PRODUCTION CROSS SECTION OF INTERMEDIATE BOSONS BY NEUTRINOS IN THE COULOMB FIELD OF PROTONS AND IRON

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This is a report on a numerical computation of the cross sections for the processes[1]

$$\nu' + Z \to W^+ + \mu^- + (Z \text{ in various states}), \tag{1}$$

and

$$\bar{\nu}' + Z \to W^- + \mu^+ + (Z \text{ in various states}), \tag{2}$$

where W is the (proposed) intermediate boson for weak interactions, ν' is the neutrino associated with the μ meson, and Z represents a target proton or nucleus. Pion production is not included. The two processes (1) and (2) have the same differential cross sections because of the following theorem:

Theorem 1. For process (2) consider a mirror image of process (1) with all momenta and helicities (i.e., polarization along direction of motion) of the corresponding particles reversed. The differential cross sections to the lowest order in e and g are identical.

To prove the theorem we perform the operation CR (where C = charge conjugation and R = space inversion) on the leptons and W, and the operation R on the nucleus. The theorem follows immediately.

Another theorem that is useful is:

Theorem 2. Consider reaction (1). The differential cross sections to the lowest order in e and g are unchanged under a mirror reflection of all momenta, if the helicities are held fixed.

To prove this theorem we perform a time-reversal operation and prove that the matrix element changes into its complex conjugate.

The interaction Lagrangian of the W with the leptons is taken to be[1]

$$\mathcal{L}_{\text{int}} = ig\psi_\mu^\dagger \gamma_4 \gamma_\lambda (1+\gamma_5)\psi_{\nu'}\phi_\lambda^* + \text{Hermitian conjugate}, \tag{3}$$

where ϕ_λ, ψ_μ, and $\psi_{\nu'}$ denote the fields describing W^+, μ^-, and ν'. The magnitude of g is[1] given by

$$g = m_W G_V^{1/2} 2^{-1/4}, \quad G_V \cong 10^{-5} m_p^{-2}. \tag{4}$$

The Lagrangian of the W particle and its interaction with the electromagnetic field A_μ is taken to be

$$\mathcal{L}_{W-A} = -\tfrac{1}{2}(\partial A_\mu/\partial x_\nu)^2 - \tfrac{1}{2} G_{\mu\nu}^* G_{\mu\nu}$$
$$- m_W^2 \phi_\mu^* \phi_\mu - ie\kappa F_{\mu\nu}\phi_\mu^* \phi_\nu, \tag{5}$$

where

$$F_{\mu\nu} = \partial A_\nu/\partial x_\mu - \partial A_\mu/\partial x_\nu,$$
$$G_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu,$$
$$\partial_\mu = \partial/\partial x_\mu - ieA_\mu, \tag{6}$$

$$e = (4\pi/137)^{1/2}. \tag{7}$$

The parameter κ is related to the magnetic moment of the W^+ (along its spin):

$$(e\hbar/2mc)(1+\kappa). \tag{8}$$

There are two Feynman diagrams that contribute in the lowest order to (1), as illustrated in Fig. 1,

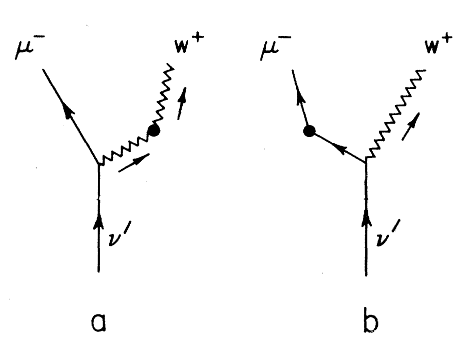

FIG. 1. Feynman diagrams.

in which the black circle represents the electromagnetic field of the nucleus. It contributes a factor V_β to the process. For the coherent process the nucleus recoils as a whole and we take

$$V_4=\frac{iZe}{q^2}F^Z(q^2),\quad V_1=V_2=V_3=0, \tag{9}$$

where Ze = nuclear charge, and[1] $q=p'-p$. The form factor F^Z is taken to be

$$F^Z=(1+\tfrac{1}{12}q^2a^2)^{-2}, \tag{10}$$

with

$$a^2=\tfrac{3}{5}\{1.3\times10^{-13}A^{1/3}\}^2\ \mathrm{cm}^2. \tag{11}$$

For incoherent processes on nuclei or for production from a free proton, we take

$$V_\beta=(ie/q^2)u_{p'}{}^\dagger\gamma_4$$
$$\times\{F_1\gamma_\beta+iF_2K\tfrac{1}{2}(\gamma_\lambda\gamma_\beta-\gamma_\beta\gamma_\lambda)(p-p')_\lambda\}u_p, \tag{12}$$

where $u_{p'}$ and u_p are the Dirac spin wave functions of the final and initial protons, normalized so that

$$u_{p'}{}^\dagger u_{p'}=u_p{}^\dagger u_p=1, \tag{13}$$

$$K=\tfrac{1}{2}(\mu_p)=0.8948,$$

and F_1 and F_2 are the form factors of the proton which we take from the Stanford experiments.[2]

The differential cross section for (1) is easily shown to be

$$d\sigma=(32\pi^5)^{-1}|\alpha_a+\alpha_b|^2$$
$$\times d^3\mu d^3W\,\delta(E_\mu+E_W+E_{p'}-E_\nu-E_p), \tag{14}$$

where α_a and α_b are the contributions to the matrix element from diagrams a and b. They are

Table I. Total cross section in 10^{-38} cm^2 for $\nu'+Z\to W^++\mu^-+\ldots$, for Z = proton and Z = Fe.

m_W	κ	E_ν/m_p	σ_p	$\frac{1}{26}\sigma_{\mathrm{Fe}}$(coherent)	$\frac{1}{26}\sigma_{\mathrm{Fe}}$(total)
$0.6m_p$	0	1	0.00564	...	0.006
		1.5	1.39	0.393	1.77
		1.8	2.95	1.23	4.13
		2	4.26	2.17	6.35
		2.5	8.01	6.35	14.12
		3	11.9	13.09	24.49
		4	19.8	35.03	53.48
		5	27.4	66.33	91.18
		6		104.6	
		8		192.3	
		10		287.4	
$1\ m_p$	0	2	0.0623	0.00739	0.0694
		2.5	0.608	0.0507	0.657
		3	1.66	0.202	1.85
		3.5	3.1	0.57	3.65
		4	4.88	1.20	6.03
		5	8.78	3.76	12.40
		6	13.0	8.2	20.88
		7		15.57	
$1\ m_p$	1	3	2.44	0.233	2.66
		4	7.18	1.41	8.54
		5	13.48	4.51	17.81
$1\ m_p$	-1	3	1.30	0.182	1.48
		4	3.77	1.05	4.78
		5	6.88	3.25	10.01
$1.4m_p$	0	3	0.0262	0.00217	0.0283
		4	0.612	0.0269	0.637
		5	1.99	0.139	2.12
		6	3.91	0.4	4.30
		7		1.12	

$$\alpha_a = -2egi\{(\nu-\mu)^2 + m_W^2\}^{-1}$$
$$\times \{2[l\phi][WV] + [LV][q\phi] - [Lq][\phi V]\},$$

and

$$\alpha_b = 2egi\{\nu - W)^2 + m_\mu^2\}^{-1}$$
$$\times \{2[l\phi][\mu V] + [lV][q\phi] - [lq][\phi V] + \det\}, \quad (15)$$

where

$$L = -(1+\kappa)l - [l\mu]\{(1-\kappa)W - \kappa q\} m_W^{-2}; \quad (16)$$

ϕ = amplitude of W:

$$(2E_W)^{1/2}\vec{\phi} = \text{unit vector} \perp \vec{W} \text{ for transverse } W$$
$$= E_W (m_W)^{-1} (\text{unit vector} \parallel \vec{W}) \text{ for longitudinal } W,$$
$$(2E_W)^{1/2}\phi_4 = 0 \text{ for transverse } W$$
$$= i\,|\vec{W}|\,(m_W)^{-1} \text{ for longitudinal } W. \quad (17)$$

The four-vector l is defined to be

$$l_\lambda = u_\mu^\dagger \gamma_4 \gamma_\lambda u_\nu, \quad (18)$$

where u_μ and u_ν are the Dirac spin wave functions of the μ^- and ν' normalized like (13). The symbol det in (15) is the determinant formed by the components of ϕ, V, l, and q:

$$\det = \epsilon_{\xi\eta\zeta\lambda} \phi_\xi V_\eta l_\zeta q_\lambda, \quad (19)$$

where ϵ is the completely antisymmetrical tensor.

The calculation was made with an IBM-7090 computer. To maximize the flexibility of the code used, the calculation was made separately for each of the two polarization states of the μ, the three polarization states of the W, and the different spin arrangements of the proton. The differential cross sections were computed and various partial sums were taken. The total 7090 time used was of the order of 10 hours. In Table I we present the results obtained for the total cross sections for the production process (1) on protons and on Fe. The results are plotted in Fig. 2. Sample energy spectra are given in Fig. 3.

The total coherent cross section for (1) is denoted by σ_Z(coherent), and the total cross section for $\mu^- + W^+$ production on a proton is denoted by σ_p. (In none of these are pion production processes included.) The sum total cross section σ_Z(total) including both coherent and incoherent processes

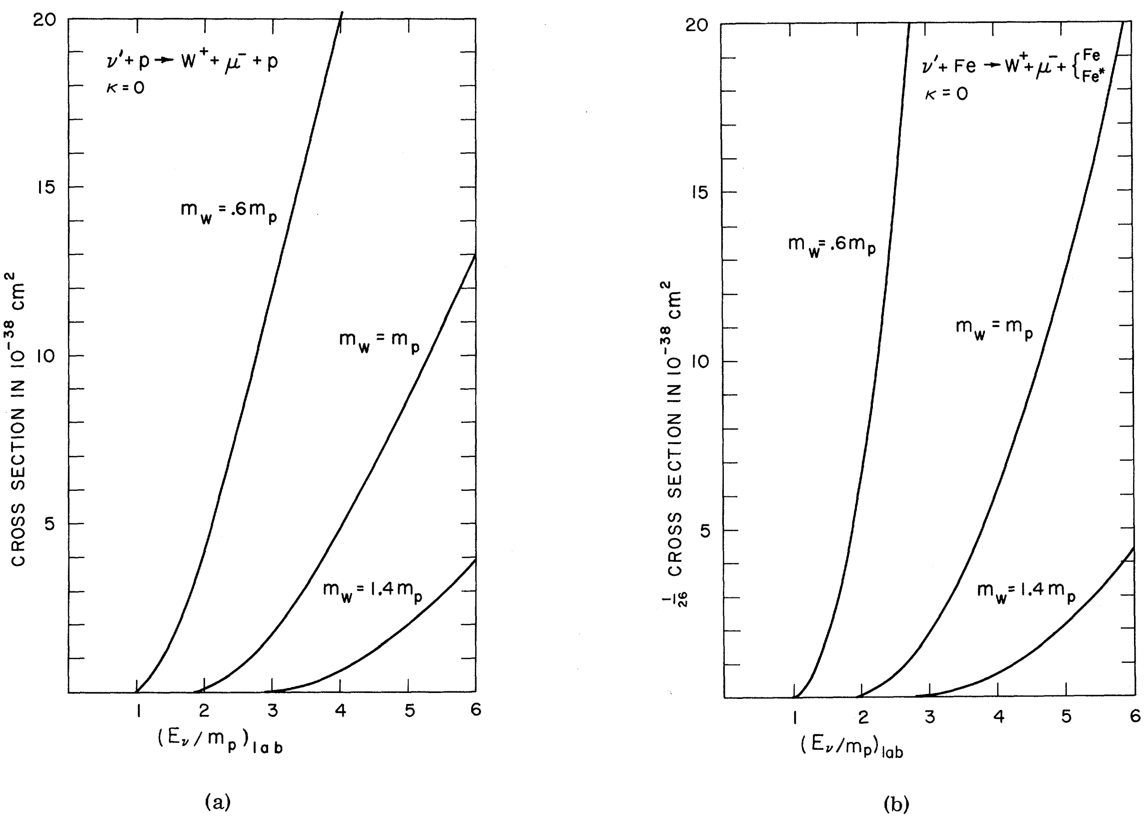

FIG. 2. Total production cross sections for process (1), (a) for protons; (b) for iron.

is computed from

$$\sigma_Z(\text{total}) \cong Z\sigma_p + (1-1/Z)\sigma_Z(\text{coherent}). \tag{20}$$

In this expression the terms $Z\sigma_p$ and σ_Z(coherent) are obvious contributions. The correction term $(-1/Z)\sigma_Z$(coherent) is to subtract out the contributions from those incoherent processes, included in $Z\sigma_p$, which give rise to small momentum transfers, and which therefore are prohibited as incoherent processes. (20) is essentially the same as the corresponding formula used by Masek, Lazarus, and Panofsky[3] for photoproduction of μ pairs.

As is evident from Table I, for high-energy ν', the contribution from the coherent process dominates. The reason for this is well known: For higher energies, the minimum momentum transfer becomes smaller, and the coherent process becomes more effective.

An examination of the energy distribution of the μ and W shows that in general the W carries most of the energy. Consequently the decay μ from the W is in general more energetic than the μ produced in company of the W.

For completeness, we list some asymptotic formulas. The total coherent cross section has been given before,[4,5]

$$\sigma_Z(\text{coherent}) \to \left(\frac{Z}{137}\right)^2 \frac{g^2}{6\pi m_W^{\,2}}(\kappa-1)^2(\ln\xi)^3, \tag{21}$$

where

$$\ln\xi = \ln[2(\sqrt{12})E_\nu/m_W^{\,2}a] \gg 1.$$

This asymptotic formula is, however, only applicable for E_ν much larger than those listed in Table I.

The asymptotic formula for the spectrum in coherent production is

$$\frac{d\sigma_Z(\text{coherent})}{dE_W} \to \left(\frac{Z}{137}\right)^2 \frac{g^2}{2\pi m_W^{\,2}}\frac{1}{E_W}\left(\ln\frac{2(\sqrt{12})E_W}{m_W^{\,2}a}\right)^2$$

$$\times[\tfrac{1}{2}E_W^{\,2} - E_W E_\nu + E_\nu^{\,2}]E_\nu^{\,-2}, \tag{22}$$

which is also applicable only at very high energies.

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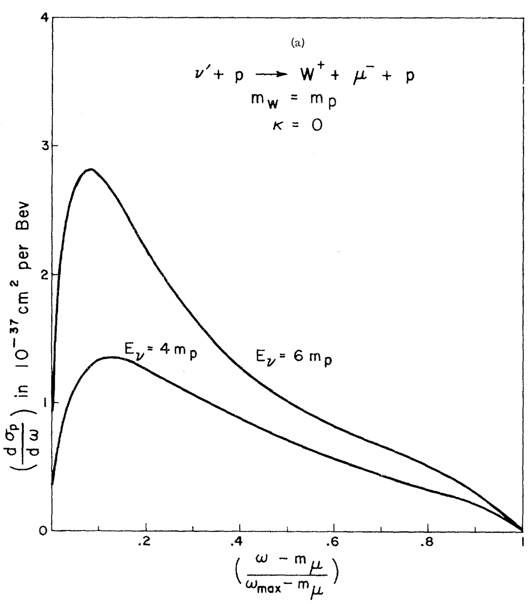
[1]We choose units so that $\hbar = c = 1$ and use the notation of T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960). A four-momentum has real space components and pure imaginary fourth component. A four-vector squared is defined so that $p^2 \equiv p_1^{\,2}+p_2^{\,2}+p_3^{\,2}+p_4^{\,2}$. All quantities are in the laboratory system. $\vec{\nu}$, $\vec{p}$, $\vec{p}'$, $\vec{W}$, and $\vec{\mu}$ denote the 3-momenta of the ν', the initial and



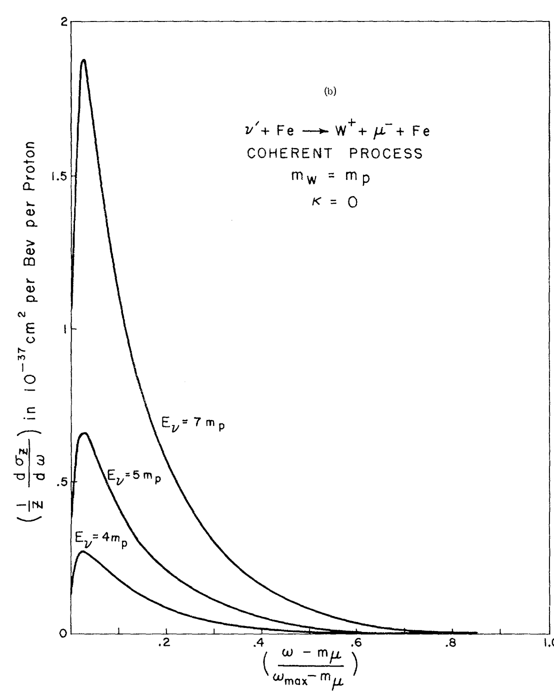


FIG. 3. Energy spectrum of the μ, (a) for protons: (b) for iron. ω = total energy of μ in the center-of-mass system. E_ν = energy of ν' in the laboratory system.

final nucleus (or proton), the W, and the μ. E_ν, E_p, $E_{p'}$, E_W, and E_μ denote their respective energies, and ν, p, p', W, and μ their respective 4-momenta. The symbol $[pq]$ denotes the 4-product of p and q. m_W, m_p, and m_μ are the masses of W, p, and μ.

[2]R. Hofstadter, F. Bumiller, and M. Croissiaux, Phys. Rev. Letters 5, 263 (1960). For momentum transfer $q^2 \geq 25$ (fermi)$^{-2}$ we use the arbitrary extrapolation: $F_1 = 0.4$ and $F_2 = 0$.

[3]G. E. Masek, A. J. Lazarus, and W. K. H. Panofsky, Phys. Rev. 103, 374 (1956).

[4]T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960).

[5]T. D. Lee, Proceedings of the 1960 Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 566.

ERRATUM

EVIDENCE FOR THE DOUBLE *F* MODEL OF THE *M* CENTER, Bruce J. Faraday, Herbert Rabin, and W. Dale Compton [Phys. Rev. Letters 7, 57 (1961)].

F. A. Kröger has kindly pointed out an error of a factor of two in the computations. At low temperature the constant K should be taken as 6 rather than 12 owing to the symmetry of the M center. This correction gives the following results:

	f_F/f_M	f_M
KBr (He temp.)	1.6	0.30
KBr (N_2 temp.)	2.0	0.24
KCl (He temp.)	1.4	0.40
KCl (N_2 temp.)	1.5	0.36

These values are now in reasonable agreement with Okura's determination (reference 9). Similarly the constant K for NaCl and LiF at room temperature becomes 3500 and 2200, respectively. Values of r/a become 5.9 for NaCl and 5.1 for LiF.