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[5]Near the threshold for process (6) the relative angular momentum of the two pions will be $l=0$ (the next permitted value is $l=2$); under these conditions the azimuthal distribution of the α recoil is again uniquely determined. It is in fact $\cos^2\varphi$ for complete polarization so that contributions of (6) can mask the asymmetry in (1).

[6]For γ-ray energies below the pion threshold there exist particular photodisintegration reactions with well-defined analyzing power for the photon polarization.

PIONIC FORM FACTOR EFFECTS IN PERIPHERAL NUCLEON-NUCLEON COLLISIONS

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There is increasing evidence that in inelastic nucleon-nucleon collisions the one-pion exchange (OPE) contribution plays an important role.[1-8] This evidence rests: (i) at a few ($\simeq$1-3) Gev on the experimental lab energy distributions[2,4] and c.m. angular distributions[3]; (ii) at many ($\gtrsim$10) Gev on the analysis of quasi-elastic scattering data,[5] of general features of the interactions[6] of emulsion experiments,[7] and of cosmic-ray interactions.[8] A further step in the analysis of such reactions consists in understanding the quantitative importance of the direct many-particle exchanges, which are expected to be increasingly important with increasing momentum transfer. The effect of these exchanges cannot, however, be revealed by comparing the OPE calculations performed till now[1,2] with experiments, because such calculations are correct only for small values of the nucleon momentum transfer.

A method for understanding the limitations of the OPE approximation has been suggested by the authors.[9] Since the matrix element for off-shell pion-nucleon scattering in the region of the 3, 3 resonance has been calculated,[9] one can evaluate a peripheral graph for single (and possibly double) pion production in nucleon-nucleon collisions, by taking full account of the virtuality of the exchanged pion. This calculation is possible for energies of the πN scattering not too far from the 3, 3 resonance, and is undetermined only by an unknown multiplicative function of the squared four-momentum of the virtual pion. This function is simply related to the pionic form factor of the nucleon. These facts make possible a check of the peripheral model and possibly allow the determination of the pionic form factor. If the experimental data at different energies were fitted with a unique choice of this function, then one might conclude that the OPE contribution is probably dominant also for high-momentum transfer, because the discrepancies generated from the many-particle exchanges are expected to depend also on the initial energy.

In this Letter we show that the existing experimental data do indeed suggest full dominance of the OPE term and we obtain a tentative behavior of the pionic form factor. The expression found in reference 9 for the off-shell $T=J=\frac{3}{2}$ πN scattering amplitude is[10]

$$f_{33}(u,\Delta^2)\simeq K(\Delta^2)\left(1+\frac{\Delta^2}{4m^2}\right)^{1/2}\frac{p_1}{q_1}\frac{B_0(u_r,\Delta^2)}{B_0(u_r,-\mu^2)}f_{33}(u,-\mu^2), \tag{1}$$

where m and μ are the nucleon and pion masses, respectively; Δ^2 is the squared four-momentum of the virtual pion; $u=(\omega-m)/m$, ω being the total c.m. energy of the πN system; $u_r=0.314$ is the u value of the 3, 3 resonance; $p_1=(p_{10}{}^2-m^2)^{1/2}$ and $q_1=(q_{10}{}^2-m^2)^{1/2}$ are the off-shell and on-shell c.m. momenta, where

$$p_{10}=(\omega^2+m^2+\Delta^2)/2\omega, \quad q_{10}=(\omega^2+m^2-\mu^2)/2\omega. \tag{2}$$

Finally, $K(\Delta^2)$ is the pionic form factor of the nucleon and $B_0(u,\Delta^2)$ is given by

$$B_0(u,\Delta^2)=\frac{4}{3}\frac{f^2}{\mu^2}\frac{u}{(u+\delta)^2}\left[1+\frac{2\delta}{u+\delta}\right], \tag{3}$$

with $f^2=0.08$ and $\delta=(\Delta^2+\mu^2)/2m^2$.

In (1) only the factor p_1/q_1 depends on both Δ^2 and u. Its u dependence is particularly strong at threshold, where it diverges like $q_1{}^{-1}$. This divergence is, however, compensated from the threshold behavior of $f_{33}(u,-\mu^2)$ which vanishes like $q_1{}^2$. Therefore, the effect of the factor p_1/q_1 on the u dependence of $f_{33}(u,\Delta^2)$ should not be too large and we put simply $p_1{}^r/q_1{}^r$ in place of p_1/q_1, r meaning "calculated at resonance."

The off-shell amplitude is then

$$f_{33}(u,\Delta^2)\simeq K(\Delta^2)\psi(\Delta^2)f_{33}(u,-\mu^2), \tag{4}$$

with

$$\psi(\Delta^2)=\frac{p_1^{\,r}}{q_1^{\,r}}\left(1+\frac{\Delta^2}{4m^2}\right)^{1/2}\frac{u_r(u_r+3\delta)}{(u_r+\delta)^3}. \tag{5}$$

The conclusion is thus that in the calculation of a peripheral graph the "pole approximation" discussed in reference 9 is approximately valid also for higher momentum transfers, apart from a multiplicative function of Δ^2, given by[11]

$$\phi(\Delta^2)=K^2(\Delta^2)K'(\Delta^2)\psi(\Delta^2), \tag{6}$$

K' representing the higher order corrections to the pion propagator. Assuming

$$\phi(\Delta^2)=[1+(\Delta^2+\mu^2)/\alpha]^{-1}, \tag{7}$$

where α is an adjustable parameter, we have tried to fit the existing experimental data[3,4,12] on the reactions

$$p+p\rightarrow p+n+\pi^+, \tag{8}$$

$$p+p\rightarrow p+p+\pi^0. \tag{9}$$

We have taken into account all the possible peripheral diagrams contributing to our reactions, according to the calculations performed in reference 2 and with the introduction of the cutoff (7) in the appropriate variable of each graph. The results for the total cross sections are shown in Table I. The comparison with the experimental data definitely shows the need for a cutoff with characteristic mass in the range 60-90 μ^2. Although the experimental total cross section seems to decrease slightly faster than the theoretical one, one sees that the discrepancy with the pole approximation can be reasonably well represented with a Δ^2-dependent function. This fact suggests an OPE contribution much more important than expected also at high Δ^2.

The differential cross sections with respect to the lab energy of the nucleons (calculated with $\alpha=60\ \mu^2$) are shown in Figs. 1 and 2 and compared with the experimental ones. The agreement is still generally good.

If we take these results seriously we can obtain the function $K^2(\Delta^2)K'(\Delta^2)$ simply by dividing the phenomenological expression for $\phi(\Delta^2)$ by $\psi(\Delta^2)$. The behavior of the function is shown in Fig. 3 and suggests that the nucleon, as seen from a π meson, looks like a sphere of radius $\sim\mu^{-1}$ in which a much smaller core is contained. We wish to stress that this conclusion is very tentative. In fact, first the formulas used are approximate and have been applied also to regions where the 3, 3 resonance is no longer dominant,[13] and secondly the existing experimental data are still rather poor. What has some chance of being not too far from reality in Fig. 3 is the steep decrease of K^2K' in the region $-\mu^2\leq\Delta^2\lesssim 10\ \mu^2$. Concerning the "core" behavior one must wait for more detailed experimental information before having any believable conclusions. The main purpose of this Letter is thus to make clear the importance of further experimental investigation of reactions (8) and (9).

The formulas for the cross sections and a more exhaustive discussion of the same problems will be contained in a forthcoming paper.

The experimental cross sections used are mainly private communications: at 0.97 Gev from Dr. D. Y. Bugg (Cambridge) and at 2.85 Gev from Dr. G. A. Smith and Professor H. Taft (Yale). We are extremely grateful to the above authors for their kindness, which made the present work possible.

Table I. Experimental and theoretical total cross sections for reactions (8) and (9) at different lab kinetic energies of the incoming proton. See references 3, 4, and 6 for the experimental data. α is the cutoff parameter defined in (7). The column $\alpha=\infty$ corresponds to calculations without cutoff.

Reaction	Energy (Gev)	Experimental cross section (mb)	Theoretical cross section (mb) $\alpha=\infty$	$\alpha=60\,\mu^2$	$\alpha=90\,\mu^2$
$p+p\rightarrow p+n+\pi^+$	0.97	18.4 ± 0.8	25.2	16.4	18.4
	2.00	16.06 ± 0.44	26.3	15.2	17.3
	2.85	11.45 ± 0.65	21.3	11.3	13.2
$p+p\rightarrow p+p+\pi^0$	0.97	3.8 ± 0.35	5.6	3.3	3.9
	2.85	2.9 ± 0.31	5.6	2.75	3.2

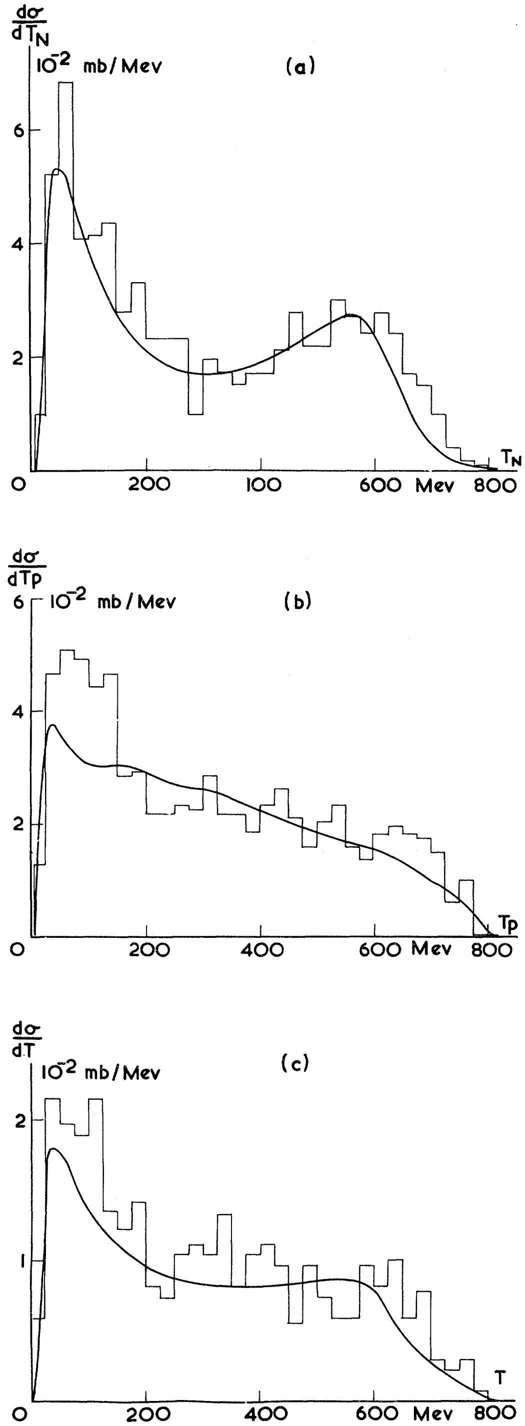

FIG. 1. Experimental (histograms) and theoretical (full lines) differential cross sections with respect to the laboratory kinetic energies at 0.97 Gev. (a) T_n: kinetic energy of the neutrons from reaction (8). (b) T_p: kinetic energy of the protons from reaction (8). (c) T: kinetic energy of the protons from reaction (9). See reference 4 for details on the experiment.

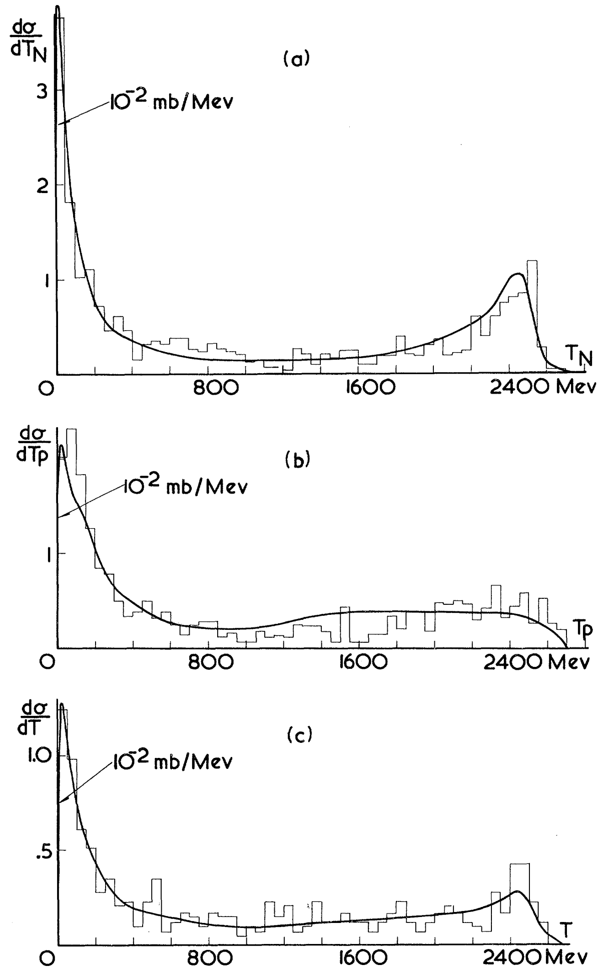

FIG. 2. Experimental (histograms) and theoretical (full lines) differential cross sections with respect to the laboratory kinetic energies at 2.85 Gev. (a) T_n: kinetic energy of the neutrons from reaction (8). (b) T_p: kinetic energy of the protons from reaction (8). (c) T: kinetic energy of the protons from reaction (9). See reference 6 for details on the experiment.

We are indebted to Dr. S. D. Drell and Professor L. Van Hove for reading the manuscript and for valuable comments.

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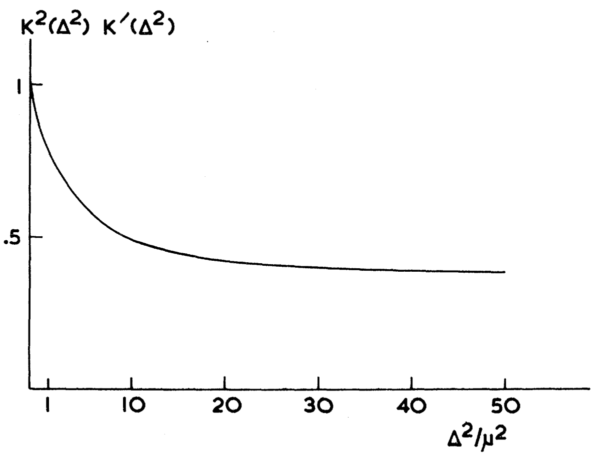

FIG. 3. The function K^2K' and its behavior with the momentum transfer, as obtained by dividing $\psi(\Delta^2)$ [see formula (5)] into $\phi(\Delta^2)$ [see formula (7)] with $\alpha = 60\mu^2$.

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[9]E. Ferrari and F. Selleri, Nuovo cimento (to be published); see also O. Iizuka and A. Klein, Progr. Theoret. Phys. (Kyoto) 25, 1018 (1961), where the same problem is discussed.

[10]Numerical estimates prove the smallness of the term containing $\cos\delta_{33}$ in formula (66) of reference 9. We therefore neglect it here.

[11]The origin of the extra factor KK' appearing in (6) should be clear: K comes from the 3-particle vertex, where the virtual π is emitted, and K' from its propagation.

[12]G. A. Smith, H. Courant, E. C. Fowler, H. Kraybill, J. Sandweiss, and H. Taft, Phys. Rev. 123, 2160 (1961).

[13]All the performed approximations (discussed also in reference 9) could eventually be avoided. The only assumption which is essential in this analysis is the dominance of the 3, 3 resonance. Its validity can be checked experimentally, for instance by comparing with the present theory the Q-value distributions of π^+p from reference 8 at fixed momentum transfer and variable initial energy.

SOLUTIONS OF THE PION-PION PARTIAL WAVE DISPERSION RELATIONS

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In 1958, Chew and Mandelstam obtained a set of integral equations for the pion-pion partial wave amplitudes.[1] The underlying physical assumption of these equations is that the exchange of a pion pair in pion-pion scattering should constitute the longest range force which, in turn, dominates the scattering amplitude at low energy. Since that time, a number of authors have published various approximate solutions to these equations based on the further assumption that the major part of the two-pion force comes from the relative S and P states of the exchanged pair.[1-5] The remaining portion of the two-pion force as well as the multipion forces are usually represented by phenomenological parameters. Unfortunately, there are mathematical difficulties even in handling the P-wave exchange force which necessitates some cutoff procedure. In principle, such cutoff procedures also require adjustable parameters. It is not clear from previous results what is the minimum number of parameters required for a realistic solution of the low-energy pion-pion problem. The purpose of this Letter is to clarify this point and to present a few numerical solutions which fall within the uncertainty of the present experimental data.[6]

The coupled S- and P-wave pion-pion dispersion equations[1] are solved numerically including left-hand cuts that satisfy crossing symmetry up to a cutoff point. The remaining left-hand cut of each partial wave amplitude is replaced by a pole together with a subtraction constant at the symmetry point $\nu = -\frac{2}{3}$, where ν is the center-of-mass momentum squared in pion units. The solutions obtained are found to depend sensitively on three parameters, and no further reduction in the number of parameters seems possible if only low partial waves are kept in all absorptive amplitudes.

The S-wave subtraction constants are related by the 5 to 2 ratio:

$$\lambda \equiv -\tfrac{1}{5}A_{l=0}^{I=0}(-\tfrac{2}{3}) = -\tfrac{1}{2}A_{l=0}^{I=2}(-\tfrac{2}{3}).$$