# EXPERIMENTAL TESTS OF MACH'S PRINCIPLE*

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In this note it will be shown that, contrary to the suggestion of Cocconi and Salpeter,[1] the extremely precise null result of the experiments of Hughes, Robinson, and Beltrow-Lopez[2] and Drever[3] is to be expected, resulting from an application of Mach's principle.

According to Mach's principle, as formulated by Bishop Berkeley,[4] Mach,[5] and Sciama,[6] the inertial forces experienced in an accelerated laboratory are gravitational, having their origin in the distant matter of the universe, accelerated relative to the laboratory. Because of the tensor character of the gravitational-inertial field, it should exhibit tensor polarization properties. In particular, as suggested by Cocconi and Salpeter, because of the flattened rotating mass distribution of our galaxy, the inertial reaction having its origin in this mass distribution should exhibit some spatial anisotropy. This should appear in the formalism as a tensor inertial mass. Cocconi and Salpeter suggested that if Mach's principle were valid, the effects of this tensor inertial mass would appear as a spatial anisotropy in certain experiments. Several experiments, designed to test Mach's principle in this way, have been performed or analyzed.[1-3,7] By far the most accurate has been that of Drever.[3]

It will be shown that the experiments do not represent a test of Mach's principle in the manner suggested by Cocconi and Salpeter. On the contrary, and in agreement with the requirements of Mach's principle, the experiments show that, with great precision, the anisotropy of the inertial mass is universal, the same for all particles.

Expressed relativistically, the suggestion of Cocconi and Salpeter[1] is that the four-momentum of a particle can be written as

$$P_i = m_{ij} u^j, \tag{1}$$

where $u^i$ is the four-velocity of the particle. In the absence of a gravitational field, the particle is assumed to obey the equations of motion,

$$dP_i/ds = F_i, \tag{2}$$

where $F_i$ is the four-force acting on the particle and as usual $F_i$ satisfies the condition

$$F_i u^i = 0. \tag{3}$$

As Mach's principle associates the inertial reaction with the matter distribution in the universe, an anisotropy in the inertial mass should be universal, the same for all particles at the same space-time location, for all particles would see the same mass distribution. With the assumption that the inertial reaction is universal, the tensor $m_{ij}$ can be expressed as

$$m_{ij} = m f_{ij}, \tag{4}$$

where $f_{ij}$ is a universal tensor field (dimensionless).

A serious objection to Eqs. (1)-(4) can be raised. Equations (2) and (3) are generally not consistent with the constraint condition,

$$g_{ij} u^i u^i = u^i u_i = 1. \tag{5}$$

Consistent equations can be obtained from a variational principle. We note that to generate a momentum such as Eq. (1), linear in a four-velocity, a first condition to be satisfied is that the Lagrangian of the particle should be a function of the invariant

$$(d\bar{s}/ds)^2 = f_{ij} u^i u^j. \tag{6}$$

Equations of motion of a free particle (gravitational forces only) are obtained from the variational principle,

$$0 = \delta \int L ds, \tag{7}$$

where variations of the coordinates are to be taken subject to the constraint, Eq. (5). With the assumption that $f_{ij}$ and $g_{ij}$ are not equivalent, the resulting expression for the four-momentum is linear in some four-velocity only for the unique choice,

$$L = m(f_{ij} u^i u^j)^{1/2}. \tag{8}$$

With this choice, the constraint condition, Eq. (5), is satisfied by the equations of motion automatically, without introducing the condition of constraint in the variational calculation. As a result, the equations of motion do not contain $g_{ij}$ explicitly.

Substituting Eq. (8) in Eq. (7), the equations of motion of a free particle are

$$\frac{d}{ds}\frac{m_{ij}u^j}{(f_{ki}u^k u^j)^{1/2}} - \frac{1}{2}\frac{m_{jk,i}u^j u^k}{(f_{kj}u^k u^j)^{1/2}} = 0. \qquad (9)$$

These are most conveniently expressed, introducing $f_{ij}$ as a new metric tensor, by defining

$$d\bar{s}^2 = f_{ij}dx^i dx^j, \qquad (10)$$

$$\bar{u}^i = dx^i/d\bar{s}. \qquad (11)$$

With these substitutions, Eq. (9) becomes

$$\frac{d}{d\bar{s}}(m_{ij}\bar{u}^j) - \frac{1}{2}m_{jk,i}\bar{u}^j\bar{u}^k = 0. \qquad (12)$$

The resulting particle trajectories are geodesics of the new geometry, with $f_{ij}$ as metric tensor. The limiting trajectories of particles with infinite energy are null geodesics of the new geometry.

Inasmuch as $g_{ij}$ does not appear explicitly in the classical equations of motion of a particle, the appropriate quantum mechanical wave equations, giving equations of motion of expectation values equivalent to the classical equations, are constructed by employing $f_{ij}$ as the metric tensor. For example, the appropriate Lagrangian density for the Klein-Gordon wave function is

$$\frac{1}{2}\hbar^2 f^{ij}\varphi_{,i}\varphi_{,j} - \frac{1}{2}m^2\varphi^2. \qquad (13)$$

It may be noted with the assumption made above, that the inertial reaction is universal, the same for all particles including photons and pions, the metric tensor $g_{ij}$ appears nowhere in the formalism. In fact, for the geometry defined by measurements in the usual way with real rods and clocks, $f_{ij}$ is the metric tensor.[8]

It should be noted also, that because of the universal character of the inertial anisotropy, being present in the same way for all particles (or fields), the spatial anisotropy is unobservable locally. The easiest way to see this is to note that a coordinate system can always be chosen in such a way as to cause $f_{ij}$ to be locally Minkowskian with vanishing first derivatives. For this coordinate system, the anisotropy of inertial mass appears explicitly nowhere in the equations.

While, with these assumptions, inertial anisotropy is not locally observable, the fact that the geometry defined by real rods and clocks is non-Euclidean may be interpreted as due in part to the effect on rods and clocks of the anisotropy of the inertial mass of the elementary particles which comprise the rods and clocks.

It is concluded finally that the extremely accurate null result of the experiment of Hughes et al. does not cast doubt upon the validity of Mach's principle. On the contrary, and in accordance with the requirements of Mach's principle, this important experiment shows, with great precision, that inertial anisotropy effects are universal, the same for all particles.

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[3]R. W. P. Drever, Phil. Mag. 6, 683 (1961).

[4]G. Berkeley, The Principle of Human Knowledge (1710), par. 111-117.

[5]E. Mach, Conservation of Energy (1872) (edition of Open Court Publishing Company, Chicago, Illinois, 1911), Note No. 1; Science of Mechanics (1883) (edition of Open Court Publishing Company, Chicago, Illinois, 1902), Chap. II, Sec. VI.

[6]D. W. Sciama, Monthly Notices Roy. Astron. Soc. 113, 34 (1953); The Unity of the Universe (Doubleday, New York, 1959), Chaps. 7-9.

[7]C. W. Sherwin, H. Frauenfelder, E. L. Garwin, E. Lüscher, S. Margulies, and R. N. Peacock, Phys. Rev. Letters 4, 399 (1960).

[8]What then is the significance of the redundant tensor $g_{ij}$? As was discussed recently [C. Brans and R. H. Dicke, Phys. Rev. 124, 925 (1961)], the choice of units of length and time are arbitrary and physical laws must be invariant under position-dependent transformation of units. As a result, considerable freedom in the definition of the metric tensor exists. While $f_{ij}$ would be the natural and simplest choice, the metric tensor can be modified at will and can be defined as $g_{ij}$ by the appropriate redefinition of units. In particular, there exist definitions of units for which the space is flat, all the Riemannian invariants being zero.