MAGNETORESISTANCE OF TRANSITION METALS IN THE HIGH-FIELD LIMIT

E. Fawcett*
Physics Department, Royal Radar Establishment, Malvern, England
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Information about the electronic structure of a metal may be obtained from a study of its magnetoresistance in the high-field limit, in which $\omega_c\bar{\tau} \gg 1$ for all cyclotron orbits on all sheets of the Fermi surface, ω_c being the cyclotron frequency and $\bar{\tau}$ the relaxation time averaged over the orbit. In odd-valent metals the resistivity tends to saturation as the high-field limit is approached, except for field directions such that open orbits on a multiply-connected sheet of the Fermi surface contribute to the conductivity, in which case the resistivity approaches quadratic variation with the magnetic field. In even-valent metals the cancellation of the linear Hall term in the conductivity tensor due to the equal numbers of electrons and holes leads to quadratic variation for general field and current orientations. This remains true for multiply-connected surfaces, but open orbits nearly perpendicular to the current direction will now lead to saturation for the corresponding field direction.[1]

The extension of these ideas to the transition metals is by no means obvious, if only for the simple reason that the chemical valency will not in general equal the number of conduction electrons, i.e., the number of occupied states in partially filled Brillouin zones. However, it is proposed here that this number is always odd or even as the atomic number[2] Z. This hypothesis appears to be true for the nontransition metals,[1] and for the transition metals it is supported by new experimental results for high-purity single crystals of Mo, Nb, and Ta. These are illustrated in Fig. 1, together with published data for W,[3] Pt,[4,5] and Rh.[5] (See Table I.) The even-Z metals Mo, W, and, for general field and current orientations, Pt exhibit approximately quadratic variation

Table I. Published data for the transition metals.

	Z	Lattice	$\mathfrak{r}$	T (°K)	θ_D(°K)[a]	Sample axis
Mo	42	bcc	1570	2.16	380	4° to [001]
Nb	41	bcc	340	10.6	250	near [110]
Ta	73	bcc	208	2.16	230	random
W[b]	74	bcc	1405	20	315	near [111]
			465	20	315	near [001]
Pt[c]	78	fcc	1400	4.2	225	10° to [001]
Pt[d]			395	4.2	(240)	polycrystal
Rh[d]	45	fcc	500	4.2	(370)	polycrystal

[a] See reference 7.
[b] See reference 3.
[c] See reference 4.
[d] See reference 5.

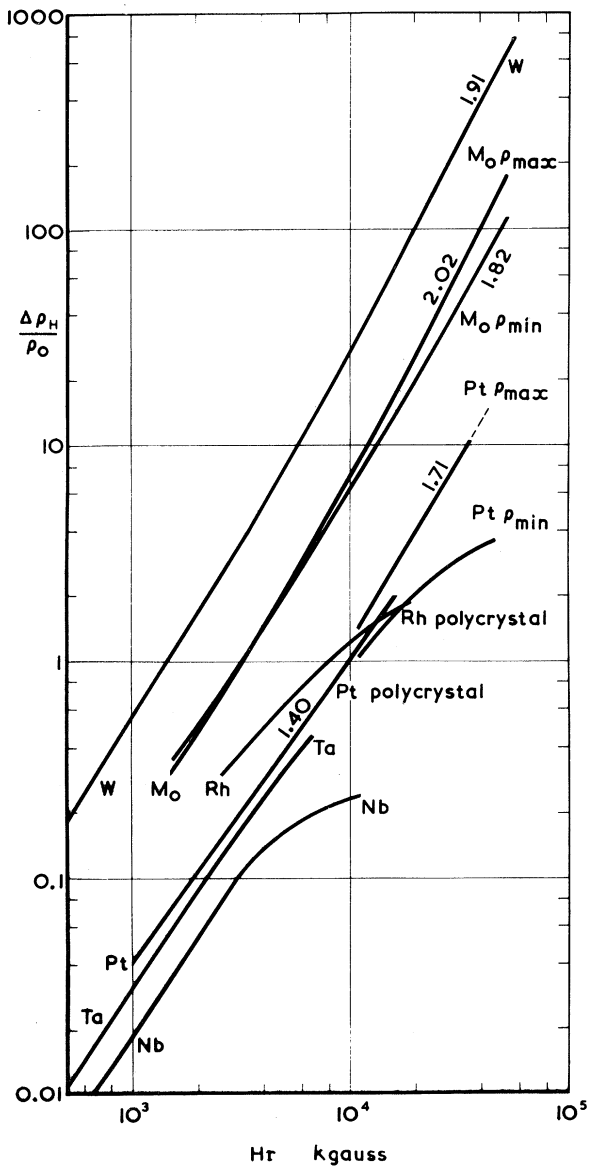

FIG. 1. Magnetoresistance of transition metals. The fractional change in resistivity, $\Delta\rho_H/\rho_0 = (\rho_H - \rho_0)/\rho_0$, is plotted against reduced magnetic field $H\mathfrak{r}$, where $\mathfrak{r} = \rho_0(\theta_D)/\rho_0(T)$ and θ_D is the Debye temperature (Kohler diagram). The exponent m in the expression $\Delta\rho_H/\rho_0 = CH^m$ is shown when the logarithmic plot is linear at the highest fields.

of the resistivity with field which indicates an even number of conduction electrons, while the odd-Z metals Nb and Rh approach saturation at the highest fields. For Ta (odd Z) the highest reduced field[6] Hr achieved may still be too low to show the onset of saturation.

Single crystals of Mo with current directions near [001], [112], and [110] have been measured in a transverse magnetic field. As the field is rotated about the current direction the resistivity exhibits a marked anisotropy, which is illustrated for a [001] sample in Fig. 2. The field dependence of the resistivity is measured in the direction of minima and maxima, and is always linear on a logarithmic plot at the highest fields (as illustrated for the [001] sample in Fig. 1), the exponent m ranging from 1.75 to 2.04. The absence of cusp-like minima with saturating resistivity shows that open orbits do not occur, which suggests that all sheets of the Fermi surface are simply connected. Further, this behavior being characteristic of even-valent metals for which $\omega_c\bar{\tau} \gg 1$ for all sheets, it is probable that the mobilities of charge carriers in the s and d bands are of the same order of magnitude. The same conclusions are true for W, and the close resemblance of the anisotropy of its magnetoresistance[8] to that of Mo suggests that the shape of the Fermi surface is very similar for the two metals.

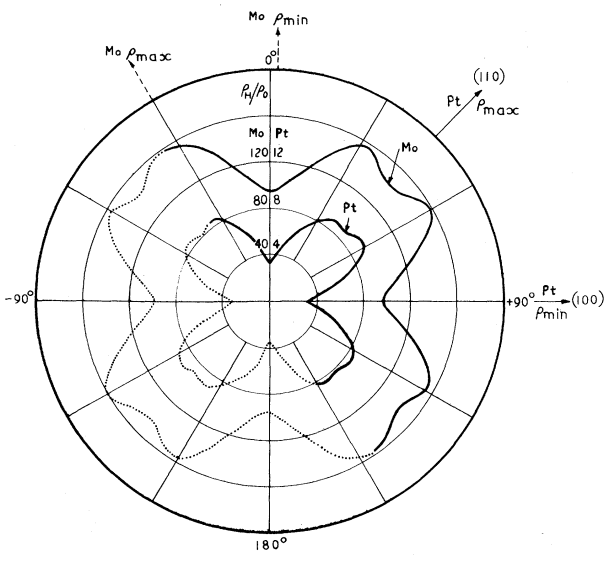

FIG. 2. Anisotropy of the magnetoresistance for Mo and Pt.[4] Mo—outer curve, $H = 31.3$ kgauss, $T = 2.16$°K; Pt (reference 4)—inner curve, $H = 23.5$ kgauss, $T = 4.2$°K. The data for these samples are given in Fig. 1 and Table I, where the field dependence of the resistivity in the directions ρ_{max} and ρ_{min} is illustrated.

The anisotropy of the magnetoresistance for the [001] sample of Pt,[4] shown in Fig. 2, is also superficially similar to that for Mo. But for Pt there is a cusp-like minimum when the field is in the [100] direction, where the resistivity saturates as illustrated in Fig. 1 (curve ρ_{min}). For the other field directions the resistivity rises approximately quadratically at the highest fields (curve ρ_{max}). This behavior indicates an even number of conduction electrons with one (or more) sheets of the Fermi surface multiply connected and giving rise to open orbits for appropriate field directions.

The resistivity for both the Nb and Ta samples in a field $H \sim 30$ kgauss is isotropic to within a few percent, indicating closed Fermi surfaces. The odd-valent behavior of the Nb samples is not conclusive evidence of a noneven number of conduction electrons, since the high electronic specific heat suggests that the effective mass of charge carriers in the d band is high so that perhaps $\omega_c\bar{\tau} < 1$ even at the highest fields. Measurements on purer samples or at higher fields showing continued saturation of the resistivity for Nb and other odd-Z metals would confirm the proposed correlation between Z and the number of conduction electrons.

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*Present address: Bell Telephone Laboratories, Murray Hill, New Jersey.

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[2]This statement is strictly correct only for metals having simple translation lattices, as have all the metals considered here. For a metal having s atoms in the primitive unit cell sZ determines the even- or odd-valent character of the magnetoresistance. For example, Ga, which is trivalent with $Z = 31$, has an orthorhombic lattice with $s = 8$. Thus sZ is even and the resistivity correspondingly increases quadratically to the highest fields for general field and current orientations [J. Yahia and J. A. Marcus, Phys. Rev. 113, 137 (1959)]; Professor Marcus informs the author that recent measurements by W. A. Reed show a saturating resistivity indicating open orbits for some orientations).

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[6]Note that $H\tau=(m_c/e\tau_{\theta D})\omega_c\bar{\tau}$, m_c being an average cyclotron effective mass and $\tau_{\theta D}$ the electron relaxation time at the Debye temperature, which is roughly the same for different metals [but see J. Ziman, Electrons and Phonons (Clarendon Press, Oxford, 1960), p. 492].

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EXPERIMENTAL EVIDENCE OF BIREFRINGENCE BY FREE CARRIERS IN SEMICONDUCTORS

K. J. Schmidt-Tiedemann
Philips Research Laboratory Hamburg, Hamburg, Germany
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Recently it has been proposed[1] that the free carriers in a semiconductor should exhibit optical birefringence if their effective mass, averaged over the ensemble of carriers, is anisotropic. It is the purpose of this Letter to present experimental evidence of this effect.

In general, the free carriers in a semiconductor make a contribution, due to their polarizability, to the relative dielectric constant of the crystal which, in the infrared region, is given by

$$\epsilon_F = -(e^2/\epsilon_0\omega^2)n\langle 1/m\rangle. \quad (1)$$

Here e is the electronic charge, ϵ_0 the dielectric constant of free space, ω the angular frequence of the light wave, n the carrier density, and $\langle 1/m\rangle$ the average of the reciprocals of the effective masses of the carriers. Equation (1) is based on several approximations which are discussed by various authors.[2-4]

The average effective mass can be made anisotropic, for example, by disturbing the equipopulation of the sub-bands in a many-valley semiconductor by means of an electric or a strain field.

The present experiments have been carried out with strained n-type germanium. The free-carrier birefringence should reveal itself by a contribution to the stress optical constant Q_{44}, which is given by

$$Q_{44} = \Delta/dX. \quad (2)$$

Here Δ is the relative phase shift of the two light waves in the crystal, d the thickness of the crystal in the direction of the light beam, and X the magnitude of an uniaxial tensile stress in the $\langle 111\rangle$ direction. The wave normal of the radiation must be perpendicular to the stress axis. The other stress optical constant, $Q_{11}-Q_{12}$, should, to a first approximation, remain unaltered by intervalley transitions. The free-carrier effect enters into this quantity only by an isotropic change of the refractive index of the crystal.

In order to obtain an expression for the contribution of the free carriers to the birefringence of the strained crystal, we started with Eq. (1) and evaluated $\langle 1/m\rangle$ by the usual deformation potential formalism. Then we evaluated the stress optical constants from the anisotropic part of the dielectric tensor.[5] It turns out that the stress optical constant Q_{44} is the sum of a contribution due to the lattice, denoted by Q_{44}^{L}, and a free-carrier part according to

$$Q_{44} = Q_{44}^{L} - AnT^{-1}F_{1/2}'(\eta_0)/F_{1/2}(\eta_0), \quad (3)$$

where

$$A = e^2[(m_0/m_t)-(m_0/m_l)]\Xi_u S_{44}\lambda/(36\pi\epsilon_0 m_0 c^2 k n_0). \quad (4)$$

Here T is the absolute temperature, m_0 the free-electron mass, m_t and m_l the transverse and longitudinal effective mass, respectively, Ξ_u the deformation potential constant and S_{44} the elastic compliance for uniaxial shear, c the velocity and λ the vacuum wavelength of the light, k the Boltzmann constant, and n_0 the refractive index of the doped crystal. $F_{1/2}(\eta_0)$ is the Fermi integral[6] of order 1/2, and the argument η_0 is the position of the Fermi level with respect to the conduction band minima in the unstrained crystal. $F_{1/2}'$ is the derivative of $F_{1/2}$. In the limiting case of small electron densities or high temperatures Maxwellian statistics applies, and in Eq. (3) the factor $F_{1/2}'/F_{1/2}$ is then replaced by unity. Absorption effects by free carriers have been neglected in the above analysis.

For the numerical calculations, the following values for the parameters involved have been used: 1.435×10^{-6} kg^{-1} cm^2 for the elastic compliance S_{44},[7] and the cyclotron resonance values[8]