MAGNETIC FLUX THROUGH A SUPERCONDUCTING RING

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London[1] recognized that the magnetic flux embraced by a superconducting ring ought to be quantized. He argued as follows: The current density is proportional to the average of

$$\vec{p} - (Q/c)\vec{A},$$

where $\vec{A}$ denotes the vector potential and Q the charge of the current-carrying particle. In the interior of the superconductor the current density vanishes, so that the condition

$$\oint \vec{p} \cdot d\vec{x} = -nh$$

for a single-valued wave function implies

$$\Phi = \oint \vec{A} \cdot d\vec{x} = -nhc/Q.$$

Substituting $Q = -e$ for the charge of the electron, he arrived at the conclusion

$$\Phi = nhc/e.$$

It is possible to recast London's discussion in a form which is completely Lorentz and gauge invariant; the details need not concern us here.

London's result inspired the suggestion[2] that the quantization of flux might be an intrinsic property of the electromagnetic field.

Not much later, Schafroth[3] pointed out that electron pairs held together by attractive interactions would obey Bose statistics and be capable of superfluid (Einstein) condensation. A likely source of the requisite attractive interaction—by way of the phonon field—had been suggested by Fröhlich[4] and by Bardeen.[5] These ideas form the basis of more detailed theories, which explain the various observed properties of superconductors so well that they have been generally accepted.

Deaver and Fairbank[6] have found that the flux embraced by a superconducting annulus varies in steps of half the size proposed by London. This is readily explained by Schafroth's theory, which requires

$$Q = -2e,$$
$$\Phi = nhc/2e.$$

The discovery of steps just this size provides a convincing direct proof for the pairing of electrons.

The notion that the electromagnetic field itself might be subject to a similar condition seems untenable now, for singly charged bosons exist (deuterons) and a condition imposed on the electromagnetic field ought to be equally compatible with all charged particles.

Instead, we arrive at the remarkable result that we can measure the magnetic flux (except for an additive undetermined multiple of $hc/2e$) embraced by a given closed path without examining the enclosed field; a superconductor placed along the path will respond with a supercurrent which compensates the fractional excess of flux. Complete Meissner effect in a multiply-connected superconductor requires coherent ring closure. We may infer that such closure will not take place unless the free energy liberated by the matching of phases exceeds the kinetic energy of the necessary supercurrent plus the added magnetic field energy. The detailed kinetic mechanism is not yet known.

I am indebted to F. Bloch for a discussion which clarified the closure problem, and to B. Deaver and W. M. Fairbank for the communication of their unpublished results.

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[2]L. Onsager, Proceedings of the International Conference on Theoretical Physics, Kyoto and Tokyo, September, 1953 (Science Council of Japan, Tokyo, 1954), p. 935.

[3]M. R. Schafroth, Phys. Rev. 96, 1149, 1442 (1954).

[4]M. Fröhlich, Phys. Rev. 79, 845 (1950); Proc. Roy. Soc. (London) A215, 291 (1952).

[5]J. Bardeen, Phys. Rev. 79, 167 (1950); 80, 517 (1950); 81, 829 (1951).

[6]B. Deaver and W. M. Fairbank, this issue [Phys. Rev. Letters 7, 43 (1961)].