ple (5) assuming Maxwellian statistics.

In these considerations, no explicit dependence on temperature of the carrier density has been taken into account. This is, at least for the samples (4) and (5), justified by the fact that the donor ionization energy tends to disappear for donor densities of the order of 10^{18} cm^{-3}.

Extending these measurements to longer wavelengths would increase their sensitivity as well as their accuracy for two reasons: First, the free-carrier effect increases proportionally to λ as long as relaxation effects may be neglected; and second, the lattice birefringence decreases as λ^{-1} in the absence of dispersion. Thus an improvement of the ratio of the electronic to the lattice contribution to birefringence proportional to λ^2 can be obtained. Experiments in this direction are in progress.

The author is indebted to Professor D. Polder, Dr. H. J. G. Meyer, and Dr. N. J. Harrick for stimulating discussions, to Miss I. Ruether and D. Restorff for carrying out the optical and the Hall measurements, and to A. Hauptmann who prepared the crystals.

[1]K. J. Schmidt-Tiedemann, Z. Naturforsch. 16a, 639 (1961).
[2]W. G. Spitzer and H. Y. Fan, Phys. Rev. 106, 882 (1957).
[3]D. Pines, Solid-State Physics, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1955), Vol. 1, p. 367.
[4]M. Cardona, W. Paul, and H. Brooks, Helv. Phys. Acta 33, 329 (1960).
[5]F. Pockels, Lehrbuch der Kristalloptik (B. G. Teubner, Leipzig and Berlin, Germany, 1906).
[6]J. McDougall and E. C. Stoner, Phil. Trans. Roy. Soc. (London) A237, 67 (1938).
[7]H. J. McSkimin, J. Appl. Phys. 24, 988 (1953).
[8]R. N. Dexter, H. J. Zeiger, and B. Lax, Phys. Rev. 104, 637 (1956).
[9]K. J. Schmidt-Tiedemann, J. Appl. Phys. 32, 2058 (1961).
[10]Y. Furukawa, J. Phys. Soc. Japan 16, 687 (1961).
[11]C. Herring and H. Vogt, Phys. Rev. 101, 944 (1956).
[12]H. Fritzsche, Phys. Rev. 115, 336 (1959).

INTERPRETATION OF THE ELECTROMAGNETIC RADIATION FROM ELECTRON PASSAGE THROUGH METAL FILMS

V. P. Silin and E. P. Fetisov
P. N. Lebedev Physical Institute, Moscow, U.S.S.R.
(Received October 18, 1961)

Recently two interesting experiments[1,2] were published, in which the electromagnetic radiation arising from the passage of fast electrons (25 kev) through thin (450-1500 A) silver films was investigated. At the light wavelength $\lambda = 3300 \pm 100$ A, the spectral distribution of the radiation was found to have a comparatively sharp peak. The strength of the peak proved to be a periodically varying function of the film thickness. Finally, the angular distribution of the radiation was investigated.

Interpreting their experimental results, the authors of those papers[1,2] consider them as corroboration of the theory of plasmons and, in particular, of the theoretical work by Ferrell[3] in 1958. In this communication we try to show that the observed electromagnetic radiation[1,2] is the transition radiation predicted by Ginzburg and Frank[4] as far back as 1946. At the same time, the regularities of the transition radiation detected in the experiments[1,2] are well described by the formula for the spectral and angular distributions of the transition radiation produced by the electrons passing through the film (normally to its surface) which was obtained by Pafomov[5] (see also Garibian and Chalikian[6]). For nonrelativistic particles and not very large values of the dielectric permeability, this formula takes the form

$$dW = (2e^2v^2/\pi c^3)d\omega d\theta \sin^3\theta \cos^2\theta \, |A(\omega,\theta)|^2, \quad (1)$$

$$A(\omega,\theta) = (\epsilon - 1)[(x+y)e^{-i\omega xd/c} + (x-y)e^{i\omega xd/c} - 2xe^{i\omega d/v}][(x+y)^2 e^{-i\omega xd/c} - (x-y)^2 e^{i\omega xd/c}]^{-1}, \quad (2)$$

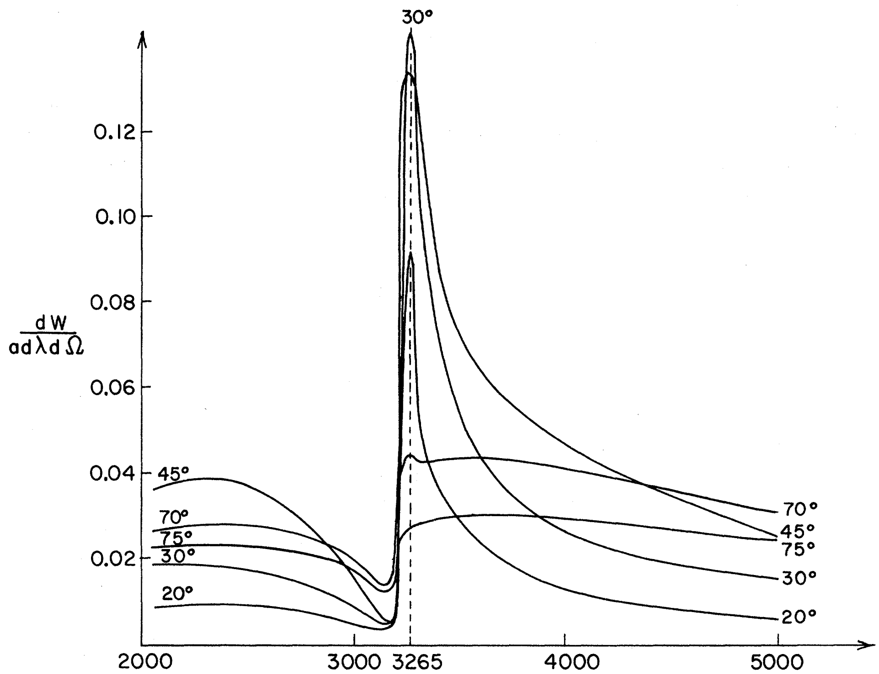

FIG. 1. Dependence of the radiation intensity on the wavelength λ calculated for a series of values of the angle θ at the following parameter values: $d = 500$ A, $v/c = 0.285$. (Coefficient $a = 2e^2v^2/\pi c^3$.)

$$x = (\epsilon - \sin^2\theta)^{1/2}, \quad y = \epsilon \cos\theta. \tag{3}$$

Here $\epsilon(\omega)$ is the complex dielectric permeability, corresponding to a field dependence on the time of $e^{-i\omega t}$. d is the film thickness.

For silver films the Wood transparency arises at the light wavelength of ~3400 A. Near such a wavelength and at not very large angles θ (for example, under conditions of the experiment of reference 2), and when $d = 500$ A, the parameter $(xd\omega/c)$ is small. Then the formula (1) can be written in the following form:

$$dW = (2e^2v^2/\pi c^3) d\omega d\theta \sin^3\theta \cos^2\theta \sin^2(\omega d/2v)$$

$$\times \left| \frac{\epsilon - 1}{\epsilon \cos\theta - i(\omega d/2c)(\epsilon - \sin^2\theta + \epsilon^2 \cos^2\theta)} \right|^2. \tag{4}$$

Now if one takes into account the fact[7] that the imaginary part of the dielectric permeability of silver near $\lambda = 3400$ A is approximately equal to 0.2-0.3, then one finds that Eq. (4) gives an angular distribution of the experimentally detected radiation (Fig. 2 of reference 2), an oscillatory dependence of the radiation intensity on film thickness with the period $(\omega d/v)$ (reference 1), and also a spectral distribution of the radiation which all have the right behavior to agree with experiment.

Since $(\omega x d/c)$ is not a small value in the whole

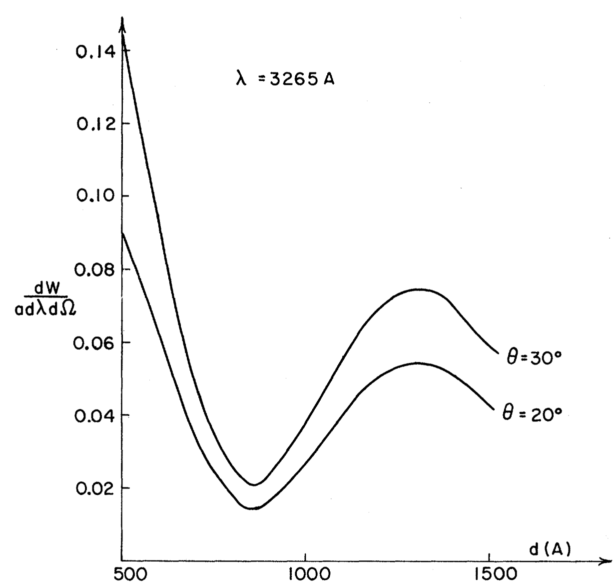

FIG. 2. Dependence of the radiation intensity on the film thickness calculated for the angles $\theta = 30°$ and $\theta = 20°$. (v/c and a are the same as in Fig. 1.)

region of frequencies and film thicknesses studied experimentally, we have in fact used Eqs. (1) and (2) for computing the yields shown in Figs. 1 and 2. It will be seen that these are in good agreement with the experiments. The values of the optical constants used in obtaining the graphs were taken from reference 7.

In the theory of transition radiation[4-6] it is assumed that only transverse waves can propagate in the medium. For investigation of propagation of longitudinal waves (plasmons) in the medium one must take into account a spatial dispersion of the dielectric permeability.[8] Since in this case existence of waves of three polarizations proves to be possible in the medium, it is necessary to make certain physical statements equivalent to additional boundary conditions,[8-11] in order to obtain solutions of the field equations.

In this communication, the following model was used for revealing the role of longitudinal waves (plasmons) for the conditions of the experiments.[1,2] Since in the conditions under consideration the product of the wavelength vector k of a light wave by the characteristic velocity of the electron is small in comparison with the frequency and we deal with the region around the Wood transparency, one can consider a weak spatial dispersion.[8,9]

In this case a spatial dispersion of the transverse dielectric permeability, owing to its small value, was neglected, and the longitudinal permeability was taken in the form: $\epsilon^{l} = \epsilon - \alpha(k/\omega)^2$. The value α, for example, in the model of free electrons, has the form

$$\alpha = \tfrac{3}{5} v_0^{\,2} \omega_L^{\,2}/\omega^2,$$

where v_0 is the velocity of electrons on the Fermi surface and ω_L is the Langmuir frequency of electrons.

The following boundary condition was used: The normal component of the field of the radiation going inside the metal is equal to zero on the surface. Such a boundary condition was obtained by a microscopic consideration of the problem of oblique incidence of the electromagnetic wave on a semi-infinite medium where the electrons of the medium suffer a mirror reflection at the surface.[10]

Then, instead of the formula (2), we obtain

$$\frac{\epsilon}{\epsilon - \alpha/v^2}(\epsilon - 1 - \alpha/v^2)\Big\{(x+y)e^{-i\omega x d/c} + (x-y)e^{i\omega x d/c} - 2x e^{i\omega d/v} + \frac{\sin^2\theta}{z}\Big[e^{-i\omega x d/c} - e^{i\omega d/v} + e^{i\omega z d/c}$$
$$- e^{i\omega d/v + i\omega z d/c - i\omega x d/c}\Big]\Big\}\Big\{(x+y)^2 e^{-i\omega x d/c} - (x-y)^2 e^{i\omega x d/c} + 2\frac{\sin^2\theta}{z}\Big[(y-x)e^{i\omega z d/c} + (x+y)e^{-i\omega x d/c}\Big]$$
$$- \frac{\sin^4\theta}{z^2}\Big[e^{2i\omega z d/c - i\omega x d/c} - e^{-i\omega x d/c}\Big]\Big\}^{-1}, \tag{5}$$

where $z = [\epsilon(c^2/\alpha) - \sin^2\theta]^{1/2}$.

The origin of the expression $\epsilon - \alpha/v^2$ in the formula (5) which makes, in particular, this formula different from formula (2), depends on the fact that, in taking into account the spatial Fourier expansion of the charge field, it is proportional to $1/[\epsilon - \alpha(k/\omega)^2]$.

Since α is roughly equal to the square of the Fermi electron velocity ($v_0 \sim 10^8$ cm/sec), it is obvious that at the incident electron energy of ~ 25 kev the value (α/v^2) is equal to the order of 10^{-4} and cannot be discerned in the experiments.[1,2] That is why it is impossible to see the change of form caused by the Coulomb field. It is clear that for revealing such an effect, it is necessary to make the energy of the incident electrons considerably less, and, also, to use targets with a smaller imaginary part of the dielectric permeability; for example, the alkali metals. Another distinction of formula (5) from formula (2) involves the terms proportional to $1/z$ and $1/z^2$. Note here that any changes in the boundary conditions lead to modifications in the coefficients in front of such terms. Nevertheless, the order of magnitude does not change.[12] On the other hand, since in our conditions z is rather large (namely, $z \sim 100$), for revealing the role of terms $1/z$ in the formula (5), measurements with an accuracy of about a percent are necessary. The main dependence of the radiation arising from electrons passing through the metal film on frequency, angle, and film thickness is determined, in the conditions of the experiments,[1,2] by the formulas (1) and (2).[13]

Thus, consideration of the possibility of propagation of longitudinal waves (plasmons) in the matter and also the possibility of their transformation into transverse electromagnetic waves radiating into vacuum does not lead, in the conditions of the experiments,[1,2] to effects which are appreciable compared to the transition radiation, the theory of which agrees with the experimental results.[1,2]

The authors wish to thank V. L. Ginzburg and I. M. Frank for interest in the work and also L. V. Parijskaya for preparing the graphs.

[1]W. Steinmann, Phys. Rev. Letters 5, 470 (1960).

[2]R. W. Brown, P. Wessel, and E. P. Trounson, Phys. Rev. Letters 5, 472 (1960).

[3]R. A. Ferrell, Phys. Rev. 111, 1214 (1958).

[4]V. L. Ginzburg and I. M. Frank, J. Exptl. Theoret. Phys. (U.S.S.R.) 16, 15 (1946).

[5]V. E. Pafomov, J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 1074 (1957) [translation: Soviet Phys. – JETP 6(33), 829 (1958)].

[6]G. M. Garibian and G. A. Chalikian, J. Exptl. Theoret. Phys. (U.S.S.R.) 35, 1282 (1958) [translation: Soviet Phys. – JETP 35(8), 894 (1959)].

[7]E. A. Taft and H. R. Philipp, Phys. Rev. 121, 1100 (1961).

[8]A. A. Rukhadze and V. P. Silin, Electromagnetic Properties of Plasma and Plasma-Like Media (Atomizdat, U.S.S.R., 1961).

[9]V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 1954 (1958).

[10]V. P. Silin and E. P. Fetisov, J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 159 (1961) [translation: Soviet Phys. – JETP (to be published)].

[11]V. L. Zhelnov, J. Exptl. Theoret. Phys. (U.S.S.R.) 40, 170 (1961) [translation: Soviet Phys. – JETP 13, 117 (1961)].

[12]Compare, for example, with reference 11.

[13]This result agrees with the conclusion obtained in reference 11 for transition radiation from one boundary.