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## SCREW INSTABILITY OF A PLASMA COLUMN

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In earlier experiments with the positive column an instability has been observed in the presence of a longitudinal magnetic field.<sup>1-3</sup> A theoretical explanation of the phenomenon has been given by Kadomtsev and Nedospasov<sup>4</sup> who discuss the growth of a screw-shaped disturbance in the plasma column. Recent experiments support the theory.<sup>5,6</sup> Even if the physical and mathematical assumptions of the theory seem to be well founded, the mechanism of the instability has not yet been fully explained. In particular, the role which the  $\vec{i} \times \vec{B}$  force plays is not very clear. In this Letter an attempt will be made to interpret the results physically.

According to the theory<sup>4</sup> the plasma is stable for a screw disturbance of the form  $\exp(ikz + im\psi - i\omega t)$  with  $m = 1$  if

$$KX^4 + FX^2 + G_d + G_r \geq 0.163 \frac{v_0}{\beta_0 D_e} X \frac{b_e}{b_i}, \quad (1)$$

where

$$K = \frac{1.28 + y}{y(y+1)}, \quad F = \frac{0.8(y+2)}{y},$$

$$G_d = \frac{0.48(1+y)}{y}, \quad G_r = 0.1 \frac{b_e/b_i}{1+y}, \quad (2)$$

and  $X = k\Omega\tau/\beta_1$ ,  $y = b_i(\Omega\tau)^2/b_e$ ,  $\Omega$  is the gyrofrequency,  $\tau$  the collision time,  $b_e$  the mobility,  $v_0$  the axial velocity, and  $D_e$  the diffusion coefficient of electrons. Further,  $b_i$  is the mobility of ions and  $\beta_0 a$  and  $\beta_1 a$  are the first zeros of  $J_0$  and  $J_1$  with  $a$  indicating the tube radius. The collision frequency of ions is assumed to be much larger than their gyrofrequency.

Examination of relation (1) and the basic equations used in the theory<sup>4</sup> shows that the right-hand member contains the imposed electric field and represents the effects which destabilize the system. The stabilizing effects are provided by the left-hand member. In the first term,  $KX^4$ , are included dissipation effects due to diffusion and conduction of electrons along the magnetic field  $\vec{B}_z$ . The term  $G_d$  represents corresponding effects in the transverse direction for electrons and ions, and  $FX^2$  contains a coupling of longitudinal and transverse dissipation effects. Finally, the term  $G_r$  arises from a rotation,  $\vec{E}_{0r} \times \vec{B}_z/B_z^2$ , of the perturbed electron distribution in the unperturbed ambipolar field  $\vec{E}_{0r}$  (see Fig. 1). Since ions are assumed to be unaffected by the magnetic field, they will lag behind the rotation of the electron distribution.

The sketch given in Fig. 1 shows a density perturbation of screw type superimposed on the unperturbed, steady-state distribution. With the directions given in the figure the axial electric field  $\vec{E}_{0z}$  tends to "lift up" the electron screw relative to the ion screw. This is equivalent to a rotation of the electron screw in the positive  $\psi$  direction. Due to a subsequent charge separation an azimuthal electric field  $\vec{E}_{\psi'}$  will arise which tends to drive the particles outwards with a speed  $\vec{E}_{\psi'} \times \vec{B}_z/B_z^2$  and to destabilize the plasma. The pitch of the screw and its angular velocity  $\vec{\omega}_r$  as given in Fig. 1 are consistent with theory<sup>4</sup> and experiments.<sup>5,6</sup> The outward drift due to charge separation resembles that discussed earlier in connection with the stability of a plasma boundary.<sup>7,8</sup> In the latter case charge separation is pro-

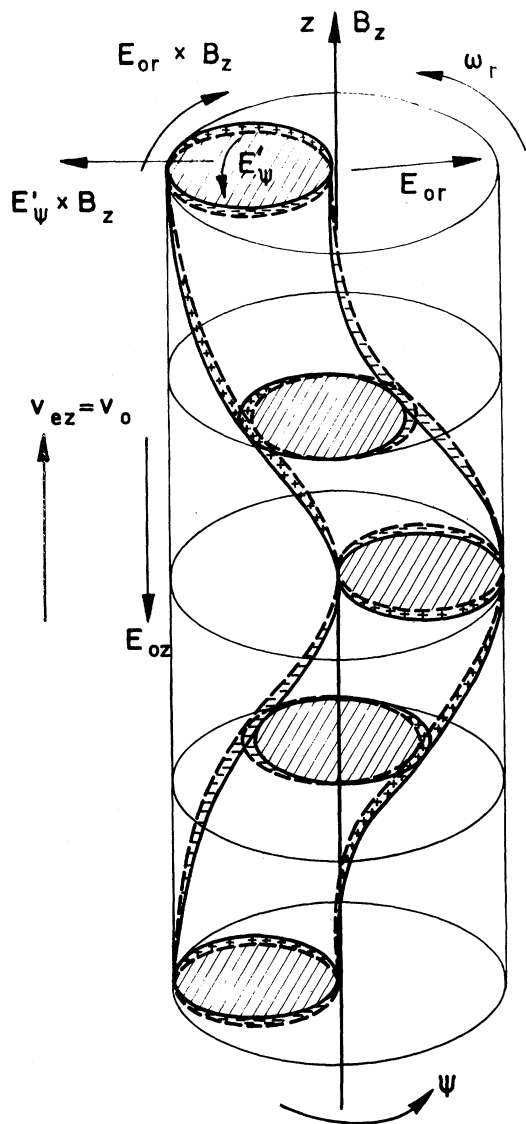


FIG. 1. Left-handed screw instability for  $m=1$ . The perturbed density distribution of ions is given by the screw-shaped body confined by full lines. The corresponding electron distribution is indicated by dashed lines.

duced by drift motions across the magnetic field, whereas the separation in the present configuration arises from motions along the field.

On the other hand, the ambipolar electric field tends to rotate the electron screw in the negative  $\psi$  direction relative to the ion screw. The field  $E_{0r}$  has the direction shown in Fig. 1 under the prevailing experimental conditions, i.e., when no field reversal takes place near the critical magnetic field. This rotation counteracts the charge separation produced by the longitudinal electric field. In the special case where these two effects balance each other the electron screw will become steady, with electrons moving in screw-shaped paths inside the ion screw. It is then also seen that the balance takes place at a certain critical strength of the magnetic field. In reality the rotation will not be uniform as assumed in this simple description but the main features of the stability situation will still be the same.

Additional stabilizing effects are provided by diffusion and conduction which tend to smooth out the perturbed density and potential distributions and to suppress their asymmetry.

According to the present interpretation it is not unimaginable that the screw instability will exist in a large number of situations where there is a sufficiently large electric field component along the magnetic field. For the mechanism just described, the destabilizing action of this component does not depend upon the ionization degree and the instability can also be expected to arise in a fully ionized plasma.

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