EVIDENCE FOR A PION-PION RESONANCE FROM PHOTOPRODUCTION OF PION PAIRS*

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In the reaction $\gamma + p \rightarrow \pi^+ + \pi^- + p$, we may expect that the behavior of the cross section will be influenced by interactions between the particles in the final state. We have made measurements of this cross section particularly designed to give evidence concerning the pion-pion interaction.

A difficulty in such an attempt is that the strong pion-proton interaction might obscure any pion-pion effect. Our approach has been that suggested by Drell and Zachariasen.[1] Let $E_{p\pi^+}$, $E_{p\pi^-}$, and $E_{\pi\pi}$ be the total energies of the indicated pairs of particles in the center-of-mass system of each pair. It is possible to vary the parameters of the reaction so that $E_{p\pi^+}$ and $E_{p\pi^-}$ are held constant, while $E_{\pi\pi}$ is varied.[2] Choosing constant values of $E_{p\pi^+}$ and $E_{p\pi^-}$ well away from any pion-proton resonance, it is hoped that the final-state nucleon-pion interaction will be kept small and substantially constant, so that a pion-pion resonance will reveal itself in a "bump" as $E_{\pi\pi}$ is varied.

The reaction is observed by detecting the proton and one of the pions in coincidence, measuring the direction and momentum of each. These parameters completely determine the kinematics, so that k, the incident γ-ray energy, and the momentum of the undetected pion are known. The experimental setup is shown in Fig. 1. The bremsstrahlung beam of the Cornell synchrotron passes through a 3-in. liquid hydrogen target and is monitored by a quantameter.[3] Particles of given direction and momentum are focussed by one of the two magnetic spectrometers, $M1$ and $M2$, into scintillation counter telescopes. Protons are unambiguously identified in $M1$ by pulse-height analysis in three scintillation counters. Pions are detected in $M2$, where protons are eliminated by requiring a Lucite Čerenkov counter in coincidence. A resolving time of 5 nanoseconds is achieved in the coincidence between proton and pion by photographing an oscilloscope trace displaying one signal from each telescope. Background from random coincidences averages about 5%. A given setting of the magnet angles and momenta determines the incident γ-ray energy at which the specified 2-pion reaction will occur; the peak bremsstrahlung energy is then set at a value well above this, but low enough so that no 3-pion process can contribute. For each point measurements are also made with the peak energy somewhat below the 2-pion energy, and this yield (typically 10%) is subtracted as background. Two measurements are made for each condition; one with the $M2$ polarity set to

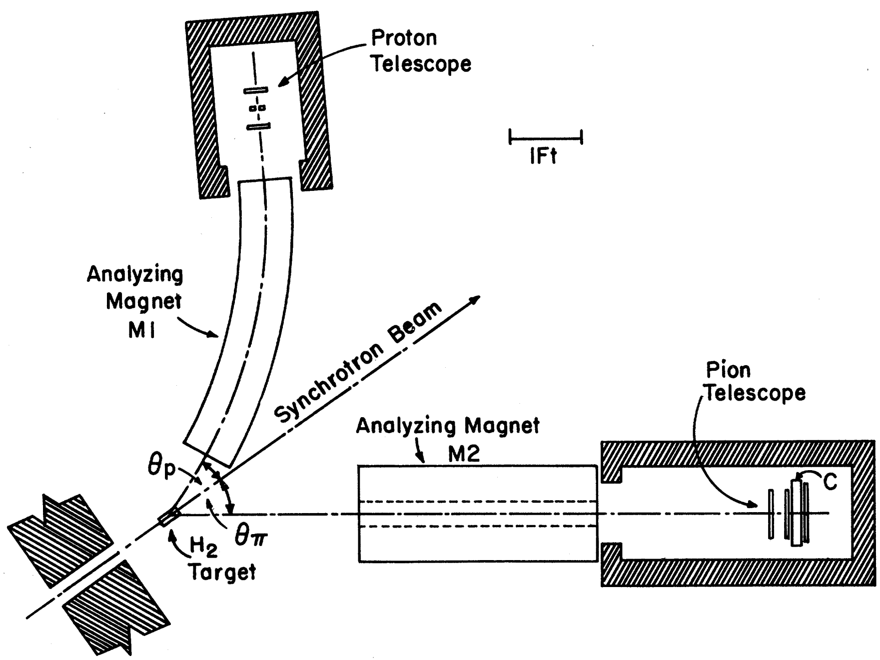

FIG. 1. Experimental setup. $M1$ is a two-lens, strong-focussing, 35° bending magnet, with parameters: $\Delta p/p = 12\,\%$; $\Delta\Omega = 4\times10^{-3}$ sr; $\Delta\theta = 5°$. $M2$ is a Panofsky-Hand type quadrupole magnet [L. N. Hand and W. K. H. Panofsky, Rev. Sci. Instr. 30, 927 (1959)] with a lead barrier in the center, as a momentum spectrometer with a line focus. Its parameters are: $\Delta p/p = 7.5\,\%$; $\Delta\Omega = 9\times10^{-3}$ sr; $\Delta\theta = 2°$. The counter marked C is a Lucite Čerenkov counter; all others are scintillation counters.

accept π^+, and the other for π^-.

Results are expressed in terms of M^2, the square of the invariant matrix element, which is related to the cross section and laboratory phase space, D, as follows. ($\hbar = c = 1$.)

$$d\sigma/(d\Omega_p d\Omega_{\pi 1} dp_p) = 2\pi M^2 D/(32\, k m_p E_p E_{\pi 1} E_{\pi 2}),$$

where

$$D = \frac{E_{\pi 1} E_{\pi 2}}{(2\pi)^6} \frac{p_{\pi 1}{}^3 p_p{}^2}{[p_{\pi 1}{}^2 (k + m_p - E_p) - E_{\pi 1} \vec{p}_{\pi 1} \cdot (\vec{k} - \vec{p}_p)]}.$$

Here k is the incident photon momentum; $\vec{p}$ and E are the laboratory momentum and energy of a final-state particle; the subscript p refers to the proton, π_1 to the detected pion, and π_2 to the other pion. In this form, M^2 has the dimensions of area.

Two series of measurements were made: one with $E_{p\pi 1} = 1.414$ Bev, $E_{p\pi 2} = 1.140$ Bev; a second with $E_{p\pi 1} = 1.414$ Bev, $E_{p\pi 2} = 1.161$ Bev. ($\pi 1$ is the pion detected in $M2$, $\pi 2$ is the other pion.) Figure 2 shows the results of the second series;

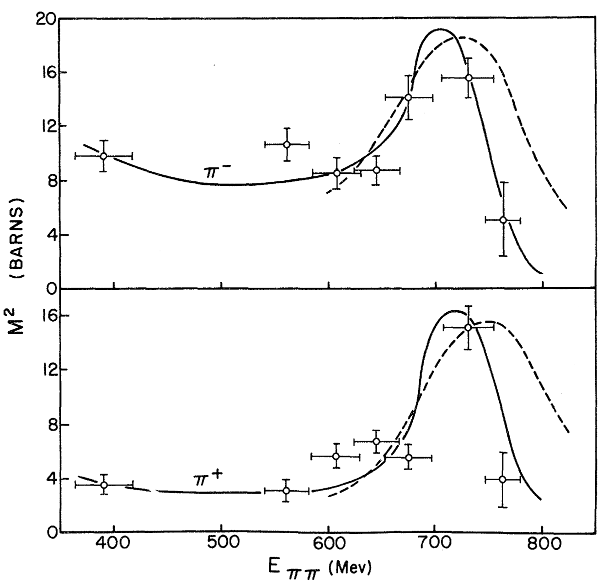

FIG. 2. M^2 vs $E_{\pi\pi}$ for $E_{p\pi^+} = 1.161$ Bev, $E_{p\pi^-} = 1.414$ Bev (upper half); and for $E_{p\pi^+} = 1.414$ Bev, $E_{p\pi^-} = 1.161$ Bev (lower half). In Figs. 2 and 3, errors indicated on M^2 are statistical, except for the special case noted in the text. Horizontal limits show the experimental resolution. The labels π^+, π^- indicate the sign of the detected pion. The curves shown are the results of the model described in the text, with the variation of the π-π phase shift determined to fit our data (solid lines), and determined from reference 4 (dashed lines). (See Fig. 4.)

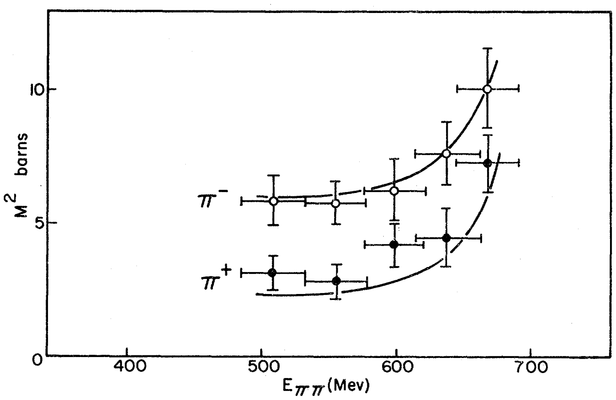

FIG. 3. M^2 vs $E_{\pi\pi}$ for $E_{p\pi^+} = 1.140$ Bev, $E_{p\pi^-} = 1.414$ Bev (open points); and for $E_{p\pi^+} = 1.414$ Bev, $E_{p\pi^-} = 1.140$ Bev (solid points).

Fig. 3, the first series. The labels π^+, π^- indicate the sign of the detected pion. Errors indicated on M^2 are counting statistics, except for the points at $E_{\pi\pi} = 760$ Mev. For these points the γ-ray energy determined by the magnet settings was very close to the peak beam energy. This led to a substantial correction factor critically dependent on the value taken for the beam energy. The indicated error includes a 1% uncertainty in energy. The horizontal bars show the experimental resolution. The maxima in Fig. 2 at $E_{\pi\pi} = 720$ Mev indicate a resonance in the π-π interaction at approximately this energy, in rough agreement with previous measurements.[4,5]

In an effort to correlate the data with features that might be expected due to final-state interactions, we considered the following simple model.[6] It is supposed that the reaction is initiated by some vertex producing π^+ and π^-, following which a single final-state scattering always occurs. The scatterings considered are p-π^+, p-π^-, and π^+-π^-. We now assume that the amplitude for each process is the product of the initiating amplitude, C (assumed constant, independent of energy), and the free-particle scattering amplitude; furthermore we assume that the π-p scattering occurs only in the 3-3 state, and the π-π scattering in a single $J = 1$ state. Then the amplitudes for the three processes are

$$CA_{p\pi^+} = C(8\pi)^{1/2} \chi_{p\pi^+} \exp(i\delta_{p\pi^+}) \sin\delta_{p\pi^+},$$

$$CA_{p\pi^-} = C(\tfrac{1}{3})(8\pi)^{1/2} \chi_{p\pi^-} \exp(i\delta_{p\pi^-}) \sin\delta_{p\pi^-}$$

$$CA_{\pi\pi} = C(12\,\pi)^{1/2} \chi_{\pi\pi} \exp(i\delta_{\pi\pi}) \sin\delta_{\pi\pi}$$

We neglect any considerations of proton spin and

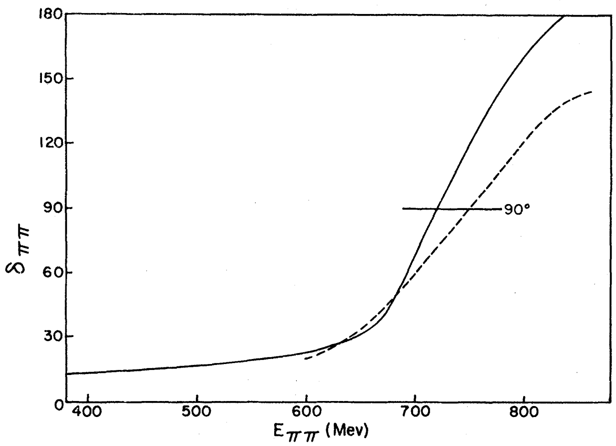

FIG. 4. $\delta_{\pi\pi}$ vs $E_{\pi\pi}$ fitting our data (solid curve), and as determined from reference 4 (dashed curve).

write the total amplitude as the sum of these amplitudes, however multiplying the π-π amplitude by an arbitrary weighting factor, η. Then we have for the production matrix element, $M^2 = C^2|A_{p\pi^+} + A_{p\pi^-} + \eta A_{\pi\pi}|^2$. For $\delta_{p\pi^+}$ and $\delta_{p\pi^-}$, we take the values from π-p scattering: $\delta(1.140) = 12°$; $\delta(1.161) = 20°$; $\delta(1.414) = 155°$.[7] Then the free parameters in the expression are $\delta_{\pi\pi}$ as a function of $E_{\pi\pi}$, C, and η. C is merely a normalizing parameter.

The solid curves in Figs. 2 and 3 are calculated from this expression with $C^2 = 0.38$ barn, $\eta = 1.6$,[8] and $\delta_{\pi\pi}(E_{\pi\pi})$ as given by the solid curve in Fig. 4. This form, which assumes a π-π resonance at $E_{\pi\pi} = 720$ Mev, of width 90 Mev,[9] reproduces the main features of the data. The dashed curves are calculated with $\delta_{\pi\pi}$ determined from the cross section given by Erwin et al.[4] (a resonance at $E_{\pi\pi} = 750$ Mev of width 150 Mev), assuming $\sigma_{\pi\pi} = 12\pi\lambda\!\!\!^{-2} \times \sin^2\delta_{\pi\pi}$. Our results indicate a resonance somewhat lower in energy and narrower in width than that of references 4 and 5. However, the lack of a reliable theory relating the resonance parameters to the measurements, and the large uncertainty in the point of highest $E_{\pi\pi}$ make our parameters for the resonance somewhat uncertain.

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[1]S. D. Drell and F. Zachariasen, Phys. Rev. Letters 5, 66 (1960).

[2]$E_{p\pi^+}$, $E_{p\pi^-}$, $E_{\pi\pi}$, and k are related: $2m_p k = E_{p\pi^+}{}^2 + E_{p\pi^-}{}^2 + E_{\pi\pi}{}^2 - 2(m_p{}^2 + m_\pi{}^2)$. Thus in executing the program described, k must be increased as $E_{\pi\pi}$ is increased. The required change is not large, and is expected to produce only a gradual variation of the cross section; thus it should not obscure a prominent $\pi\pi$ effect. k varied from 920 to 1225 Mev as $E_{\pi\pi}$ was varied from 400 to 760 Mev.

[3]R. R. Wilson, Nuclear Instr. 1, 101 (1957).

[4]A. R. Erwin, R. March, W. D. Walker, and E. West, Phys, Phys. Rev. Letters 6, 628 (1961); also D. Stonehill, C. Baltay, H. Courant, W. Fickinger, E. C. Fowler, H. Kraybill, J. Sandweiss, J. Sanford, and H. Taft, Phys. Rev. Letters 6, 624 (1961).

[5]E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters 7, 192 (1961).

[6]We are not aware of the existence of any treatment of final-state interactions involving three interacting particles.

[7]H. A. Bethe and F. de Hoffmann, Mesons and Fields (Row, Peterson and Company, Evanston, Illinois, 1955), Vol. 2, p. 125; W. D. Walker, J. Davis, and W. D. Shephard, Phys. Rev. 118, 1612 (1960).

[8]Unfortunately, the fit with $\eta = 1$ is not as good.

[9]This width is about equal to the experimental resolution; thus our data are consistent with a narrower width for the resonance. The calculated curves do not have the experimental resolution folded in.