\frac{\sigma}{(\rho_Q - \rho_N)^2}\right)^{1/3} \simeq (4.7\text{ fm})\left(\frac{\sigma}{\sigma_0}\right)^{1/3}\left(\frac{\rho_Q - \rho_N}{\rho_0}\right)^{-2/3} , \tag{7}$$

and the surface and Coulomb energy per unit volume is given by

$$\epsilon_{S+C} = \left(\frac{9\pi}{32}\sigma^2(\rho_Q - \rho_N)^2\right)^{1/3} \simeq (8\text{ MeV}\cdot\text{fm}^{-3})\left(\frac{\sigma}{\sigma_0}\frac{\rho_Q - \rho_N}{\rho_0}\right)^{2/3} . \tag{8}$$

To estimate length scales and energy densities, we need the surface tension of quark matter and the charge densities in the two phases. A rough estimate of the surface tension is the bag constant B times a typical hadronic length scale ~1 fm. Estimates of the bag constant range from 50 to 450 MeV$\cdot$fm^{-3} [6]. The kinetic contribution to the surface tension at zero temperature has been calculated in the bag model in Ref. [7]. Only massive quarks contribute because relativistic particles, unlike nonrelativistic ones, are not excluded near the surface due to the boundary conditions. The kinetic contribution to σ from a quark species depends strongly on its mass and chemical potential. For $m_s \ll \mu_s$ it behaves as $(3/4\pi^2)\mu_s^2 m_s$, and it vanishes as m_s approaches μ_s. If we adopt for the strange quark mass the value $m_s \simeq 150$ MeV, and for the quark chemical potentials one-third of the baryon chemical potential, which generally is slightly larger than the nucleon mass, $\mu_s \simeq \mu_B/3 \gtrsim m/3$, we obtain from Ref. [7] $\sigma \simeq 10$ MeV$\cdot$fm^{-2}, which is close to the maximum value it can attain for any choice of m_s. We conclude that the surface tension for quark matter is poorly known, but lies most probably in the range 10–100 MeV$\cdot$fm^{-2}. [Lattice gauge theory estimates of σ at high temperatures and zero quark chemical potentials lie in the range $\sigma \simeq (0.14–0.28)T_c^3 \sim 10–60$ MeV$\cdot$fm^{-2} [8] for $T_c \sim 150–200$ MeV, comparable to our estimates for cold quark matter, but it is unclear to what extent this agreement is accidental.]

We turn now to charge densities. Consider quark matter immersed in a uniform background of electrons. β equilibrium insures that $\mu_d = \mu_s = \mu_u + \mu_e$, and therefore in the absence of quark-quark interactions, one finds the total electric charge density in the quark matter phase is given for $\mu_e \ll \mu_u \sim \mu_d \equiv \mu_q$ and $m_s \ll \mu_q$ by

$$\rho_Q = \frac{e}{3}(2n_u - n_d - n_s - 3n_e) \simeq \frac{e}{\pi^2}\left(\frac{1}{2}m_s^2\mu_q - 2\mu_e\mu_q^2\right) . \tag{9}$$

Assuming $m_s \simeq 150$ MeV and $\mu_q \simeq m/3$ the second term dominates except for small μ_e and so the droplet is negatively charged and for $\mu_e \simeq 170$ MeV the density is about $-0.4e$ fm^{-3}, the characteristic scale of densities adopted in making estimates above.

Because of the high quark density, ρ_N is small compared with ρ_Q in Eq. (5) when quark matter occupies a small fraction of the volume. The electron chemical potential in neutron stars depends strongly on the model for the nuclear equation of state, but generally one finds $\mu_e \lesssim 170$ MeV. Consequently, for $\sigma \simeq 10$ MeV·fm^{-2} we find from Eq. (5) a radius of $R \gtrsim 3.1$ fm, whereas $\sigma \simeq 100$ MeV·fm^{-2} gives $R \gtrsim 6.6$ fm. For f close to unity one finds nuclear bubble radii which are comparable with those for quark droplets, and for the layerlike structures expected for $u \simeq 0.5$, half the layer thickness is of comparable size. Estimates of characteristic scales for rodlike structures give similar values.

Detailed calculations show that the effects of nonuniformity of the charge distribution affect estimates of Coulomb energies significantly if the characteristic lengths R and a exceed the Debye screening length. The estimates of screening lengths made above show that screening will be not be dominant for surface tensions below about 100 MeV, if the charge-density difference is ρ_0, but for higher values the simple picture of coexisting uniform bulk phases would become invalid, and the droplet phase would increasingly resemble two electrically neutral phases in equilibrium.

We now consider whether the droplet phase has a lower energy than two coexisting phases, each of which is electrically neutral. The energy of the droplet phase is the sum of the bulk contributions, plus the surface and Coulomb energies calculated earlier. Our approach is first to calculate the bulk energy, and then to estimate the surface and Coulomb energies. We adopt a simple form for the energy density of nuclear matter consisting of a quadratic compressional term, a symmetry term, and an electron energy density:

$$\begin{aligned}\epsilon_N &= n[m + E_{\mathrm{comp}}(n) + S(n)(1-2x)^2] + \epsilon_e \\ &= n\left[m + \frac{K_0}{18}\left(\frac{n}{n_0} - 1\right)^2 + S_0\left(\frac{n}{n_0}\right)^\gamma (1-2x)^2\right] \\ &\quad + \frac{\mu_e^4}{12\pi^2}.\end{aligned} \tag{10}$$

Here n is the baryon density, $n_0 = 0.16$ fm^{-3} is the nuclear saturation density, and x is the proton fraction. The compressibility we choose as $K_0 \simeq 250$ MeV and for the symmetry term we take that of Ref. [9] with $S_0 \simeq 30$ MeV and $\gamma \simeq 1$. The electron chemical potential is never much above the muon mass and therefore muons may be ignored. For quark matter we assume the bag model equation of state,

$$\epsilon_Q = \left(1 - \frac{2\alpha_s}{\pi}\right)\left(\sum_{q=u,d,s} \frac{3\mu_q^4}{4\pi^2}\right) + B + \frac{\mu_e^4}{12\pi^2} , \tag{11}$$

with the QCD fine structure constant $\alpha_s \simeq 0.4$ and bag constant $B \simeq 120$ MeV·fm^{-3}. We have taken all quark masses to be zero. In the absence of surface and Coulomb effects the equilibrium conditions for the droplet phase are that the quark and nuclear matter should have equal pressures, and that it should cost no energy to convert a neutron or a proton in nuclear matter into quarks in quark matter. The last condition amounts to $\mu_n = 2\mu_d + \mu_u$ and $\mu_p = \mu_d + 2\mu_u$. The electron density is the same in quark and nuclear matter, and we assume that matter is electrically neutral and in β equilibrium, that is $\mu_n = \mu_p + \mu_e$ and $\mu_d = \mu_u + \mu_e$. The chemical potentials are related to the Fermi momenta by $\mu_q = p_{F,q}(1 - 2\alpha_s/\pi)^{-1/3}$. Electrons contribute little to pressures, but they play an important role through the β equilibrium and charge neutrality conditions.

Figure 1(a) shows the density dependence of the energy density of the droplet phase calculated neglecting surface and Coulomb energies ($\sigma = 0$). The energy of uniform, electrically neutral bulk nuclear matter in β equilibrium is also shown, together with the corresponding result for quark matter. The double-tangent construction gives the energy density for densities at which the two bulk neutral phases coexist. This corresponds to the standard treatment of the phase transition between nuclear matter and quark matter, in which the pressure remains constant throughout the transition, and consequently neutron stars have a core of quark matter and a mantle of nuclear matter, with a sharp density discontinuity at the phase transition. As one sees, if surface and Coulomb effects may be ignored, the transition from nuclear matter to the droplet phase occurs at a lower density than the transition to two bulk neutral phases, a feature also apparent in Ref. [3]. In addition, droplets of nuclear matter survive up to densities above those at which bulk neutral phases can coexist. We also observe that bulk contributions to the energy density of the droplet phase are always lower than those for coexisting bulk neutral phases. While detailed properties of the droplet phase depend strongly on the bulk energies, the qualitative picture we find persists over a wide range of possible bulk matter properties.

We now estimate surface and Coulomb energies. When quark matter occupies a small fraction of space, f, one can show that the difference in energy between the droplet phase and bulk neutral nuclear matter varies as

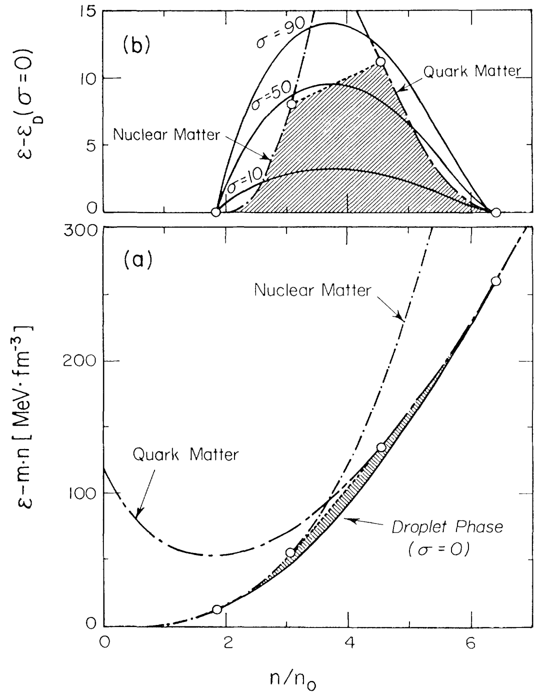

FIG. 1. (a) The full line gives the energy density of the droplet phase without surface and Coulomb energies ($\sigma = 0$). Also shown are the energy densities of electrically neutral bulk nuclear matter, quark matter in β equilibrium, and the double-tangent construction (dashed line) corresponding to the coexistence of bulk electrically neutral phases. (b) Energy densities of the droplet phase relative to its value for $\sigma = 0$ for $\sigma = 10$, 50, and 90 MeV$\cdot$fm^{-2}. When the energy density of the droplet phase falls within the hatched area it is energetically favored.

f^2. In contrast to this, the contributions to the energy density from surface and Coulomb energies are linear in f [see Eq. (6)]. Similar results apply for f close to unity. This shows that the transitions to the droplet phase must occur via a first-order transition. However, if the surface and Coulomb energies are sufficiently large, the droplet phase may never be favorable. The energy-density difference between the droplet phase, neglecting surface and Coulomb effects, and two coexisting neutral phases is at most 10 MeV$\cdot$fm^{-3}, as may be seen from Fig. 1. This is very small compared with characteristic energy densities which are of order 1000 MeV$\cdot$fm^{-3}. In Fig. 1(b) we show the energy density of the droplet phase for various values of the surface tension, relative to the value for $\sigma = 0$. In these calculations the geometry of the droplets was characterized by a continuous dimensionality d as described in Ref. [5], with $d = 3$, 2, and 1 corresponding to spheres, rods, and plates, respectively. For the droplet phase to be favorable, its energy density must lie below those of nuclear matter, quark matter, and coexisting electrically neutral phases of nuclear and quark matter. That is, the droplet phase will be favored if its energy lies within the hatched region in Figs. 1(a) and 1(b). We see that whether or not the droplet phase is energetically favorable depends crucially on properties of quark matter and nuclear matter. For our model the droplet phase is energetically favorable at some densities provided $\sigma \lesssim 70$ MeV$\cdot$fm^{-2}. However, given the large uncertainties in estimates of bulk and surface properties, one cannot at present claim that the droplet phase is definitely favored energetically.

Should the quark-droplet phase exist in neutron stars, it could have important observational consequences. First, as Glendenning showed, the pressure difference across the droplet phase can be large, of order 250 MeV$\cdot$fm^{-3}. This is also seen from Fig. 1(a), since the pressure is the negative intercept of the tangent to the curve. Consequently a large portion of a neutron star could consist of matter in the droplet phase. Second, phases with isolated droplets would be expected to be solid. The melting temperature is $\sim Z^2e^2f^{1/3}/(170R)$ [10], typically some hundreds of MeV, while spaghettilike and lasagnalike structures would exhibit anisotropic elastic properties, being rigid to some shear strains but not others in much the same way as liquid crystals. This could be important for quake phenomena, which have been invoked to explain observations in a number of different contexts. Third, neutrino generation and hence cooling of neutron stars could be influenced. This could come about because nuclear matter in the droplet phase has a higher proton concentration than bulk, neutral nuclear matter, and this could make it easier to attain the threshold condition for the nucleon direct Urca process [9]. Another is that the presence of the spatial structure of the droplet phase might allow processes to occur which would be forbidden in a translationally invariant system. Finally, one should bear in mind the possibility that even if the droplet phase were favored energetically, it would not be realized in practice if the time required to nucleate were too long.

To summarize, we have shown that whether or not a droplet phase consisting of quark matter and nuclear matter can exist in neutron stars depends not only on bulk properties, but also on the surface tension. In order to make better estimates it is important to improve our understanding of the transition between bulk nuclear matter and bulk quark matter. For the droplet phase to be possible, this must be first order. If the transition is indeed first order, better estimates of the surface tension are needed to determine whether the droplet phase is favored energetically.

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