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STABLE PLASMA CONFINEMENT BY MULTIPOLE FIELDS*

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Multipole magnetic fields increase with distance from and are convex toward the center of symmetry, providing a central equilibrium position and correct field curvature for stability of confined plasma. Braginskii and Kadomtsev[1] and Grad[2] have treated stability only for this convex portion of field, and Tuck[3] pointed out the utility of joining lines of force exterior to the multipole conductors to eliminate plasma escape along lines of force.

We describe a multipole configuration for which it has been possible to derive the conditions for plasma stability over the completely contained tubes of flux. These conditions can conveniently be met. The structure has only r and θ fields; thus eliminating the z field of Stellarators and pinches, and critical surfaces which break up into multiple pressure-reducing flux bundles under the influence of field errors.[4] A large volume of high $\beta(\equiv 2p/B^2)$ plasma is contained with no plasma surfaces in the sense of the cusp model.[1-3] The configuration is stable due to the externally supplied fields alone, no matter how little plasma is present. There is no requirement for persistence of special plasma current distribution as employed in pinches.

Figure 1 shows the cross section of a multipole field configuration. The z direction can be linear or along the circumference of a torus. Toroidal effects were found sufficiently small to be omitted in the calculations. Current flows in one direction through the rods and in the opposite direction on the wall. Briefly, the stability of the exterior of the multipole results from the flux tube connection between Region I where the field is convex toward the plasma, thus providing stability, and Region II where the field outside the plasma is concave toward the plasma contributing instability to the flux tube. The plasma on flux lines which circle a rod is in a completely stable situation. Conditions for net stability follow from the considerations sketched below which will be fully published elsewhere.

The energy principle[5] is used with the flux function, ψ, and orthogonal coordinate, χ, chosen so that

$$B_\theta = -\partial\psi/\partial r,$$

$$B_r = (1/r)(\partial\psi/\partial\theta),$$

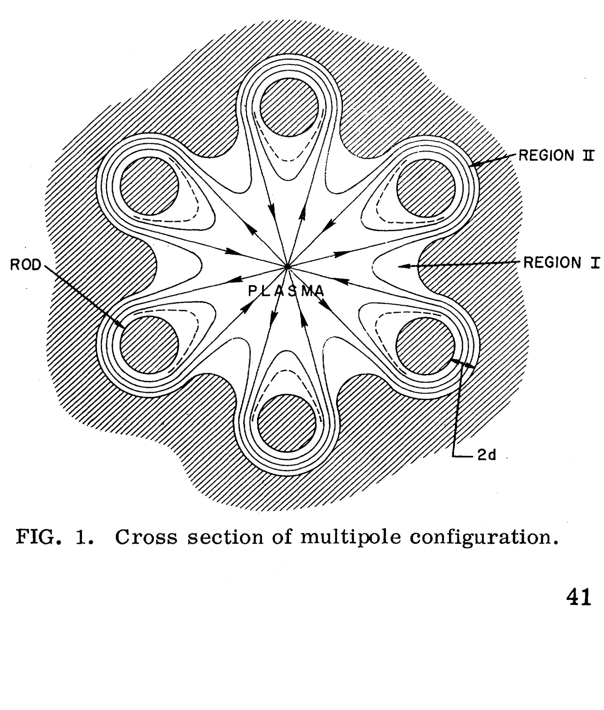

FIG. 1. Cross section of multipole configuration.

and so that a line element, ds, is

$$(ds)^2 = (1/B^2)(d\psi)^2 + J^2B^2(d\chi)^2 + (dz)^2.$$

We minimize the energy, δW, with respect to displacements in the χ and z directions, eliminating terms containing these components. The sufficient condition for stability that results is that the function,

$$S = \oint d\chi \{B^{-2}J^{-1}(\partial X_k/\partial\psi)^2 - X_k^{\,2}JB^{-2}(\partial p/\partial\psi)[\partial(p+\tfrac{1}{2}B^2)/\partial\psi]\},$$

be positive for all flux lines, where X_k is defined by

$$\xi_\psi = \sum_{k=0} (1/B)X_k \cos kz,$$

and ξ_ψ is the ψ component of the displacement. Consider first a uniform X_k along the arcs in Fig. 2. Then the stability condition becomes

$$(B_1^{\,2}/B_2^{\,2}) < \phi_1/\phi_2.$$

This condition is easy to satisfy since B_1 may be an order of magnitude less than B_2.

A worse displacement is one which does not occur in Region I where stability is provided, but rather the displacement is only in Region II. Stability would then have to come from bending the lines of force. The stability condition for this type of displacement is

$$(dp/dr_2) < (B_2^{\,2}/2r_2)(\pi/\phi_2)^2,$$

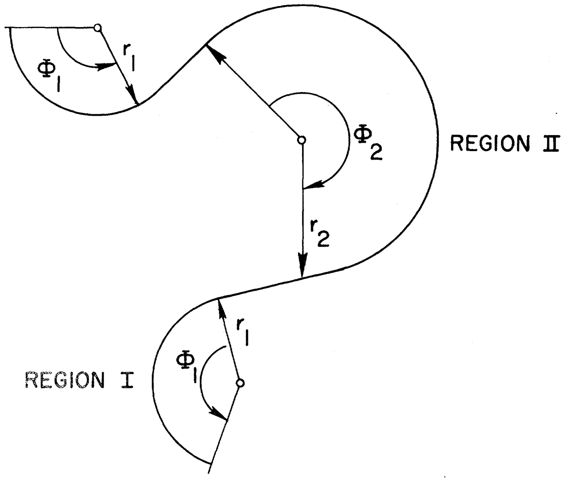

FIG. 2. Geometry of flux line for stability calculation.

or approximately

$$\beta_2 < (d/r_2)(\pi/\phi_2)^2.$$

This condition can also be satisfied with convenient proportions of rod and rod cavity.

A general result for plasma containment time is

$$\tau \sim \psi_{\max}^{\,2}/(\eta P_0),$$

where η is plasma resistivity in emu, or

$$\tau = 10^{-7} T_{\mathrm{ev}}^{\,3/2} d^2 (B_2/B_1)^2 \text{ seconds.}$$

This time is generally long compared to the time for other processes to affect the plasma.

Even at low density and in the collisionless case, pressure is scalar and constant along a field line. A spherically symmetric velocity distribution in the central low field is maintained in the high-field region where the orbit magnetic moment is an invariant. In case the ion gyroradius is less than d, electrostatic ion reflection due to electron space charge occurs in the high-field region and in this case the action is invariant. This insures constancy of pressure on the central field line.

A free rod can have a stable equilibrium position, but with supports for the toroidal case, plasma lifetimes useful for some purposes are possible.

Linear arrays allow rods to be connected outside the plasma. Termination in a point cusp requires the cusp flux to join the rod flux and to wind its way along the multipole, producing a B_z. There is a confluence of cusp flux, as it starts around the rod, where a singular line is generated.

Injection of plasma through the convex surface of Region I allows entry by the instability caused by reversal of the pressure gradient.[6] Once within, the plasma is confronted with a stable configuration which confines it.

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[1]S. Braginskii and B. Kadomtsev, Plasma Physics and the Problem of Controlled Thermonuclear Reactions (Pergamon Press, New York, 1961), Vol. III, p. 356.

[2]J. Berkowitz et al., Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1958), Vol. 31, p. 171.

[3]J. L. Tuck, Nature 187, 863 (1960).

[4]T. Ohkawa and D. W. Kerst, Bull. Am. Phys. Soc. 6, 290 (1961).

[5]W. Bernstein, E. A. Frieman, M. D. Kruskal, and R. Kulsrud, Proc. Roy. Soc. (London) A244, 17 (1957).

[6]J. L. Tuck, Phys. Rev. Letters 3, 314 (1959).