New Mechanism for Electron Heating in Shocks

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Electron trajectories diverge exponentially in a sufficiently small-scale electrostatic field with a static external magnetic field. This electron trajectory instability results in electron heating in the field structure typical for the collisionless shock front ramp. The magnitude of the effect is sufficient to explain the observed heating at the Earth's bow shock and interplanetary shocks. The heating features are consistent with the observations.

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Collisionless shocks are one of the fundamental phenomena in plasma physics, space physics, and astrophysics. The problem of electron heating is one of the main problems of collisionless shock physics, since the heating is an important mechanism of the direct flow energy redistribution and is intimately related to the shock structure and formation process. Shock heated electrons are believed to be responsible for the radiation of a number of various astrophysical objects, such as supernova remnants, "hot spots" in jets, etc. The problem of electron heating at quasiperpendicular collisionless shocks has attracted attention for some time. A large body of data has been collected in *in situ* satellite experiments. A variety of mechanisms have been proposed, based on the electron interaction with different modes and instabilities (see, e.g., [1]). However, all of them encounter difficulties when compared with the experimental data; either the effective collision frequency is too low or the turbulence and heating regions are clearly separated spatially. In [2] it was shown that reversible electron dynamics in macroscopic quasisteady fields in the shock is apparently responsible for the heating magnitude and electron distribution shape. Adiabatic heating appears to be insufficient for most observed shocks [3]. Quasiparallel electric field acceleration and subsequent energy redistribution among the parallel and perpendicular degrees of freedom [4] was proposed as a heating mechanism. However, data show no correlation between the heating and the angle between the shock normal and the upstream magnetic field [3,5] as could be expected in this case.

In this paper we present a brief analysis of the new mechanism of the electron heating in the quasiperpendicular collisionless shock front, based on the phenomenon of the electron trajectories instability [6] in the typical shock front stationary field structure. The idea of the mechanism was proposed in [6–9]. The mechanism differs in principle from all the mechanisms proposed earlier, since it is not related to any kind of turbulence but is kinematic by its nature. We define temperature for any particle distribution simply as follows:

$$\frac{3}{2}T = \frac{m}{2}\langle(\mathbf{v} - \langle\mathbf{v}\rangle)^2\rangle = \frac{m}{2n}\int f(\mathbf{v})(\mathbf{v} - \langle\mathbf{v}\rangle)^2\, d\mathbf{v}\,, \tag{1}$$

where angle brackets denote averaging. This definition works even in the case when the distribution is not Maxwellian. Heating, therefore, can be described as an effective increase of the velocity-space volume, occupied by electrons. The electron trajectory instability [6] can be responsible for this increase. It is shown below that this effect occurs in the electric field, transverse to the magnetic field, when the electric field gradient is sufficiently large. These electric field gradients in the supercritical quasiperpendicular shock front ramp are produced in the steepening process [7–11]. Laboratory observations showed the existence of such fields in the form of isomagnetic jump [12]. There is also space observational evidence of large electric field gradients at the terrestrial bow shock front [13].

Electron motion in a perpendicular shock front is governed by the equations of motion

$$\dot{v}_x = -\frac{e}{m}E_x(x,t) - \frac{e}{mc}v_y B_z(x,t)\,, \tag{2}$$

$$\dot{v}_y = -\frac{e}{m}E_y + \frac{e}{mc}v_x B_z(x,t), \quad E_y = \text{const}\,. \tag{3}$$

It is impossible to solve the equations exactly in the general case. Instead, we analyze the variations of the velocity-space volume, occupied by the electrons. For this we consider the behavior of two trajectories $(\mathbf{r}_1, \mathbf{v}_1)$ and $(\mathbf{r}_2, \mathbf{v}_2)$ which are assumed to be initially close to one another. We expect coordinates and velocities to be smooth functions of time (far from possible separatrix), so that one can linearize the equations of motion at small times. It is straightforward to obtain the following equations for $\delta\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $\delta\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$:

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$$\dot{\delta x} = \delta v_x \ , \tag{4}$$

$$\dot{\delta y} = \delta v_y \ , \tag{5}$$

$$\dot{\delta v_x} = -\frac{e}{m}\frac{\partial E_x}{\partial x}\delta x - \Omega \delta v_y - \frac{\partial \Omega}{\partial x}\delta x \, v_y \ , \tag{6}$$

$$\dot{\delta v_y} = +\Omega \delta v_x + \frac{\partial \Omega}{\partial x}\delta x \, v_x \ . \tag{7}$$

Here we are searching for the time dependence faster than that related to the field variations, so we restricted ourselves with the first terms in the Taylor expansion and treat all the coefficients, $\partial E_x/\partial x$, $\partial \Omega \partial x$, and $\Omega = eB_z/mc$, as approximately constant.

The set is linear and $\delta x, \delta \mathbf{v} \propto \exp(\lambda t)$ with

$$\lambda^3 + \lambda \left(\frac{e}{m}\frac{\partial E_x}{\partial x} + v_y \frac{\partial \Omega}{\partial x} + \Omega^2 \right) + \Omega \, v_x \frac{\partial \Omega}{\partial x} = 0 \ . \tag{8}$$

When a solution $\lambda > 0$ exists the trajectories diverge exponentially.

If the terms of the form $v\partial\Omega/\partial x \ll \Omega^2$ they can be neglected and the instability criterion takes the form

$$\frac{\lambda^2}{\Omega^2} = \frac{e}{m\Omega^2}\left| \frac{\partial E_x}{\partial x} \right| - 1 > 0 \tag{9}$$

if $\partial E_x/\partial x < 0$ (remember that $E_x < 0$).

We estimate the ratio of the terms $v(\partial\Omega/\partial x)$ and Ω^2 as follows:

$$v\frac{\partial \Omega}{\partial x} \Big/ \Omega^2 \sim \frac{v_T}{\Omega L} < \frac{v_T}{v_A}\sqrt{\frac{m_e}{m_i}} \ , \tag{10}$$

where it is assumed that the typical scale $L \geq c/\omega_{pe}$, and the typical velocity v_T is in fact thermal velocity. Therefore, when $v_T/v_A < \sqrt{m_i/m_e} \approx 42$ the term Ω^2 dominates. In the opposite case $v_T \partial\Omega/\partial x > \Omega^2$, $(e/m)|\partial E_x/\partial x|$ the instability should cease and the adiabatic heating mechanism should work. When $v_T \partial\Omega/\partial x \sim (e/m)|\partial E_x/\partial x| - \Omega^2$ the instability criterion is smeared out and the trajectory can diverge even if (9) is not satisfied. In this case the heating (see below) will be above the adiabatic value but considerably weaker than when (9) is satisfied.

The found trajectory instability is known to indicate possible dynamical chaos in the system [14]. In the case when the magnetic field is constant λ is a Lyapunov exponent. A positive Lyapunov exponent is one of the basic indications of stochastic dynamics [14]. Thus, a transition to stochastic dynamics occurs in our Hamiltonian system. We emphasize that this transition is due to the interaction with a nonperiodic stationary regular field structure.

It can be shown directly that the trajectory divergence results in an increase of the differential velocity space volume $dv_x \, dv_y$ and effective heating should be expected.

The analytical analysis shows the possibility of the kinematic heating due to the electron trajectory divergence in a strong electric field gradient with an external magnetic field and provides the heating criterion in the form (9). Quantitative analytical calculation of the temperature increase is very complicated because of difficulties in converting time dependence into space dependence. Here we numerically determine some quantitative features of the heating mechanism.

The goal of the simulation is a simple and clear demonstration of the described mechanism's capability of producing a substantial temperature increase, comparable to the observed one. Since the field structure in the shock front is determined by the ions primarily, we study the electron motion in a given stationary field structure as test particles, neglecting the back influence on the fields. It should be mentioned that a stationary problem is analyzed in the simulation. In the simulations the dimensionless form of the equations of motion was used, where time was measured in Ω_e^{-1}, coordinate x was measured in c/ω_{pe}, velocities were measured in v_A, and the magnetic field was measured in the upstream magnetic field B_u. The model dimensionless magnetic field is chosen as follows: $B = 1$ for $x < -D$, $B = 3$ for $x > D$, and

$$B = 2 + 0.125(3z^5 - 10z^3 + 15z), \qquad z = x/D \, , \tag{11}$$

in order to produce necessary smoothness and exclude undesirable numerical edge effects.

The electric field $E_y = V_u B_u/c$ is constant. The electric field E_x is zero outside the region $|x| < D$, while inside it is chosen as follows:

$$E_x = \Delta\phi_0 15(z^2 - 1)^2/16D \ . \tag{12}$$

The chosen form of the electric field is predicted by the two-fluid hydrodynamics. The potential drop across the shock ramp $\Delta\phi$, however, is not determined by theory and is taken from the experimental data. The field geometry is shown in Fig. 1.

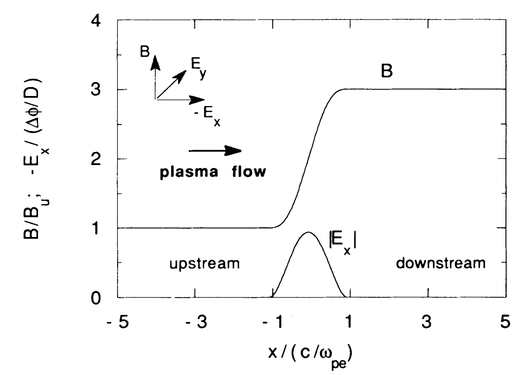

FIG. 1. The model magnetic field B_z and electric field E_x are shown. The electric field E_y is constant and is not shown. The mutual directions of the components are shown in the top of the figure.

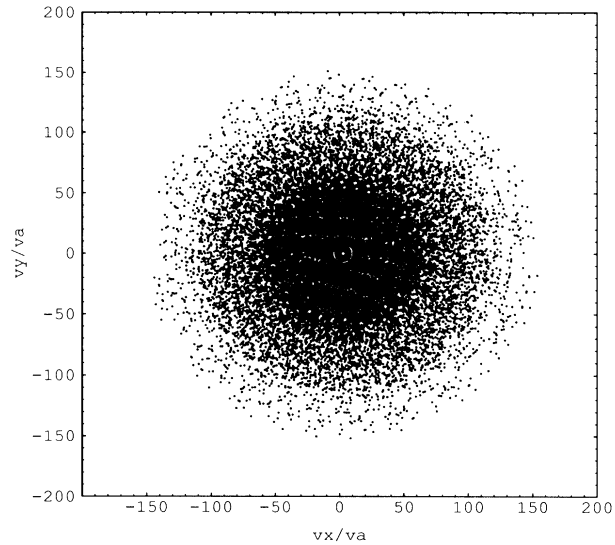

FIG. 2. Far downstream electron distribution in the v_x, v_y plane for $D = 7$, $e\Delta\phi = 500$ eV. Clear adiabatic heating is seen with final temperature $T_d = 30$ eV.

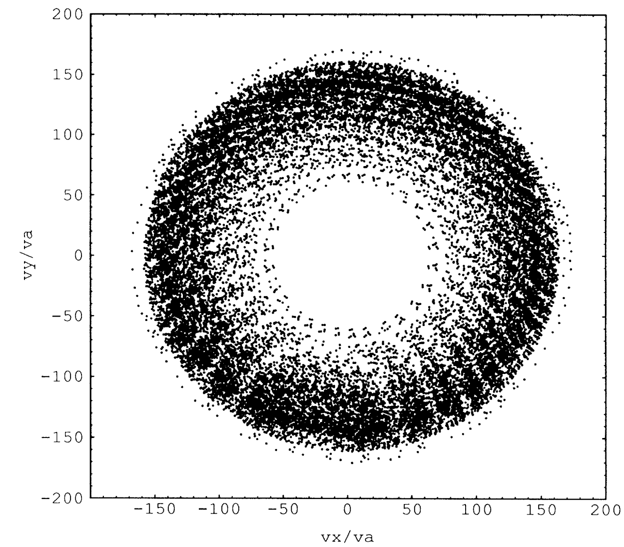

FIG. 3. Far downstream electron distribution in the v_x, v_y plane for $D = 3$, $e\Delta\phi = 500$ eV. Clear trajectory instability heating is seen with final temperature $T_d = 125$ eV.

The basic parameters are upstream magnetic field $B_u = 5 \times 10^{-5}$ G, upstream temperature $T_e = 10$ eV, upstream plasma velocity $V_u = 400$ km/s, and upstream density $n = 5$ cm^{-3}. For these parameters the Mach number is $M = 8$. The parameters are in the range typical for observed shocks.

In the simulations the Maxwellian distribution for the initial 1000 points was generated far upstream. Each particle was traced using the equations of motion. Remember that the stationary problem is analyzed. In the far downstream, where the distribution is already independent of the coordinate x, the v_x,v_y distribution is constructed and the temperature calculated as an average according to (1).

In Fig. 2 the final distribution for $D = 7$, $e\Delta\phi = 500$ eV is shown. In this case the threshold (9) is not achieved and the heating is purely adiabatic: the final temperature is $T_d = 30$ eV$= T_u(B_d/B_u)$.

In Fig. 3 the final distribution for the case of instability $D = 3$, $e\Delta\phi = 500$ eV is shown. The final temperature $T_d = 125$ eV is well above the adiabatic value. One can expect that the ringlike distribution is unstable and after relaxation the temperature will be lower. However, one can expect that the final temperature will be not less than half the value obtained in the simulation.

The final temperatures for a number of runs with different model potentials $\Delta\phi$ and ramp width D are shown in Fig. 4 together with the threshold contour. Under the threshold, adiabatic heating is clearly distinguished from the strong trajectory instability heating above the threshold (9). Note several cases of the instability threshold smearing. One can see that T_d monotonically increases with increasing potential and $D =$ const and when D decreases while the potential is constant. Figure 4 shows that the heating mechanism based on the electron trajectory instability can provide the whole spectrum of the observed downstream temperatures at planetary bow shocks and interplanetary shocks (cf. [3, 5]).

Several runs were done with different initial temperatures. It was found that the heating efficiency decreases with the initial temperature increase and it becomes adiabatic for the initial temperatures larger than $T_u = 30$ eV. This result is in good agreement with the observed feature [3].

In conclusion, we discovered the electron trajectory in-

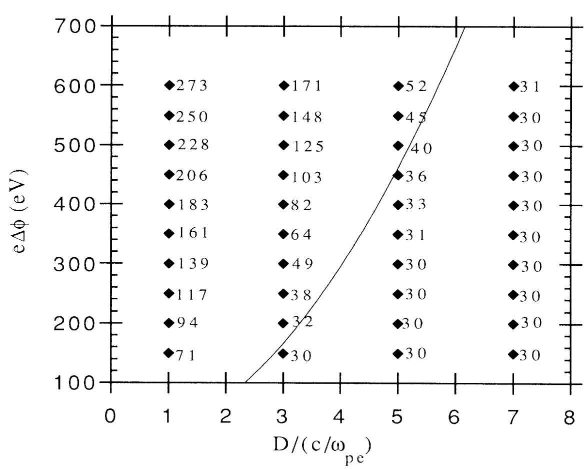

FIG. 4. Final temperatures for various ramp widths D (in c/ω_{pe} units) and cross-potential drops $e\Delta\phi$ (in eV) in the D-$\Delta\phi$ plane. Initial temperature $T_u = 10$ eV, $B_d/B_u = 3$. The threshold (9) is contoured.

stability and showed that a transition to the stochastic dynamics can occur in a regular electromagnetic field structure. The trajectories diverge in a sufficiently steep electrostatic field with the external magnetic field. Such a field structure is typical for quasiperpendicular collisionless shocks fronts. The instability makes it possible to transfer a substantial part of the cross-shock potential energy into the electron thermal energy, thus providing an effective mechanism of heating in *regular* stationary fields. It is shown that such heating in shock fronts is sufficiently strong to produce the heating values that are observed at the terrestrial bow shock and interplanetary shocks. The results can also be applied to electron heating in astrophysical shocks and other nonlinear structures.

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