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EXPERIMENTAL EVIDENCE OF BIREFRINGENCE BY FREE CARRIERS IN SEMICONDUCTORS

K. J. Schmidt-Tiedemann
Philips Research Laboratory Hamburg, Hamburg, Germany
(Received October 9, 1961)

Recently it has been proposed[1] that the free carriers in a semiconductor should exhibit optical birefringence if their effective mass, averaged over the ensemble of carriers, is anisotropic. It is the purpose of this Letter to present experimental evidence of this effect.

In general, the free carriers in a semiconductor make a contribution, due to their polarizability, to the relative dielectric constant of the crystal which, in the infrared region, is given by

$$\epsilon_F = -(e^2/\epsilon_0\omega^2)n\langle 1/m\rangle. \quad (1)$$

Here e is the electronic charge, ϵ_0 the dielectric constant of free space, ω the angular frequence of the light wave, n the carrier density, and $\langle 1/m\rangle$ the average of the reciprocals of the effective masses of the carriers. Equation (1) is based on several approximations which are discussed by various authors.[2-4]

The average effective mass can be made anisotropic, for example, by disturbing the equipopulation of the sub-bands in a many-valley semiconductor by means of an electric or a strain field.

The present experiments have been carried out with strained n-type germanium. The free-carrier birefringence should reveal itself by a contribution to the stress optical constant Q_{44}, which is given by

$$Q_{44} = \Delta/dX. \quad (2)$$

Here Δ is the relative phase shift of the two light waves in the crystal, d the thickness of the crystal in the direction of the light beam, and X the magnitude of an uniaxial tensile stress in the $\langle 111\rangle$ direction. The wave normal of the radiation must be perpendicular to the stress axis. The other stress optical constant, $Q_{11}-Q_{12}$, should, to a first approximation, remain unaltered by intervalley transitions. The free-carrier effect enters into this quantity only by an isotropic change of the refractive index of the crystal.

In order to obtain an expression for the contribution of the free carriers to the birefringence of the strained crystal, we started with Eq. (1) and evaluated $\langle 1/m\rangle$ by the usual deformation potential formalism. Then we evaluated the stress optical constants from the anisotropic part of the dielectric tensor.[5] It turns out that the stress optical constant Q_{44} is the sum of a contribution due to the lattice, denoted by Q_{44}^{L}, and a free-carrier part according to

$$Q_{44} = Q_{44}^{L} - AnT^{-1}F_{1/2}'(\eta_0)/F_{1/2}(\eta_0), \quad (3)$$

where

$$A = e^2[(m_0/m_t)-(m_0/m_l)]\Xi_u S_{44}\lambda/(36\pi\epsilon_0 m_0 c^2 k n_0). \quad (4)$$

Here T is the absolute temperature, m_0 the free-electron mass, m_t and m_l the transverse and longitudinal effective mass, respectively, Ξ_u the deformation potential constant and S_{44} the elastic compliance for uniaxial shear, c the velocity and λ the vacuum wavelength of the light, k the Boltzmann constant, and n_0 the refractive index of the doped crystal. $F_{1/2}(\eta_0)$ is the Fermi integral[6] of order 1/2, and the argument η_0 is the position of the Fermi level with respect to the conduction band minima in the unstrained crystal. $F_{1/2}'$ is the derivative of $F_{1/2}$. In the limiting case of small electron densities or high temperatures Maxwellian statistics applies, and in Eq. (3) the factor $F_{1/2}'/F_{1/2}$ is then replaced by unity. Absorption effects by free carriers have been neglected in the above analysis.

For the numerical calculations, the following values for the parameters involved have been used: 1.435×10^{-6} kg^{-1} cm^2 for the elastic compliance S_{44},[7] and the cyclotron resonance values[8]

for the effective-mass parameters.

The measurements were carried out with infrared radiation of a vacuum wavelength of $2.2 \pm 0.05\ \mu$. The thickness d of the n-type germanium samples in the direction of the light beam was 2 mm. An uniaxial tensile stress up to 300 kg/cm^2 was applied along the $\langle 111 \rangle$ direction. The phase shift was detected by means of a Senarmont-type compensator, and Q_{44} was derived from Eq. (2). Details of the apparatus have been given elsewhere.[9]

Figure 1 shows the experimental results. The uppermost curve (1), taken from an undoped sample, represents the normal photoelasticity of the germanium lattice which is nearly independent of temperature. The following curves are enumerated in the order of increasing doping density.

In order to check the quantitative agreement between theory and experiment, the carrier density n has been determined by Hall measurements.

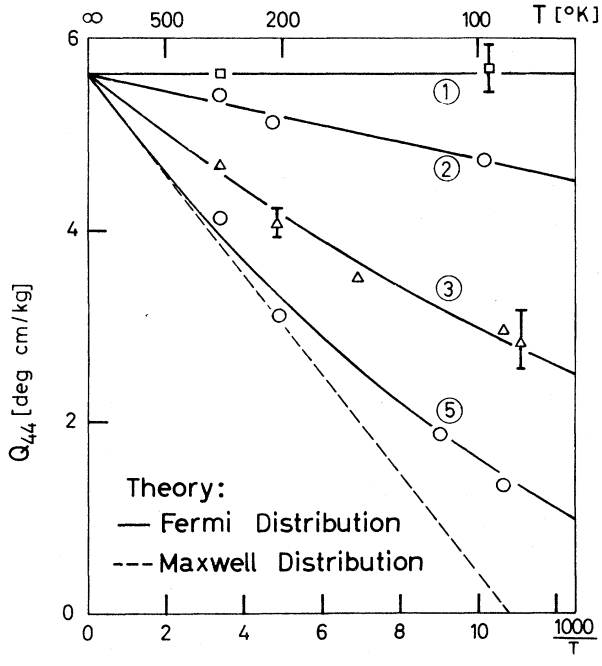

FIG. 1. Temperature dependence of the stress optical constant Q_{44} for n-type germanium samples of different carrier densities. The curves are denoted by the same numbers as their corresponding samples in Table I. The full lines are theoretical curves calculated from the data given in Table I by means of Eqs. (3) and (4). For the sake of clarity, the curve corresponding to sample (4) has been omitted. The broken line represents a calculated relationship corresponding to sample (5) assuming a Maxwellian carrier distribution function.

Additionally, it has been calculated from the resistivity using the resistivity vs carrier density relationship established by Furukawa[10] by means of Hall measurements. The ratio of Hall to drift mobility has been set equal to 0.926 for the samples (2) and (3) corresponding to Maxwellian statistics,[11] and equal to 0.784 for the samples (4) and (5) corresponding to strong degeneracy.[11] The influence of impurity scattering, though presumably not negligible in this range of carrier densities, has not been taken into account.

From these carrier density values and the measured stress optical constants, the deformation potential constant Ξ_u has been calculated by means of Eq. (3) for a lattice temperature of 90°K. The results as given in Table I yield a value of 18.9 $\pm$1.7 ev, which is in fair agreement with Fritzsche's value of 18.5 ev determined from piezoresistance measurements.[12] Additionally, the full curves in Fig. 1, which have been calculated from Eq. (3) using corresponding values of n and Ξ_u, give a satisfying representation of the temperature dependence of Q_{44}. We regard this agreement between theory and experiment as a confirmation of the proposed concept of free-carrier birefringence.

If the Maxwellian approximation is introduced for the carrier distribution function, we get a good fit for the sample (2). However, this approximation obviously fails to apply for more strongly doped samples, as can be seen from the broken line in Fig. 1 which corresponds to sam-

Table I. Resistivities ρ in 10^{-3} ohm cm, carrier densities n_H in 10^{18} cm^{-3}, and deformation potential constants Ξ_u in ev as derived from birefringence measurements, for different n-type germanium samples. All data are taken with antimony-doped samples.

Sample	ρ	n_H [a]	n_H [b]	Ξ_u [c]	Ξ_u [d]
(1)	$>10^3$	...	...	...	...
(2)	17.7	0.20	...	20.6	...
(3)	7.3	0.89	0.74	17.8	20.6
(4)	5.1	1.26	1.1	16.4[e]	18.3
(5)	4.6	1.33	1.26	17.2	18.5

[a] Author's Hall measurements.
[b] Furukawa's data (reference 10).
[c] Calculated from values of n_H given in the third column.
[d] Calculated from values of n_H given in the fourth column.
[e] This sample showed strong inhomogeneities of impurity distribution.

ple (5) assuming Maxwellian statistics.

In these considerations, no explicit dependence on temperature of the carrier density has been taken into account. This is, at least for the samples (4) and (5), justified by the fact that the donor ionization energy tends to disappear for donor densities of the order of 10^{18} cm^{-3}.

Extending these measurements to longer wavelengths would increase their sensitivity as well as their accuracy for two reasons: First, the free-carrier effect increases proportionally to λ as long as relaxation effects may be neglected; and second, the lattice birefringence decreases as λ^{-1} in the absence of dispersion. Thus an improvement of the ratio of the electronic to the lattice contribution to birefringence proportional to λ^2 can be obtained. Experiments in this direction are in progress.

The author is indebted to Professor D. Polder, Dr. H. J. G. Meyer, and Dr. N. J. Harrick for stimulating discussions, to Miss I. Ruether and D. Restorff for carrying out the optical and the Hall measurements, and to A. Hauptmann who prepared the crystals.

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INTERPRETATION OF THE ELECTROMAGNETIC RADIATION FROM ELECTRON PASSAGE THROUGH METAL FILMS

V. P. Silin and E. P. Fetisov
P. N. Lebedev Physical Institute, Moscow, U.S.S.R.

(Received October 18, 1961)

Recently two interesting experiments[1,2] were published, in which the electromagnetic radiation arising from the passage of fast electrons (25 kev) through thin (450-1500 A) silver films was investigated. At the light wavelength $\lambda = 3300 \pm 100$ A, the spectral distribution of the radiation was found to have a comparatively sharp peak. The strength of the peak proved to be a periodically varying function of the film thickness. Finally, the angular distribution of the radiation was investigated.

Interpreting their experimental results, the authors of those papers[1,2] consider them as corroboration of the theory of plasmons and, in particular, of the theoretical work by Ferrell[3] in 1958. In this communication we try to show that the observed electromagnetic radiation[1,2] is the transition radiation predicted by Ginzburg and Frank[4] as far back as 1946. At the same time, the regularities of the transition radiation detected in the experiments[1,2] are well described by the formula for the spectral and angular distributions of the transition radiation produced by the electrons passing through the film (normally to its surface) which was obtained by Pafomov[5] (see also Garibian and Chalikian[6]). For nonrelativistic particles and not very large values of the dielectric permeability, this formula takes the form

$$dW = (2e^2v^2/\pi c^3)d\omega d\theta \sin^3\theta \cos^2\theta |A(\omega,\theta)|^2, \quad (1)$$

$$A(\omega,\theta) = (\epsilon - 1)[(x+y)e^{-i\omega xd/c} + (x-y)e^{i\omega xd/c} - 2xe^{i\omega d/v}][(x+y)^2 e^{-i\omega xd/c} - (x-y)^2 e^{i\omega xd/c}]^{-1}, \quad (2)$$