sion with highly excited fragments of low kinetic energy and asymmetric fission with moderately excited fragments of high kinetic energy—might be associated with the existence of two barriers and two saddle-point configurations in the liquid drop calculations of Cohen and Swiatecki.[5] The height of the second barrier relative to the first falls rapidly as x increases above a critical value in the vicinity of 0.7 and its relatively low energy and apparent stability against asymmetry[5] suggest identifying this barrier with symmetric fission.

All three of the fissioning species showed fine structure in the mass yield between masses 134 and 146. This fine structure shows up in the total yields[6] but it is perhaps more instructive to look at the contour diagram, Fig. 2. Here it is clearly seen that the structure appears predominantly at high kinetic energies.[7] It seems to be more a result of inhibition than of preference and must be closely related to the fact that the events in this region are "running out of energy" so that the nuclei formed are nearly in their ground states. The contours along the northeast face of the mountain are nearly parallel to the line of maximum energy release. The structure is less pronounced in Pu^{239} and scarcely discernible in Cf^{252}. In these cases the mountain is further away from the limiting energy.

A full account of this work will shortly be submitted to the Canadian Journal of Physics. I am indebted to Dr. W. J. Swiatecki for pointing out the connection with the liquid drop calculations.

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DEUTERONS FROM HIGH-ENERGY PROTON BOMBARDMENT OF MATTER

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The mass analysis of the secondary particles from targets exposed to the 25-Bev proton beam of the CERN proton synchrotron has revealed an appreciable fraction of high-energy deuterons.[1] In addition large numbers of deuterons have been reported among the "grey" tracks in nuclear emulsions.[2-4]

Recently Hagedorn[5] has suggested that these deuterons are produced in elementary nucleon-nucleon collisions. However, the results obtained from a statistical model are strongly energy dependent and especially at lower energies it is hard to reconcile them with the experimental results. For deuterons with momenta below 2 Bev/c they disagree with experiment by a factor greater than 10^3.

In interpreting the emulsion results the Bristol group suggested that the deuterons are formed when a knock-on nucleon pairs with a nucleon in the tail of the Fermi distribution of the excited nucleus. Because of their high energy, however, it seems more likely that they result from the pairing of two cascade nucleons. The angular distribution of the knock-on "shower" nucleons has a strong forward peak and in many cases there will be pairs of particles which have small relative momenta. It is from such pairs that we believe deuterons are formed.

To calculate the ratio of deuterons to protons expected from such a process, we have considered the following model. An incident beam of free protons and neutrons is scattered by an effective nuclear potential, which represents the interaction of the high-energy cascade nucleons with the remainder of the nucleus. The protons and neutrons can interact with each other during the scattering. The incident particles are not parallel or monoenergetic, but have a certain momentum distribution, say $P(\vec{k})d\vec{k}$, which is the same for neutrons and protons and fits the experimental distribution for cascade protons. The flux of particles in the incident beam is chosen to yield n_0 nucleons of each type contained in the nuclear volume at any one time, where n_0 is the experimental average number of high-energy protons from a struck nucleus.

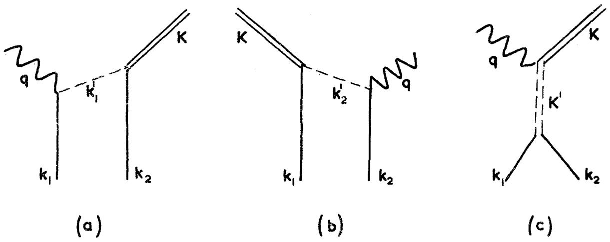

FIG. 1. Diagrams (a), (b), and (c) of Fig. 1 illustrate the simplest methods of deuteron formation. $\vec{k}_1, \vec{k}_2$ are the momenta of the proton and neutron in the initial state, $\vec{q}$ the recoil of the nucleus, and $\vec{K}$ the deuteron momentum in the final state. In case (a) the proton is scattered into an intermediate state by an interaction with the nucleus. The scattered proton and an unscattered neutron then interact with each other to form a deuteron. In case (b) a scattered neutron pairs with an unscattered proton. In case (c) the neutron and proton interact first with each other to form an intermediate deuteron state. This deuteron is then scattered by the nucleus into the final state.

To form a deuteron, a proton and a neutron must interact with each other and with the nuclear field, so that in terms of Feynman-like diagrams the simplest schemes of deuteron production can be illustrated as in Fig. 1. Here $\vec{k}_1, \vec{k}_2$ represent the momenta of the proton and neutron, $\vec{K}$ the deuteron momentum, and $\vec{q}$ the recoil of the nucleus. In diagrams (a) and (b) one of the nucleons has been scattered before the two interact to form a deuteron; in (c) they interact first with each other and the virtual deuteron formed is scattered into the final state.

The contribution from these diagrams has been calculated using a relativistic second order perturbation theory developed for this problem which will be described in a later paper. In the relativistic region the free-particle states are represented by the positive-energy solutions of the Klein-Gordon equation and the nuclear field by the effective potential calculated by Duerr[6] for such particles. In the nonrelativistic region the nucleus is represented by the well-known optical potential.

Since the formation of deuterons arises only from pairs of particles with small relative momenta, the interaction between the neutron and proton can be represented in their c.m. frame by the ordinary deuteron potential.

To simplify the calculation we have chosen $\vec{K}$ to be in the forward direction, and for comparison with the experimental results we have calculated the ratio of the number of deuterons expected in a solid angle $d\Omega$ about $\vec{K}$ with the number of free protons of momentum K which have passed through the nuclear potential and which emerge in the same solid angle $d\Omega$.

In the calculation the following parametric form for the momentum distribution $P(\vec{k})$ was chosen:

$$P(\vec{k}) \propto k^{-\alpha}(\cos\theta)^{\beta}, \tag{1}$$

where α and β are parameters and θ is the angle of the vector $\vec{k}$ with respect to the direction of the incident nucleon which initiates the nucleon shower. This momentum distribution is normalized such that the average number of nucleons of each type within the nuclear volume at any one time, and of momenta exceeding a minimum value k_0, is the number n_0. In the CERN experiment, for example, the high-energy "cascade" protons have energies mostly exceeding 500 Mev, and their momentum distribution above this energy can be fitted by a distribution of type (1); in this case therefore we choose k_0 to correspond to an energy of 500 Mev.

In general, from an accumulation of experimental emulsion observations[4,7-10] the parameter values listed in Table I were chosen. The values of n_0 in this table refer to the silver and bromine nuclei of emulsions and would be expected to vary somewhat with the size of the nucleus.

Table I. Values of the parameters α, β, and n_0 for different minimum energies in the nucleon cascade. These values have been chosen from an accumulation of experimental emulsion observations.[a]

Minimum energy ($\hbar^2 k_0^2/2m$) in the nucleon cascade	α	β	n_0	Range of deuteron energies calculated
30 Mev	3.5	1	6	60-250 Mev
125 Mev	3.5	2	4	250-500 Mev
250 Mev	4	3	2	>500 Mev

[a]See references 4, 7-10.

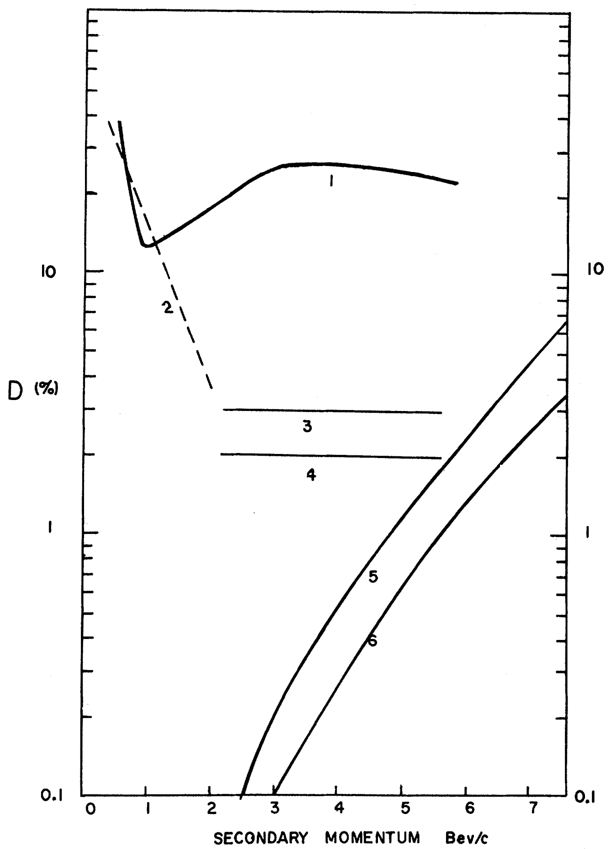

FIG. 2. A comparison of the observed and calculated deuteron to proton ratio for protons and deuterons of the same momentum. Curve 2 represents the experimental results from emulsions.[2-4] Curves 3 and 4 are the deuteron to proton ratios found in the CERN experiment. Curves 5 and 6 are the results calculated by Hagedorn for the two different deuteron factors $\Omega_d = 0.2$, 0.1, respectively. Curve 1 represents the deuteron to proton ratio calculated from the model described above.

The results of this calculation are plotted in Fig. 2 and compared with the experimental results of emulsion groups and the CERN group. The important conclusion is that there are ample numbers of deuterons formed by this process to account for the experimental observations. Indeed, in the high-energy region the numbers yielded by the present calculation are considerably too large; this is almost certainly due to the large repulsive Duerr potential[6] which we have simply taken at face value. The deuteron ratio is proportional to the square of the modulus of the effective nuclear potential, and is therefore quite sensitive to this potential. It would in fact be possible that the present deuteron formation mechanism can be used to provide a direct and sensitive measurement of the strength of the effective nuclear potential for relativistic particles.

The results of Hagedorn[5] are also shown in Fig. 2 for comparison purposes. It is difficult to reconcile the strong energy dependence of this model with the rather slow energy variation of the experimental results. Moreover, it appears to the present authors extremely probable that the statistical model employed by Hagedorn would strongly overestimate the deuterons produced in elementary nucleon-nucleon collisions at all energies, thereby impairing the "local" high-energy agreement of Hagedorn results with experiment.

In the CERN experiment moreover the percentage of deuterons detected increases with the size of the target nucleus roughly as the nuclear radius. If the deuterons are produced in nucleon-nucleon collisions, this is difficult to explain. On the present model involving the recombinations of cascade nucleons themselves such an increase of the deuteron ratio with the size of the target nucleus is to be expected.

Full details of the present calculation will be described in a later paper.

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