Critical Current of Josephson-Coupled Systems in Perpendicular Fields

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We calculate the magnetic field and temperature dependence of the critical current density j_c along the c axis in Josephson-coupled layered systems when the applied magnetic field is parallel to the c axis. j_c decreases with field and temperature because of the thermal fluctuations of the vortices which induce a phase difference across the junctions. The anisotropy ratio $\gamma \propto 1/\sqrt{j_c}$ acquires a field and temperature dependence. The decoupling phase transition line above which the superconducting current along the c axis vanishes is determined.

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High-T_c layered superconductors are well described by the Lawrence-Doniach (LD) model, the essence of which is the Josephson nature of the interlayer coupling. The value of the interlayer critical current j_c can be used as a measure of the coupling strength and, as such, determines the 3D electrodynamic and thermodynamic properties of these compounds. Below we show that at high temperatures, when a magnetic field is applied perpendicular to the layers, j_c is strongly suppressed. Indeed, recent measurements of the critical current of Bi- and Tl-based tapes at high temperature have shown a rapid decrease (approximately exponential) as a function of magnetic field when the magnetic field is applied along the c axis [1–3]. Although the nominal current direction in these measurements is in the a-b plane, the platelike morphology and known Josephson-junction behavior of high angle grain boundaries has suggested the "brick-wall" model in which intergranular current transfer is across large area c-axis Josephson junctions [4, 5]. This implies that measured critical currents are determined by c-axis currents flowing parallel to the applied field between the platelets. In single crystals a suppression of the supercurrent along the c axis when the magnetic field is parallel to the c axis was seen in resistivity measurements [6–8] and was interpreted [7] as a decoupling of the layers due to the application of a magnetic field.

These experimental facts can be accounted for if we consider that the interlayer Josephson critical current density j_c in perpendicular fields has to be renormalized. In other words, due to the nonlinear dependence of the current on the gauge-invariant phase difference the interlayer Josephson critical current j_c becomes field and temperature dependent.

We believe the physical mechanism responsible for the renormalization of the critical current in the c direction to be the following. A magnetic field applied parallel to the c axis will create "pancake" vortices in the a-b planes [9]. If the pancake vortices are aligned from one layer to the next and the vortices are straight, parallel stacks of pancake vortices, they do not generate a difference in the phase of the order parameter from one layer to the next and do not suppress the critical current [10]. At large temperature, however, the pancake vortices are no longer aligned from one layer to the next because of thermal disorder. The degree of misalignment depends on the elastic properties of the flux lattice. The misalignment between pancake vortices in adjacent layers generates a phase difference which depresses the critical current density locally [11]. As a result, the thermally averaged j_c is depressed (and consequently the penetration length for the current along the c axis $\lambda_c \propto j_c^{-1/2}$ and anisotropy ratio $\gamma = \lambda_c/\lambda_{ab}$ increase). This modifies the elastic constants (which depend on j_c via λ_c) and therefore the ability of the flux lattice to withstand thermal distortions. Thus, j_c must be calculated self-consistently. The reduction in critical current caused by the thermal fluctuations increases with magnetic field (i.e., with vortex density) and temperature. The corresponding self-consistency equation for the critical current shows that the supercurrent along the c axis vanishes above some decoupling field, $B_D(T)$. Recently, Glazman and Koshelev [12] concluded that the thermal fluctuations of the pancake vortices suppress the superconducting long-range order in the direction of the applied field causing the decoupling of the layers, but they did not study the behavior of the critical current or the anisotropy parameter.

In the following we consider a layered superconductor in a magnetic field perpendicular to the layers. This external field is such that $H_{c1} \ll H \ll H_{c2}$. Our analysis is restricted to high temperatures, or, more precisely, to the reversible region of the H-T plane so that pinning effects are unimportant. Within the framework of the LD model, we evaluate the dependence of the critical current density along the c axis on magnetic field and temperature. We then show how our results apply to superconducting tapes and how they complement the brick-wall model [4] when the magnetic field is applied along the c axis.

In the LD formalism, the interlayer Josephson current density is given by

$$j_{z;n,n+1}(\boldsymbol{\rho}) = j_0 \sin[\varphi_{n,n+1}(\boldsymbol{\rho})], \qquad (1)$$

$$\varphi_{n,n+1}(\boldsymbol{\rho}) = \phi_n - \phi_{n+1} - \frac{2\pi}{\Phi_0}\int_{ns}^{(n+1)s} A_z(\mathbf{r})dz, \qquad (2)$$

where $\varphi_{n,n+1}(\boldsymbol{\rho})$ is the gauge-invariant phase difference. $\phi_n(\boldsymbol{\rho})$ is the phase of the order parameter in the nth layer, s is the interlayer spacing, and $A_z(\mathbf{r})$ is the c-axis component of the vector potential. $\boldsymbol{\rho} = (x, y)$ designates coordinates in the a-b plane and $\mathbf{r} = (\boldsymbol{\rho}, z)$. In the presence of vortices, the gauge-invariant phase difference can be obtained easily if we neglect the Josephson current between the layers. (We will comment on this approximation later.) The phase difference is then given by the solution to the two-dimensional Laplace equation [13]:

$$\nabla_\rho^2\, \varphi_{n,n+1}(\boldsymbol{\rho}) = 0. \tag{3}$$

The solution to Eq. (3) must obey the boundary condition that the line integral of $\nabla_\rho \varphi_{n,n+1}(\boldsymbol{\rho})$ along a closed contour encircling the vortex be equal to 2π. This solution is

$$\varphi_{n,n+1}(\boldsymbol{\rho}) = \sum_{\mathbf{m}} \left(\arctan \frac{x - x_{n\mathbf{m}}}{y - y_{n\mathbf{m}}} - \arctan \frac{x - x_{n+1,\mathbf{m}}}{y - y_{n+1,\mathbf{m}}} \right), \tag{4}$$

where $\boldsymbol{\rho}_{n\mathbf{m}}$ is the position of a pancake vortex in the nth layer and $\mathbf{m}$ is a set of two indices labeling the pancake vortices in a given layer.

In order to determine the effect of the thermal motion of the pancake vortices on the total critical current, $I_{n,n+1}$ between layers n and $n+1$, we evaluate the following statistical average:

$$I_{n,n+1} = j_0 \left| \int d\boldsymbol{\rho}\, \langle \exp[i\varphi_{n,n+1}(\boldsymbol{\rho})] \rangle \right| . \tag{5}$$

Here $\langle \cdots \rangle$ denotes thermal averaging:

$$\langle A \rangle = \frac{\int \mathcal{D}\boldsymbol{\rho}_{n,\mathbf{m}}\, A\{\boldsymbol{\rho}_{n,\mathbf{m}}\} \exp(-\beta \mathcal{F}\{\boldsymbol{\rho}_{n\mathbf{m}}\})}{\int \mathcal{D}\boldsymbol{\rho}_{n\mathbf{m}} \exp(-\beta \mathcal{F}\{\boldsymbol{\rho}_{n\mathbf{m}}\})}, \tag{6}$$

where $\mathcal{F}\{\boldsymbol{\rho}_{n\mathbf{m}}\}$ is the free energy functional of the lattice in terms of pancake coordinates. To carry out the integration in Eq. (5) explicitly we describe the flux lattice distortions in the harmonic approximation. Let $\mathbf{u}_{n\mathbf{m}}$ be the displacement of a pancake vortex from its equilibrium position $\boldsymbol{\rho}_{\mathbf{m}}$ in the Abrikosov lattice. In terms of the Fourier components $\mathbf{u}(q, \mathbf{k})$ of $\mathbf{u}_{n\mathbf{m}}$,

$$\mathbf{u}(q, \mathbf{k}) = \frac{s\Phi_0}{B} \sum_{n,\mathbf{m}}{}' \mathbf{u}_{n\mathbf{m}} \exp(i\mathbf{k}\cdot\boldsymbol{\rho}_{\mathbf{m}} + iqn), \tag{7}$$

the free energy functional is

$$\mathcal{F}\{\mathbf{u}_{n\mathbf{m}}\} = \frac{B}{2s\Phi_0} \sum_{q,\mathbf{k}} \sum_{i,j} (c_{11} k^2 \mathcal{P}_{L,ij} + c_{66} k^2 \mathcal{P}_{T,ij} + \delta_{ij} c_{44} Q^2) u_i(q,\mathbf{k}) u_j^*(q,\mathbf{k}), \tag{8}$$

where $\mathbf{k} = (k_x, k_y)$, $Q^2 = 2(1 - \cos q)/s^2$, and $i, j = x, y$. $\mathcal{P}_{L,ij} = k_i k_j / k^2$ and $\mathcal{P}_{T,ij} = \delta_{ij} - \mathcal{P}_{L,ij}$ are longitudinal and transverse projection operators. c_{66}, c_{11}, and c_{44} are the flux lattice shear, compression, and tilt moduli, respectively. Ignoring the thermal distortions, i.e., in the standard mean-field approach, they are given by [12, 14]

$$c_{66} = \frac{\Phi_0 B}{(8\pi\lambda_{ab})^2}, \quad c_{44} = \frac{B^2}{4\pi[1 + \lambda_c^2 k^2 + \lambda_{ab}^2 Q^2]} + \frac{B\Phi_0}{32\pi^2\lambda_c^2} \ln \frac{\xi_{ab}^{-2}}{K_0^2 + \lambda_J^{-2}} + \frac{B\Phi_0}{32\pi^2 \lambda_{ab}^4 Q^2} \ln\left(1 + \frac{Q^2}{K_0^2}\right),$$
$$c_{11} = \frac{B^2[1 + \lambda_c^2(k^2 + Q^2)]}{4\pi[1 + \lambda_{ab}^2(k^2 + Q^2)][1 + \lambda_c^2 k^2 + \lambda_{ab}^2 Q^2]}, \tag{9}$$

where $\lambda_c^2 = c\Phi_0 / 8\pi^2 s j_0$, and $\lambda_J = \gamma s$ is the Josephson length. For simplicity, the summation over $\mathbf{k}$ is performed over a circular Brillouin zone of radius $K_0^2 = 4\pi B/\Phi_0$. Beyond this approach the thermal fluctuations of the vortices affect the moduli c_{11} and c_{44} due to a weakening of the Josephson coupling between the layers. Now the moduli should be calculated from the free energy $F(\mathbf{u}_{i,n\mathbf{m}})$ in the presence of an imposed vortex distortion $\mathbf{u}_{i,n\mathbf{m}}$ as the second derivatives of F with respect to $\mathbf{u}_{i,n\mathbf{m}}$. In this case the terms in c_{11} and c_{44} which come from the Josephson part of the free energy functional $\mathcal{F}_J$ are obtained by expanding $\mathcal{F}_J$ in terms of $\mathbf{u}_{i,n\mathbf{m}}$,

$$\mathcal{F}_J\{\mathbf{u}_{t,n\mathbf{m}} + \mathbf{u}_{i,n\mathbf{m}}\} = \frac{\Phi_0 j_0}{2\pi c} \sum_n \int d\boldsymbol{\rho}\, [1 - \cos(\varphi_{n,n+1}\{\mathbf{u}_{t,n\mathbf{m}} + \mathbf{u}_{i,n\mathbf{m}}\})], \tag{10}$$

at given thermal distortions $\mathbf{u}_{t,n\mathbf{m}}$ [$\mathcal{F}_J$ should be integrated over $\mathbf{u}_{t,n\mathbf{m}}$ to obtain the free energy $F(\mathbf{u}_{i,n\mathbf{m}})$]. In the expansion of $\mathcal{F}_J$ in $\mathbf{u}_{i,n\mathbf{m}}$ the terms where j_0 appears are of the form $j_0 \cos[\varphi_{n,n+1}(\mathbf{u}_{t,n\mathbf{m}})]$. Using the self-consistent approach we replace this factor by its thermal average (below we show that this average, $j_0\langle \cos[\varphi_{n,n+1}(\mathbf{u}_{t,n\mathbf{m}})]\rangle$ depends weakly on $\boldsymbol{\rho}$). This means that in Eqs. (9) we replace j_0 by the effective critical current density $j_c(B)$ and λ_c by $\lambda_c(B)$ given by

$$\lambda_c^2(B) \equiv \frac{c\Phi_0}{8\pi^2 s j_c(B)}, \quad j_c(B) = \frac{I_{n,n+1}(B)}{\int d\boldsymbol{\rho}}, \tag{11}$$

where $I_{n,n+1}(B)$ is given by Eq. (5). Equations (5)–(11) allow one to obtain j_c and the effective anisotropy parameter $\gamma(B) = \lambda_c(B)/\lambda_{ab}$ self-consistently.

We note that $\lambda_c(B)$ defined by Eq. (11) indeed determines the penetration depth for the current along the c axis for a weak component of the external field $H_{e,x}$ parallel to the layers in the presence of a large component $H_{e,z}$ perpendicular to the layers. To show this we use Maxwell's equation for h_x:

$$\frac{\partial h_x}{\partial y} = \frac{\partial h_{t,x}}{\partial y} + \frac{\partial h_{e,x}}{\partial y} = \frac{4\pi}{c} j_0 \sin\left[\phi_n - \phi_{n+1} - \frac{2\pi s}{\Phi_0}(A_{t,z} + A_{e,z})\right], \tag{12}$$

where $h_{e,x} = \partial A_{e,z}/\partial y$ is induced by the component of the external field $H_{e,x}$, and $\mathbf{h}_t = \nabla \times \mathbf{A}_t$ is caused by the distortions of the vortices; the gauge $\nabla \cdot \mathbf{A} = 0$ is chosen. Expanding in $A_{e,z}$ and averaging over thermal distortions we obtain the usual London equation for $h_{e,x}$:

$$\frac{\partial^2 h_{e,x}}{\partial y^2} = \frac{8\pi^2 s}{c\Phi_0} j_0 \langle \cos[\varphi_{n,n+1}(\mathbf{u}_{t,nm})]\rangle h_{e,x} = \lambda_c^{-2}(h_{e,z}) h_{e,x}. \tag{13}$$

Here we used the same self-consistent approach as mentioned above for the moduli, namely, we replaced $\cos[\varphi_{n,n+1}(\mathbf{u}_{t,nm})]$ by its average value.

We simplify further the evaluation of $I_{n,n+1}$ by linearizing the gauge-invariant phase difference $\varphi_{n,n+1}$ in the displacements $\mathbf{u}_{n\mathbf{m}}$:

$$\varphi_{n,n+1}(\boldsymbol{\rho}) = \sum_{\mathbf{m}} (\mathbf{u}_{n\mathbf{m}} - \mathbf{u}_{n+1;\mathbf{m}}) \nabla_\rho \tan^{-1}\frac{x - x_{\mathbf{m}}}{y - y_{\mathbf{m}}}. \tag{14}$$

This expression is valid under the condition $|\boldsymbol{\rho}_{n+1,\mathbf{m}} - \boldsymbol{\rho}_{n,\mathbf{m}}| \ll |\boldsymbol{\rho} - \boldsymbol{\rho}_{n,\mathbf{m}}| \ll \lambda_J$. For distances $|\boldsymbol{\rho} - \boldsymbol{\rho}_{\mathbf{m}}|$ larger than λ_J, the Josephson current provides 3D screening and makes the phase difference smaller. In terms of $\mathbf{u}(\mathbf{k}, q)$, the phase difference can be expressed as

$$\varphi_{n,n+1}(\boldsymbol{\rho}) = \frac{B}{s\Phi_0} \sum_{\mathbf{k},q} e^{-iqn}(1 - e^{iq}) \boldsymbol{\mathcal{D}}(\boldsymbol{\rho}, \mathbf{k}) \mathbf{u}(q, \mathbf{k}), \tag{15}$$

where, for a square flux lattice,

$$\boldsymbol{\mathcal{D}}(\boldsymbol{\rho}, \mathbf{k}) = \frac{2\pi i B}{\Phi_0} \sum_{\mathbf{G}} \left(\frac{k_y + G_y}{|\mathbf{k} + \mathbf{G}|^2}\hat{\mathbf{x}} - \frac{k_x + G_x}{|\mathbf{k} + \mathbf{G}|^2}\hat{\mathbf{y}} \right) \times \exp[i(\mathbf{k} + \mathbf{G}) \cdot \boldsymbol{\rho}], \tag{16}$$

$\mathbf{G}$ are the reciprocal lattice vectors. After integration over $\mathbf{u}(q, \mathbf{k})$ in Eq. (5) the final result is

$$I_{n,n+1} = j_0 \int d\boldsymbol{\rho}\, \exp[-S(\boldsymbol{\rho})], \tag{17}$$

where

$$S(\boldsymbol{\rho}) = \frac{TB}{s\Phi_0} \sum_{\mathbf{k},q} \frac{1 - \cos q}{k^2} \left[\frac{|\mathcal{D}_x k_y - \mathcal{D}_y k_x|^2}{c_{66} k^2 + c_{44} Q^2} + \frac{|\mathcal{D}_x k_x + \mathcal{D}_y k_y|^2}{c_{11} k^2 + c_{44} Q^2} \right]. \tag{18}$$

We consider in the following the coordinate-independent term S_0 which is the contribution in Eq. (18) diagonal in reciprocal lattice vectors. The nondiagonal terms which determine the coordinate dependent part of $S(\boldsymbol{\rho})$ are much smaller than S_0. The sums over $\mathbf{k}$ and $\mathbf{G}$ in Eq. (18) diverge logarithmically because of the approximations used to obtain Eq. (14). Taking into account the condition $|\boldsymbol{\rho}_{n+1,\mathbf{m}} - \boldsymbol{\rho}_{n,\mathbf{m}}| \ll |\boldsymbol{\rho} - \boldsymbol{\rho}_{n,\mathbf{m}}| \ll \lambda_J$ under which Eq. (14) is valid, we see that the upper limit for the summation over $\mathbf{G}$ is $1/u_T$, where $u_T^2 = \langle(\mathbf{u}_{n+1,\mathbf{m}} - \mathbf{u}_{n,\mathbf{m}})^2\rangle$. Notice that the condition $u_T \ll \lambda_J$ is fulfilled because $u_T^2 \lesssim s(4\pi\lambda_c)^2 T/\Phi_0^2$. The lower limit for the summation over $\mathbf{k}$ is $1/\lambda_J(B) = \lambda_{ab}/s\lambda_c(B)$. The dependence of λ_J on j_c is another reason that j_c should be calculated self-consistently.

The self-consistency equation for the effective anisotropy parameter $\gamma(B)$ is

$$\gamma^2 = \gamma_0^2 \exp[S_0(\gamma)], \tag{19}$$

where γ_0 is the anisotropy ratio at $B = 0$. The solution to this equation shows that the c-axis critical current j_c decreases approximately exponentially with B up to some decoupling field $B_D(T)$ above which $j_c = 0$. When $B \gg \Phi_0/\lambda_J^2, \Phi_0/\lambda_{ab}^2$, the decoupling field coincides with the field at which the vortex lines dissociate into a gas of pancake vortices. This field is given by the condition $K_0^2 u_T^2 \approx 1$ [7,12,15]. For $B > B_D$ the pancake vortices in neighboring layers become almost uncorrelated and the c-axis critical current vanishes.

If $\xi_{ab}(0)/s \ll \gamma_0 \ll \lambda_{ab}/s$ [moderate anisotropy, $\xi_{ab}(T)$ is the correlation length [16]], the renormalization of the elastic moduli drives the decoupling phase transition. The decoupling field is given by

$$B_D(T) \approx \frac{\Phi_0^3}{16\pi^3 T s e \lambda_{ab}^2(T)\gamma_0^2} \quad (e = 2.718...)\ . \tag{20}$$

As B increases from zero to $B_D(T)$, the critical current decreases from j_0 to j_0/e and then drops suddenly to zero. The phase transition is first order with a latent heat (per unit volume) given by $\Delta Q = \Phi_0^2/32e\pi^3\lambda_{ab}^2 s^2\gamma_0^2$.

For large anisotropy, $\gamma_0 \gg \lambda_{ab}/s$, only the third term in c_{44} is important (it describes the electromagnetic interaction of pancake vortices in different layers). The renormalization of λ_J drives the transition, and the decoupling field is

$$B_D(T) \approx \frac{\Phi_0^3 s}{32\pi^3 T \lambda_{ab}^4(T)} \ln\left(\frac{8\pi^2 T\lambda_{ab}^4}{\Phi_0^2 s^3}\right). \tag{21}$$

As B increases from zero to $B_D(T)$, the critical current density decreases almost exponentially from j_0 to zero. All the thermodynamical quantities are continuous at the phase transition. The crossover from the moderate anisotropy behavior to the high anisotropy behavior occurs for a value of γ_0 equal to $\gamma_{\rm cr}$ of the order λ_{ab}/s. For the parameters used in Fig. 1, we estimated $\gamma_{\rm cr} \approx 60$.

The magnetic field dependence of the critical current and the temperature dependence of B_D are shown in Fig. 1 for $\gamma_0 = 30$ and 55. The decoupling field of approximately 2 T at $T = 30$ K obtained by Latyshev and Volkov [8] via critical current measurements is in reasonable agreement with our estimates. Notice that the decoupling line lies above the irreversibility line. This is consistent with our assumption that pinning is unimportant. Finally, although we have assumed an ordered vortex lattice for our calculations, the results obtained above can also be applied to the case of a flux liquid (if such a state exists) because j_c depends on c_{44} mainly

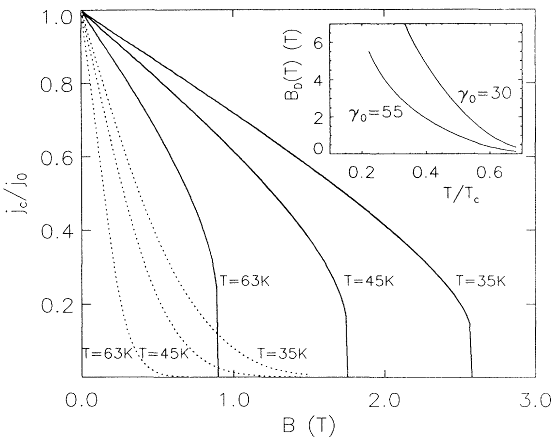

FIG. 1. Magnetic field dependence of the critical current along the c axis in a single crystal. The solid lines are for $\gamma_0 = 55$; the dashed lines are for $\gamma_0 = 200$. The inset shows the temperature dependence of the decoupling field for two values of γ_0 and $s = 15$ Å, $\lambda_{ab}(0) = 1500$ Å, and $\xi_{ab}(0) = 25$ Å.

while its dependence on c_{66} is very weak.

The same mechanism for the reduction of j_c applies to tapes. In superconducting tapes, the brick-wall model assumes that a transport current flowing along the tape is transferred from one brick to another along the c axis of the crystallites ("bricks") making up the tape. The difference between a single crystal and a tape is that the bricks have a finite length L along the a-b planes. Also, j_0 now has a value appropriate for the weak links between the bricks and is much smaller than the interlayer critical current density. Hence, as a rough approximation, we can use the results obtained for a single crystal in the limit $\gamma_0 \gg \lambda_{ab}/s$ replacing λ_J by L in the lower limit for the summation over $\mathbf{k}$ in Eq. (18) (we assume that $L < \lambda_J$). The critical current depends approximately

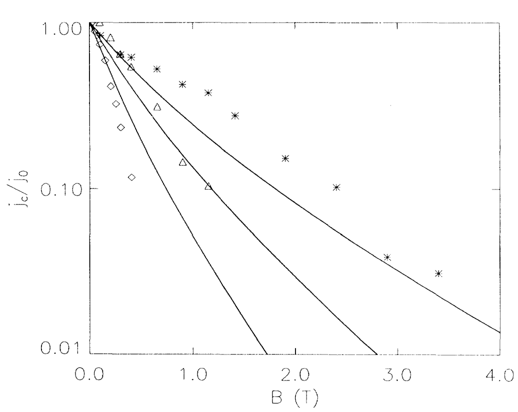

FIG. 2. Magnetic field dependence of the critical current of a superconducting tape Bi 2:2:2:3 for different temperatures and $\lambda_{ab}(0) = 1200$ Å, $s = 18$ Å, $L = 1.0$ μm, and $T_c = 110$ K. The experimental data were taken from Ref. [2].

exponentially on B and there is no decoupling transition because of the finite size of the system [17]. The magnetic field dependence of j_c/j_0 in the limit considered here (electromagnetic coupling) is shown in Fig. 2 for various temperatures.

In conclusion, we have obtained the temperature dependence of the decoupling field when the applied field is along the c axis in Josephson-coupled layered superconductors. In perpendicular fields, the anisotropy parameter is now field and temperature dependent because of the vortex fluctuations (contrary to the Ginzburg-Landau model for which γ is a constant).

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[17] If the crystal is larger than a certain critical dimension, R_c of the order of the Josephson length, self-field effects can lead to the penetration of vortices inside the sample. They limit the critical current along the c axis. More precisely, we have $2\pi j_c R_c/c = H_{c1,\parallel}$ in the absence of an applied magnetic field. The effect is important for single crystals, but is negligible for tapes where $L \lesssim R_c$.