$\pi^+ + p$ AND $\pi^- + p$ TOTAL CROSS SECTIONS FROM 8 TO 20 Bev/c*

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Previous experiments[1,2] by von Dardel et al. have reported measurements of the $\pi^+ + p$ and $\pi^- + p$ total cross sections over the incident pion momentum range of 4-10 Bev/c. These investigators have found that both the $\pi^+ + p$ and $\pi^- + p$ total cross sections decrease slowly and monotonically (by $\approx 10\%$) going from 4.5 Bev/c to 10 Bev/c incident pion momentum.

At 10 Bev/c, $\sigma_t(\pi^- + p) = 27.0 \pm 0.4$ mb, $\sigma_t(\pi^+ + p) = 25.2 \pm 0.4$ mb, and $\sigma_t(\pi^- + p) - \sigma_t(\pi^+ + p) = 1.8 \pm 0.6$ mb, where the subscript t denotes total cross section. As those authors concluded, the data do not satisfy the necessary conditions for application of the Pomeranchuk theorem[3] which is that both $\sigma_t(\pi^+ + p)$ and $\sigma_t(\pi^- + p)$ become and remain constant with increasing energy. As a matter of fact, neither cross section appears to be consistent with this condition.

In a recent publication[4] the authors of this communication have investigated the antiproton-proton and the proton-proton total cross section from 4 to 20 Bev/c. $\sigma_t(p+p)$ appeared to become and remain constant within the errors from 10 to 20 Bev/c $[\sigma_t(p+p) \approx 39.5 \pm 1$ mb], but the total antiproton-proton cross section continued to decrease monotonically from 10 Bev/c where $\sigma_t(\bar{p}+p) = 58 \pm 4$ mb to 20.3 Bev/c where $\sigma_t(\bar{p}+p) = 48 \pm 4$ mb, and hence the Pomeranchuk theorem could not be verified by this investigation. Therefore it appeared to be of considerable interest to extend the investigation of the $\sigma_t(\pi^- + p)$ and the $\sigma_t(\pi^+ + p)$ to the so far uninvestigated energy range of 10-20 Bev/c. The π^--π^+ particle-antiparticle pair is especially interesting to compare since, in addition to the general Pomeranchuk theorem,[5] there is another specific argument which can be applied to the behavior of the difference $\sigma_t(\pi^- + p) - \sigma_t(\pi^+ + p)$ at high enough energies which cannot be applied to the behavior of the difference $\sigma_t(\bar{p}+p) - \sigma_t(p+p)$ or the difference $\sigma_t(K^- + p) - \sigma_t(K^+ + p)$. This argument, which was proposed by Okun and Pomeranchuk,[6] can be summarized as follows. At high enough energies one would expect the charge-exchange cross section to become small compared to the total inelastic cross section, since charge exchange represents only one inelastic channel of what one might expect to be an arbitrarily large number of possible inelastic channels at sufficiently high energy. Therefore the $T = \frac{3}{2}$ state complex scattering amplitude must approximately equal the $T = \frac{1}{2}$ state complex scattering amplitude, since the charge-exchange amplitude is proportional to the difference of these two amplitudes. This then implies that the $\pi^- + p$ total elastic scattering cross section equals the $\pi^+ + p$ total elastic scattering cross sections and hence also that

$$\sigma_t(\pi^- + p) = \sigma_t(\pi^+ + p).$$

Of course the above argument does not depend on the constancy of cross sections at high energy which is a necessary requirement for the Pomeranchuk theorem, and also does not depend on the validity of the forward dispersion relations which are employed to prove the theorem.

Weinberg[7] has recently presented a generalized Pomeranchuk theorem which relaxes the conditions required to prove that $\sigma_+ - \sigma_- \to 0$ as $E \to \infty$; σ_+ is the particle total cross section and σ_- is the antiparticle total cross section. The essential assumptions are: (a) that the difference $\sigma_+(E) - \sigma_-(E)$ does not change sign an infinite number of times; (b) that the cross sections $\sigma_\pm$ do not grow faster than $(\ln E)^{1/2}$ as $E \to \infty$. Then, using the forward dispersion relations, he demonstrates that $\sigma_+ - \sigma_- \to 0$ as $E \to \infty$.

The present experiment to determine $\sigma_t(\pi^+ + p)$ and $\sigma_t(\pi^- + p)$ from 8 to 20 Bev/c was run during the same period and with the same experimental arrangement as the previously reported work on $\sigma_t(\bar{p}+p)$ and $\sigma_t(p+p)$,[4] the only difference being that the gaseous Čerenkov counter was tuned to a pressure which selected pions rather than protons, or antiprotons. The previous method of using six counters of different area to extrapolate the measured transmission to zero area was again employed. The measured transmissions and hence also the values of the cross sections were corrected for the following effects:

(a) Due to the π-μ and K-μ_2 decays there is a muon contamination of the beams. Using the known pion and K-meson production spectra and angular distributions from previous work,[8] the known K-meson and pion lifetimes, and the beam setup geometry and magnetic geometry, we were able to estimate the muon contamination of the pion beam to range from 2 to 3%, with an error of ±1%; this correction was applied to the data. The uncertainty in the difference of $\sigma_t(\pi^- + p) - \sigma_t(\pi^+ + p)$

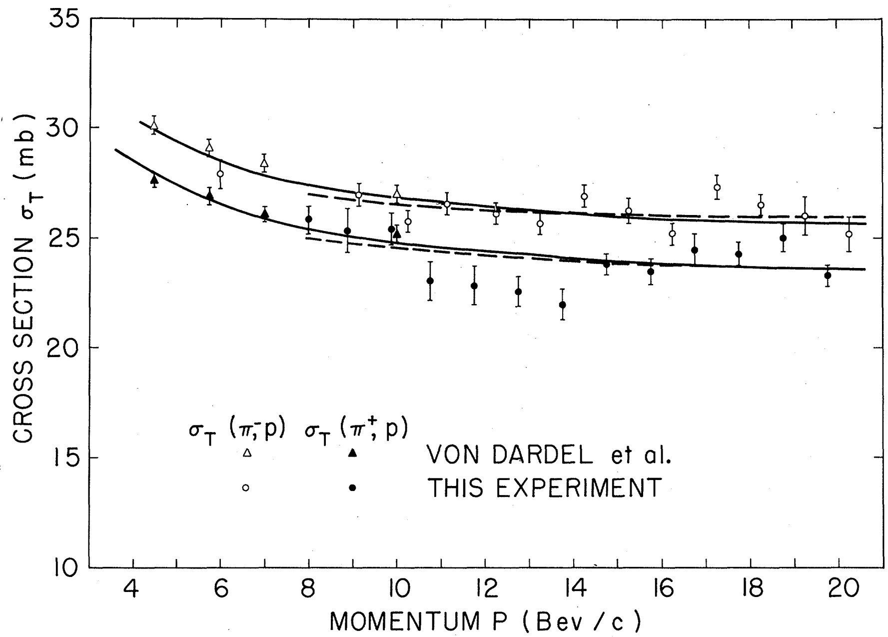

FIG. 1. π^-+p and π^++p total cross sections. The solid lines are Fit 2′ and the dashed lines are Fit 2 (see text). The momentum of the system is defined to $\sim\pm2.5\%$, and the absolute scale of the momentum is uncertain by 2%.

due to muon contamination is only a fraction of a percent.

(b) The electron contamination of the beam was estimated to be $\sim 0.3\%$ and a correction was made.

(c) Accidentals mainly in the end counter array were monitored during the run and the appropriate correction (~1%) to the cross sections was made with an uncertainty of a fraction of 1%.

(d) The proton contamination correction to the cross section is less than 1% below 15 Bev/c and rises to approximately 3.5% at 20 Bev/c. The uncertainty in the cross section due to this correction is less than 1%.

The results are shown in Fig. 1. The errors on each point predominantly represent point-to-point errors. In addition we estimate that the absolute scale uncertainty may be as much as 2% (i.e., 0.5 mb); however, this absolute uncertainty has a negligible effect on the difference $\sigma_t(\pi^-+p) - \sigma_t(\pi^++p)$.

From Fig. 1 it is clear that our results agree very well with those of von Dardel et al. at the lower momenta. Considering the errors, it appears that there is no convincing evidence from the present data for other than a smooth slow variation of the total cross section with energy. Therefore we have attempted three least-squares fits of the momentum dependence of the total cross sections from 8 to 20 Bev/c. Fit 1: $\sigma_t=a$; Fit 2: $\sigma_t=a+b/p$; Fit 3: $\sigma_t=a+b/\ln p$; where both a and b are constants. It was found that Fit 2 has the lowest χ^2, and on inspection of the lower energy data of von Dardel et al. it appears that Fit 2 and Fit 3 are the only fits capable of reasonably fitting the data from 4.5 to 20 Bev/c.

Fit 2′ is the Fit 2 momentum dependence $\sigma_t=a+b/p$ but applied to the sum of our data and the data of von Dardel et al. (i.e., a fit from 4.5 to 20 Bev/c). Both Fit 2 and Fit 2′ are shown in Fig. 1. It is clear that this functional dependence can be made to fit both our own and the lower energy data reasonably well. The parameters a and b for these fits are given in Table I.

The calculated differences $\Delta=\sigma_t(\pi^-+p)-\sigma_t(\pi^++p)$

Table I. Fit of $\sigma_t=a+b/p$, where σ_t is in mb and p is pion lab momentum in Bev/c.

	Fit 2		Fit 2′	
σ_t	a	b	a	b
$\sigma_t(\pi^++p)$	22.59 ± 0.63	19.39 ± 8.21	22.26 ± 0.33	25.10 ± 2.83
$\sigma_t(\pi^-+p)$	25.26 ± 0.46	13.03 ± 5.27	24.37 ± 0.29	24.94 ± 2.65

for all of the fits 1, 2, 2′, and 3 show that Δ is probably constant or at best slowly decreasing from 4.5 to 20 Bev/c. On the other hand, the total cross sections decrease monotonically by about 10% from 4.5 Bev/c to 10 Bev/c, and the additional decrease from 10 Bev/c to 20 Bev/c is only about 4%. If the assumed functional dependence $\sigma = a + b/p$ were to be correct for higher energies, our results imply that the limiting values of both total cross sections at very high energies would decrease from the values at 20 Bev/c by about 5% but the same order of difference ($\Delta \sim 2$ mb) would still remain. We also found this to be the case for the other fits.

Although extrapolation of the observed momentum dependence of the cross section in the observed energy range to much higher energies is not to be taken very seriously, it is, on the other hand, strongly implied by our data that at 20 Bev/c we are still very far from the energy region where the Pomeranchuk or Weinberg theorems, etc., can be applied or even apparently verified. Furthermore the substantial difference $\sigma_t(\pi^- + p) - \sigma_t(\pi^+ + p)$ and its slow decrease with energy that we have observed also imply that the number of inelastic channels which are effective is much too small and is growing much too slowly with energy to allow us to believe that we are approaching the necessary energy to demonstrate the high-energy limit theorems considered.

In view of our previous work on antiproton-proton and proton-proton total cross sections[1] and other previous work on K^+-proton and K^--proton total cross sections, we now believe that it is likely that even a possible experimental verification of the high-energy limit theorems (i.e., $\sigma_+ - \sigma_- \to 0$ as $E \to \infty$) will have to await the next large increase in proton accelerator energy.

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[5]From Pomeranchuk's theorem, one concludes that if $\sigma_t(\pi^+ + p)$ and $\sigma_t(\pi^- + p)$ become and remain constant beyond an energy $E > \epsilon$ then $\sigma_t(\pi^+ + p) = \sigma_t(\pi^- + p)$ for $E > \epsilon$.

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