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K^* AND $K^+-3\pi$ DECAY

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It is well known[1] that the spectrum of the unlike pion in the τ^+-decay mode of K^+ mesons deviates noticeably from the pure statistical distribution. Many people[2] have tried to explain this deviation as due to pion-pion "final state" interaction. On the basis of their analysis of the τ-decay spectrum, these people,[2] however, obtained pion-pion S-wave scattering lengths which do not agree with those obtained[3] on the basis of the crossing relations developed by Chew and Mandelstam.[4] In this note we wish to show that the observed deviation of the spectrum of the unlike pion in τ^+ decay from the statistical distribution can be simply explained if τ^+ decay proceeds via K^* (see Fig. 1) provided that K^* has spin unity, in favor of which there is some slight evidence.[5] On this model the spectrum of the unlike pion in $\tau^{+\prime}$ decay comes out to be the same as given by Weinberg[6] on the basis of $|\Delta\vec{I}|=\frac{1}{2}$ rule and is consistent with experiment.[7] Below we give the details of the calculation.

Denote the 4-momenta of the three emerging pions from K-meson decay by k_1, k_2, and k_3, where k_3 will always refer to the unlike pion in the τ or τ' decay mode. Let K denote the 4-momentum of the K-meson ($K^2=-m_K^2$). We introduce the

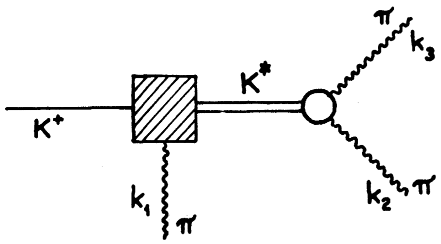

FIG. 1. Feynman diagram for the τ decay mode of the K meson via K^*.

three scalar variables,

$$s_1 = -(K - k_1)^2,$$
$$s_2 = -(K - k_2)^2,$$
$$s_3 = -(K - k_3)^2, \tag{1}$$

where

$$s_1 + s_2 + s_3 = m_K^2 + 3m_\pi^2. \tag{2}$$

Let T_1, T_2, and T_3 denote the kinetic energies of the three pions in the rest system of the K meson. In a nonrelativistic treatment of the decay spectrum, it is convenient to introduce the Dalitz[1] variables

$$x = \sqrt{3}(T_1 - T_2)/Q,$$
$$y = (3T_3 - Q)/Q, \tag{3}$$

where $Q = T_1 + T_2 + T_3$. Then, aside from a constant factor, the decay spectrum is given by

$$dw(x, y) = |M|^2 dx dy, \tag{4}$$

where M is the matrix element for $K - 3\pi$ decay and $x^2 + y^2 \leq 1$.

Gell-Mann and Rosenfeld[8] showed that a good empirical fit to the τ^+-decay data, obtained from a plot compiled by Dalitz,[1] is obtained with

$$M_\tau \sim 1 + \tfrac{1}{10} y. \tag{5}$$

We now show that the Feynman diagram shown in Fig. 1 leads to a matrix element of the form (5) for τ^+ decay provided that K^* has spin unity. If K^* has spin unity, Fig. 1 gives, for τ^+ decay, a matrix element of the form

$$g_{K^{0*}} f_{K^{0*}} (s_2 - s_3)/(-s_1 + m_{K^*}^2),$$

where $g_{K^{0*}}$ is the strong coupling constant for the reaction $K^* \to K + \pi$ and $f_{K^{0*}}$ is the weak coupling constant for the decay $K^{0*} \to \pi^+ + \pi^-$. Hence the matrix element for τ^+ decay is given by

$$M_\tau(s_1, s_2, s_3) = \lambda + g_{K^{0*}} f_{K^{0*}} (s_2 - s_3)/(-s_1 + m_{K^*}^2) + \text{same expression with } s_1 \text{ and } s_2 \text{ interchanged}$$

$$= \lambda + g_{K^{0*}} f_{K^{0*}} \left[\frac{s_2 - s_3}{-s_1 + m_{K^*}^2} + \frac{s_1 - s_3}{-s_2 + m_{K^*}^2} \right], \tag{6}$$

where λ is a constant which represents the contribution from intermediate states of higher energy (and corresponds to the subtraction constant in dispersion relations). If we consider the matrix element M_τ at the symmetric point $s_1 = s_2 = s_3 = s_0 = \frac{1}{3} m_K^2 + m_\pi^2$, we find $\lambda = M_\tau(s_0, s_0, s_0)$. Now the maximum value of s_1 or s_2 in τ decay is $(m_K - m_\pi)^2 \approx 6.25\, m_\pi^2$, while $m_{K^*}^2 \approx 40\, m_\pi^2$. Therefore we can neglect s_1 and s_2 as compared with $m_{K^*}^2$ in the denominators of the above expression, so that (6) becomes

$$M_\tau = \lambda + g_{K^{0*}} f_{K^{0*}} (s_1 + s_2 - 2s_3)/m_{K^*}^2,$$

which, on using (2) and (3), becomes

$$M_\tau = \lambda \left[1 + \frac{g_{K^{0*}} f_{K^{0*}}}{\lambda} \left(\frac{2m_K Q}{m_{K^*}^2} \right) y \right] \tag{7}$$

Now putting in the experimental values of m_K, m_{K^*}, and Q (≈ 75 Mev), we get $2m_K Q/m_{K^*}^2 \approx \frac{1}{10}$. Hence if we take $\lambda \approx g_{K^{0*}} f_{K^{0*}}$ (or, what is the same thing, $\lambda \approx f_{K^{0*}}$ since $g_{K^{0*}}$ has been shown to be of the order of unity by Chan[5]), Eq. (7) reduces to (5) which has been shown to fit the τ^+-decay data by Gell-Mann and Rosenfeld[8] and Weinberg.[6]

Note that if K^* has spin zero, then in Eq. (6), $(s_2 - s_3)$ and $(s_1 - s_3)$ do not appear and if we neglect s_1 and s_2 as compared with $m_{K^*}^2$, the matrix element is constant and the spectrum will not deviate from the statistical distribution.

For $\tau^{+\prime}$ decay, we get the matrix element (corresponding to the diagram in Fig. 1)

$$M_{\tau'} = \lambda' + g_{K^{+*}} f_{K^{+*}} (2m_K Q/m_{K^*}^2) y,$$

where $\lambda' = M_{\tau'}(s_0, s_0, s_0)$. It is well known that the $|\Delta \vec{I}| \leq \frac{3}{2}$ rule[2,6,8] gives $\lambda' = \lambda/2$. It can be shown that

$$g_{K^{+*}} = -g_{K^{0*}}/\sqrt{2},$$

and, if we assume the $|\Delta \vec{I}| = \frac{1}{2}$ rule,

$$f_{K^{+*}} = \sqrt{2} f_{K^{0*}},$$

so that

$$M_{\tau'} = \tfrac{1}{2}\lambda\left[1 - 2\frac{g_{K^{0*}} f_{K^{0*}}}{\lambda}\left(\frac{2m_K Q}{m_{K^*}^2}\right) y\right]. \tag{8}$$

Note that the factors multiplying y in Eqs. (7) and (8) are in the ratio 1:-2 which is the same result as obtained by Weinberg[6] on the basis of the $|\Delta \vec{I}| = \frac{1}{2}$ rule. Now taking as before $g_{K^{0*}} f_{K^{0*}} \approx \lambda$, Eq. (8) becomes

$$M_{\tau'} = \tfrac{1}{2}\lambda[1 - 2\tfrac{1}{10}y], \tag{9}$$

which is consistent with the experiment.[6,7]

Note that for $\tau^{+\prime}$ decay another diagram, which is obtained from Fig. 1 by replacing $k_1(\pi^0)$ with $k_3(\pi^+)$, is also possible. But it is easy to see that on symmetrization between k_1 and k_2 this gives zero.

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VECTOR MESONS AND POSSIBLE VIOLATIONS OF CHARGE SYMMETRY IN STRONG INTERACTIONS

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The two vector bosons, $\rho \to 2\pi$ [1] and $\omega \to 3\pi$,[2] needed to understand the main features of the electromagnetic form factors of the nucleon, have been recently observed in several experiments, the masses of the ρ and ω being very close but the observed width of the ρ being much larger than that of ω.

A very important result has recently been obtained by the Berkeley group[3] which found that the wide ρ^0 resonance is split into two levels ρ_1^0, ρ_2^0 whose experimental widths are of the same order of magnitude as that of ω^0. The first tentative attribution of quantum numbers identifies the vector boson with the higher level ρ_2^0.

So the present very preliminary values of the masses of ρ^0 and ω^0 are the following:

$$m_{\omega^0} = 787 \pm 10 \text{ Mev}, \quad m_{\rho^0} = 775 \pm 10 \text{ Mev}.$$

The observed widths, ~10 Mev, coincide with the experimental resolution.

The equality, within experimental errors, of m_{ρ^0} and m_{ω^0} is quite intriguing since both particles have the same quantum numbers except