2 to 4 kc/sec $[\omega = \omega_H \cos\theta(1+\alpha R^2 N_A^2 \cos^2\theta)^{-1}]$.

The SCS radiate A waves in the band $\omega < (v_A/u)\Omega \ll \Omega$, and B waves in the band $(v_A/u)\Omega < \omega \ll \Omega$ by the cyclotron mechanism. (See Fig. 1.) But at $\Omega \gg \omega$ the group velocity of A waves is directed, regardless of the orientation of the wave vector $\vec{k}$, along the magnetic field H; so at the earth's surface the wave will have the polarization and frequency of an A wave (neglecting the possibility of wave-type transformation, due to inhomogeneities of the medium or of the field). Such polarization was observed by Sugiura.[14]

However, micropulsations[14] might be explained by one more type of instability (considered for electron streams by Zhelezniakov[15]) — transverse impulse instability.[16]

When $\omega > \Omega$ the B wave only remains, so micropulsations and radio emission should have elliptical polarization of opposite sense.

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STABILITY CRITERION FOR ARBITRARY HYDROMAGNETIC EQUILIBRIA*

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(Received November 13, 1961)

A condition which is both necessary and sufficient for the stability of general hydromagnetic equilibria with respect to localized displacements has been derived. Such localized instabilities have been studied for many special configurations,[1-5] and are often severely limiting.[6]

In a hydromagnetic fluid model, an equilibrium configuration is determined by the equations

$$\vec{J} \times \vec{B} = \vec{\nabla} p, \tag{1}$$

$$\vec{J} = \vec{\nabla} \times \vec{B}, \tag{2}$$

$$\vec{\nabla} \cdot \vec{B} = 0. \tag{3}$$

The general solutions of these equations for systems which are spatially bounded have the property that the lines of force lie on surfaces, called magnetic surfaces, which are topologically nested toroids.[7] On some surfaces the magnetic lines of force may close on themselves after n revolutions the short way around the toroid and m revolutions the long way. We define the rotational transform for such a surface by

$$\iota = 2\pi n/m. \tag{4}$$

It is the same for all lines of force on the surface. The definition of the rotational transform can be extended so that ι is continuous. We assume that the rotational transform varies from surface to surface. Then both the set of surfaces for which $\iota/2\pi$ is rational and the lines close on themselves, and the set for which $\iota/2\pi$ is irrational and the lines are ergodic on the surface, are dense sets. We also assume that $\vec{\nabla} p$ does not vanish.

The stability problem is tractable only in a suitably chosen coordinate system. We adopt a nonorthogonal coordinate system in which a magnetic surface is labeled by the volume V which it contains and a line of constant θ and ζ intersects each surface once. Hamada[8] has shown that θ and ζ can be chosen such that

$$\vec{\nabla} V \cdot \vec{\nabla}\theta \times \vec{\nabla}\zeta = 1, \qquad (5)$$

for a system in which the magnetic field is given by

$$\vec{B} = \psi'(V)\vec{\nabla}V \times \vec{\nabla}\theta + \chi'(V)\vec{\nabla}\zeta \times \vec{\nabla}V. \qquad (6)$$

Here ψ is the flux through a constant ζ cross section of the surface, χ is the flux through a constant θ ribbon, one side of which lies in the surface and the other on the magnetic axis ($V=0$), and primes denote derivatives with respect to V.

We use the energy principle of Bernstein et al.,[9] which states that the system is stable if and only if the functional

$$2\delta W = \int \left\{ \left[\vec{\nabla} \times (\vec{\xi} \times \vec{B}) + \frac{\vec{J} \times \vec{\nabla} V}{|\vec{\nabla} V|^2} \vec{\xi} \cdot \vec{\nabla} V \right]^2 + \gamma p (\vec{\nabla} \cdot \vec{\xi})^2 + K(\vec{\xi})^2 \right\} dV d\theta d\zeta, \qquad (7)$$

where

$$K = -2[\vec{J} \times \vec{\nabla} V \cdot (\vec{B} \cdot \vec{\nabla}) \vec{\nabla} V] / |\vec{\nabla} V|^4, \qquad (8)$$

can not be made negative for some $\vec{\xi}(V, \theta, \zeta)$. For localized displacements this can be reduced by a series of algebraic minimizations to a functional of $\vec{\xi} \cdot \vec{\nabla} V$. From this form it can be shown that the necessary and sufficient condition for stability with respect to localized displacements is that

$$F = \left\{ \left[\tfrac{1}{2}(\psi'\chi'' - \psi''\chi') \oint \frac{dl}{|\vec{B}|} - \oint \frac{\vec{J} \cdot \vec{B}}{|\vec{\nabla} V|^2} \frac{dl}{|\vec{B}|} \right]^2 \Big/ \oint \frac{|\vec{B}|^2}{|\vec{\nabla} V|^2} \frac{dl}{|\vec{B}|} \right\} + \oint K \frac{dl}{|\vec{B}|} > 0, \qquad (9)$$

on each closed line of force, where the integrals are evaluated along that line of force.

Mercier[10] has given a necessary condition for the stability of these equilibria with respect to localized perturbations which corresponds to Eq. (9) but with the line integrals replaced by integrals over the magnetic surfaces. For helically invariant systems, which represent the most general two-dimensional configurations, each line integral is the same for all lines of force on the surface and Eq. (9) reduces to Mercier's condition.

Details of the calculation will be forthcoming.

*Work supported by the U. S. Atomic Energy Commission.

†On loan from Atomic Power Department, Westinghouse Electric Corporation, Pittsburgh, Pennsylvania.

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