ANOMALY IN MESON PRODUCTION IN $p+d$ COLLISIONS*

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In an earlier Letter[1] we reported the results of some preliminary measurements of the momentum spectra of He^3 and H^3 nuclei produced in collisions of high-energy (624- to 743-Mev) protons with deuterium. In addition to the single-pion reactions

$$p+d \rightarrow \begin{cases} He^3+\pi^0 & (1a) \\ H^3+\pi^+, & (2a) \end{cases}$$

we looked for the reactions

$$p+d \rightarrow \begin{cases} He^3+\omega^0 & (1b) \\ He^3+\pi^++\pi^- & (1c) \\ He^3+\pi^0+\pi^0, & (1d) \end{cases}$$

and

$$p+d \rightarrow \begin{cases} H^3+\omega^+ & (2b) \\ H^3+\pi^++\pi^0, & (2c) \end{cases}$$

where ω may be a particle of mass between 1 and 2.8 pion masses. For reactions resulting in a He^3, the two pions (or particle) can be in isotopic spin states 0 or 1; if a H^3 results, only $I=1$ is allowed. We found an anomalous peak in the He^3 spectra which appeared to behave kinematically like a particle or resonance of mass approximately 310 Mev. At that time we were unable to give a definite isotopic spin assignment to the anomaly, and we considered a P-wave π-π resonance as a possible explanation.[2]

We have since repeated the experiment with a new arrangement which enabled us to measure both the He^3 and H^3 spectra with improved resolution and accuracy. With the new data we have been able to assign an isotopic spin $I=0$ to the anomaly, and subsequently to rule out the P-wave π-π resonance hypothesis. It is possible to explain the anomaly by means of a strong S-wave π-π attraction in the $I=0$ state, with a scattering length between 2 and 3 pion Compton wavelengths.

Figure 1 is a schematic drawing of the experi-

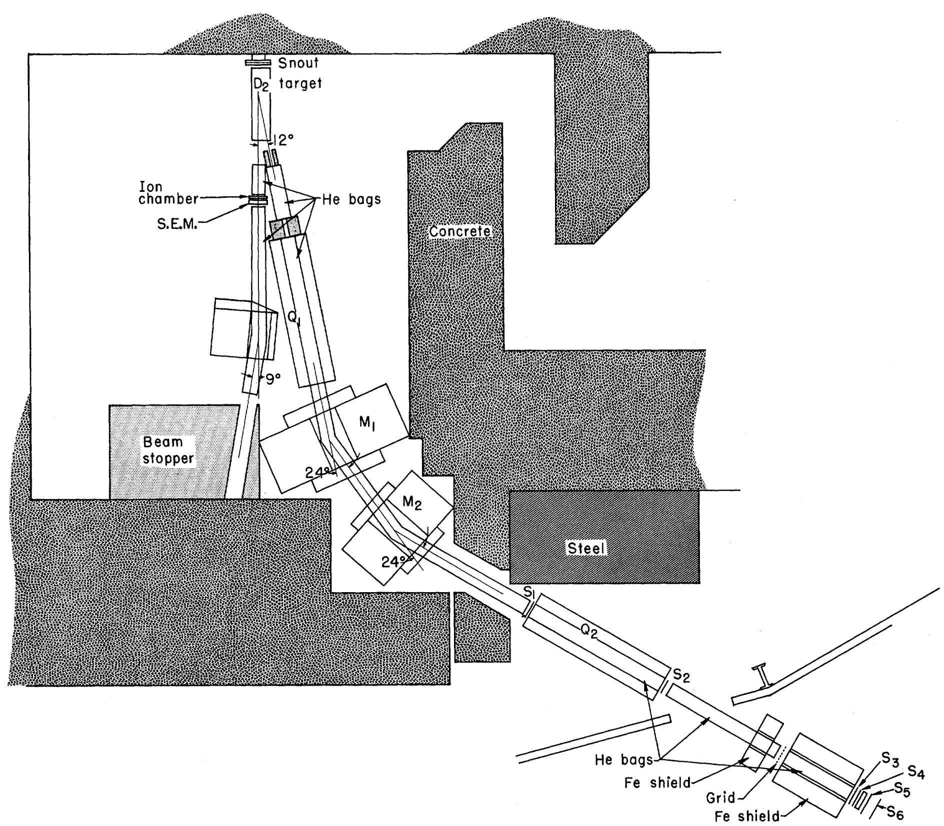

FIG. 1. Experimental arrangement.

mental arrangement. The proton beam extracted from the 184-inch cyclotron was passed through a gaseous deuterium target operated at liquid nitrogen temperature and at about 300 psi. Particles produced at 11.8 deg were collimated by a system of slits, and focused at infinity by the quadrupole Q_1. Momentum analysis was accomplished by the bending magnets M_1 and M_2. Quadrupole Q_2 focused the particles at a grid consisting of six $\frac{1}{2}$-inch-wide counters, each of which defined a momentum bite $\Delta p/p$ of 0.45%. He^3 and H^3 were selected from other particles by time of flight, range, and dE/dx. Backgrounds were measured by using hydrogen gas in the target.

The results of the measurements[3] of the He^3 and H^3 spectra at the full proton energy of 743 Mev are shown in Figs. 2(a) and 2(b). We were unable to observe He^3 of momenta less than 1000 Mev/c because of their low range. The peaks at 1530 Mev/c correspond to Reactions (1a) and (2a) at 50 deg c.m. for the heavy particle. The peak at 820 Mev/c in the H^3 spectrum corresponds to 156 deg c.m. in Reaction (2a). Double-pion production for this laboratory-system angle is kinematically possible between the limits of 910 and 1440 Mev/c.

If we assume the anomaly is not due to a pion-nucleon interaction, and that charge independence holds, we can analyze Reactions (1b), (1c), (1d), (2b), and (2c) in terms of the isotopic spin of the two pions (or particle). Since the $p+d$ system has $I=\frac{1}{2}$ and $I_z=\frac{1}{2}$, we can form with He^3 or H^3 the isotopic spin eigenfunctions having $I=\frac{1}{2}$, $I_z=\frac{1}{2}$:

$$\Pi_0{}^0 He^3, \qquad (3a)$$

$$(\tfrac{2}{3})^{1/2}\Pi_1{}^1 H^3 - (\tfrac{1}{3})^{1/2}\Pi_1{}^0 He^3, \qquad (3b)$$

where, for two pions,

$$\Pi_1{}^1 = (\tfrac{1}{2})^{1/2}(\pi^+\pi^0 - \pi^0\pi^+), \qquad (4a)$$

$$\Pi_1{}^0 = (\tfrac{1}{2})^{1/2}(\pi^+\pi^- - \pi^-\pi^+), \qquad (4b)$$

and

$$\Pi_0{}^0 = (\tfrac{1}{3})^{1/2}(\pi^+\pi^- + \pi^-\pi^+ - \pi^0\pi^0). \qquad (4c)$$

Thus, with H^3 we have only $I=1$ production; but with He^3 we have both $I=0$ and $I=1$, the amount of $I=1$ being $\frac{1}{2}$ as large as in the H^3 case. To get the $I=0$ spectrum shown in Fig. 3, we drew a smooth curve through the H^3 spectrum, divided it by two, and subtracted it from the He^3 spectrum.

Before looking carefully into possible explana-

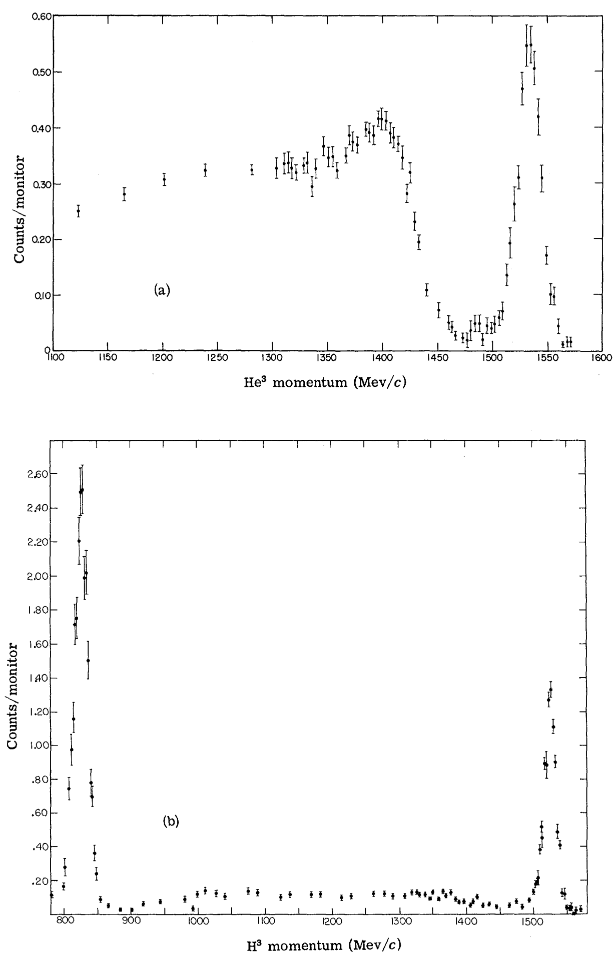

FIG. 2. (a) Momentum spectrum of He^3 at 11.8 deg, laboratory system. (b) Momentum spectrum of H^3.

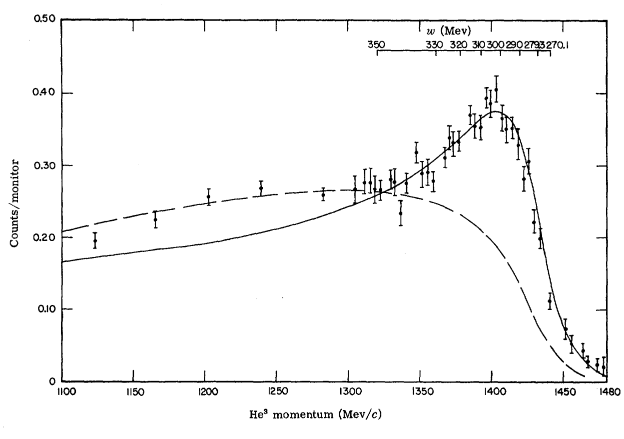

FIG. 3. $I=0$ part of He^3 spectrum. The dashed curve is the phase-space volume fitted to the points below 1300 Mev/c. The solid curve is the phase-space volume multiplied by the pion-pion enhancement factor for a scattering length $a_{s0}=2.8\hbar/\mu c$. The experimental resolution has been folded into both curves. The w scale gives the total energy in the two-pion barycentric system.

tions for the bump that appears in the $I=0$ spectrum, one must know the momentum resolution of the experiment. This resolution was calculated by taking into account such effects as finite angular definition, beam dimensions, image and grid sizes, multiple scattering, angular divergences, and energy spread of the proton beam. By making reasonable assumptions about the energy spread of the proton beam, we were able to reproduce the shapes and widths of the three single-pion production peaks. Figure 4 shows the resolution function for He^3 momenta near 1400 Mev/c. The full width at half maximum is 35 Mev/c, with an estimated uncertainty of $^{+6}_{-2}$ Mev/c.

As a starting point for comparing the data with theory, we computed the Lorentz-invariant phase-space volume element in the laboratory system:

$$\phi_S = \frac{d^2\rho}{dp_3 d\Omega_3} = \frac{p_3^{\,2}}{\omega_3}\left(1 - \frac{4\mu^2}{w^2}\right)^{1/2}, \qquad (5)$$

where

$$\rho \propto \int \frac{d\vec{p}_3}{\omega_3}\frac{d\vec{p}_4}{\omega_4}\frac{d\vec{p}_5}{\omega_5}\,\delta(\vec{p}_3+\vec{p}_4+\vec{p}_5-\vec{p}_1)\,\delta(\omega_3+\omega_4+\omega_5-W_L),$$

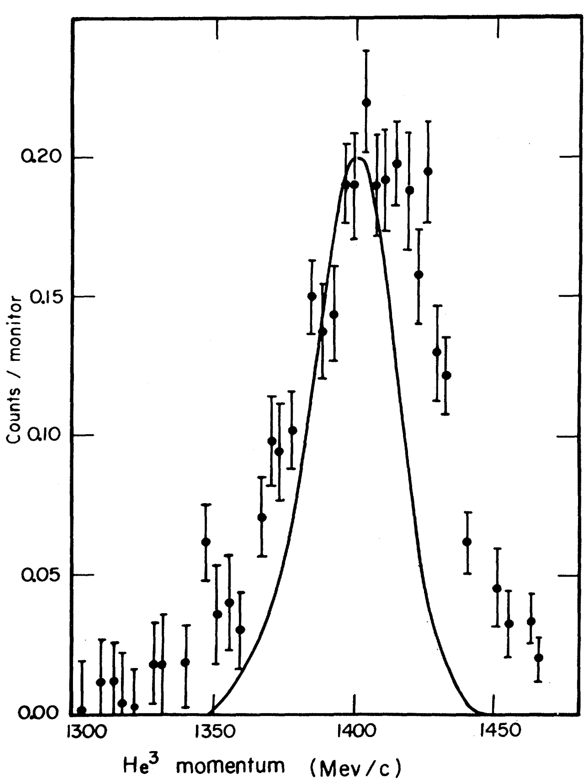

FIG. 4. Effect of subtracting the phase-space volume from the $I=0$, He^3 data. The solid curve is the computed experimental resolution at 1400 Mev/c.

$\vec{p}_1$ is the momentum of the incoming proton, W_L is the total energy in the laboratory system, p_3 and ω_3 are the respective momentum and total energy of the He^3, and w is the total energy in the barycentric system of particles 4 and 5, the two pions of mass μ. We have assumed the transition matrix element to be a constant, and restrict ourselves to the relativistically invariant form of the volume in phase space. The calculations were done for charged and neutral pions and combined according to Eq. (4c). With the resolution folded in and normalized to the experimental data at momenta below 1350 Mev/c, ϕ_S is shown as the dashed curve in Fig. 3.

A promising explanation at the moment is that the anomaly is due to a strong S-wave π-π interaction[4] that can be characterized by a scattering length. The conditions for the validity of the theory of final-state interactions[5] are: that the mechanism of the primary reaction be a short-range interaction, that the final-state interaction be strong and attractive, and that we consider only low relative energies of the two pions. Under these conditions, which we will see can be satisfied here, the volume element in phase space [Eq. (5)] for a given value of the pion-pion energy w is enhanced by a factor proportional to the pion-pion scattering cross section at the energy w. To obtain the energy dependence of the pion-pion cross section we go to Eq. (V.22) of Chew and Mandelstam,[6] where we define a scattering length a_{S0} as the amplitude at zero energy. We have

$$\left(\frac{\nu}{\nu+1}\right)^{1/2}\cot\delta_0^{\,0} = \frac{1}{a_{S0}} + \frac{2}{\pi}\left(\frac{\nu}{\nu+1}\right)^{1/2}\ln\left[\nu^{1/2}+(\nu+1)^{1/2}\right], \qquad (6)$$

where $\delta_0^{\,0}$ is the S-wave π-π phase shift in the $I=0$ state and ν is the square of the momentum (in pion mass units) in the two-pion barycentric system. This gives an enhancement factor (normalized to unity at $\nu=0$),

$$F(\nu) = \left\{\left[1 + \frac{2a_{S0}}{\pi}\left(\frac{\nu}{\nu+1}\right)^{1/2}\ln\left[\nu^{1/2}+(\nu+1)^{1/2}\right]\right]^2 + a_{S0}^{\,2}\left(\frac{\nu}{\nu+1}\right)\right\}^{-1}. \qquad (7)$$

This is exact for the S-dominant solutions of the π-π equations[7] where the π-π coupling constant is

$$\lambda = -\tfrac{1}{5}\frac{a_{S0}}{1+0.554\,a_{S0}}.$$

For the P-dominant solutions[8] Eq. (7) is still a very good approximation, but to evaluate λ we must know ν_R and Γ, the position and width parameters, respectively, of the P-wave π-π resonance.[9] We obtained good fits to our $I=0$ data at momenta above 1350 Mev/c with a_{S0} between 2.2 and $3.0\hbar/\mu c$ with a best-fit value of $2.5\hbar/\mu c$. The solid curve of Fig. 3 shows the fit obtained for $a_{S0}=2.8\hbar/\mu c$. We do not expect the computed curve to fit the experimental points below 1350 Mev/c for the following reason: The final-state interaction picture is valid only at low relative energies of the two pions where their attraction is large compared to other effects, such as details of the production mechanism and final-state π-He^3 interactions.

In Fig. 4 we show the result of subtracting the phase-space volume from the data. The peak occurs at a mass value of about 300 Mev, and unfolding the resolution gives a width of about 25 Mev. This yields a lifetime of the same order as the interaction time, and the concept of a particle becomes vague. It therefore seems doubtful that this is the vector meson of Nambu.[10] We intend to settle this question independently of the resolution in a subsequent experiment.

We cannot completely rule out the possibility that the anomaly is due to final-state interactions between the He^3 and one of the pions. The $I=\frac{3}{2}$, $J=\frac{3}{2}$, pion-nucleon state can occur in the H^3 case, but not as strongly as in the He^3 case. However, in the region of the anomaly we are below the energy of the $\frac{3}{2}$, $\frac{3}{2}$ resonance. Also, in other experiments in which the final state consists of two pions and a nucleon, the energy spectrum of the nucleon is usually not strongly influenced by the $\frac{3}{2}$, $\frac{3}{2}$ resonance.[11] We can dismiss the S-wave pion-nucleus interaction because it is known to be small at these energies.[12]

We have also considered the symmetrization of the wave function for the two pions. Details of the derivation are given in reference 3. We present here only the result, which is to multiply Eq. (5) by the factor $B(R)=1+\exp\{-[(w/\mu)^2-4]\times(R/2.15)^2\}$, where R is the radius of interaction in units of $1/\mu$. The effect of the symmetrization on the shape of ϕ_S is small for all values of R and cannot reproduce the observed bump. We will therefore neglect $B(R)$, although including it would decrease the value of the scattering length derived above.

Another effect we mention only briefly is the He^3 wave function. Roughly speaking, the three nucleons stick together to form a He^3 more easily when they have low relative energies because the He^3 wave function has fewer high-momentum components. This effect favors low He^3 momenta. From both the H^3 and He^3 spectra, we conclude that this effect is small or else compensated for by something else.

We plan to repeat the experiment at another laboratory-system angle. It should then be possible to make the correct interpretation.

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[1]A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters 5, 258 (1960).

[2]A P-wave rather than an S-wave resonance seemed more plausible. See G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960), footnote 19; W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 365 (1959); Phys. Rev. 117, 1609 (1960).

[3]These data also appear in N. E. Booth, A. Abashian, and K. M. Crowe, Revs. Modern Phys. (to be published) and are available in tabular form in the Lawrence Radiation Laboratory Report UCRL-9599, 1961 (unpublished). Some of the plotted points are slightly in error because of needed corrections that had not been made. The tabulated data, however, have been corrected.

[4]This approach was first suggested to us by Dr. Kenneth M. Watson of this Laboratory as one of several different explanations. Recently, Truong has succeeded in fitting some of our earlier data this way. See T. N. Truong, postdeadline paper at the New York Meeting of the American Physical Society, February, 1961, and Phys. Rev. Letters 6, 308 (1961).

[5]K. M. Watson, Phys. Rev. 88, 1163 (1952).

[6]G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

[7]G. F. Chew, S. Mandelstam, and H. P. Noyes, Phys. Rev. 119, 478 (1960).

[8]G. F. Chew and S. Mandelstam, Nuovo cimento 19, 752 (1961).

[9]B. Desai, Phys. Rev. Letters 6, 497 (1961). Desai has calculated the enhancement factors to be expected if the resonance parameters are assumed to be $\nu_R=3.5$ and $\Gamma=0.3$. With these values, our $a_{S0}=2.5$ gives $\lambda=-0.19$. For the S-dominant case, Eq. (8) gives $\lambda=-0.21$. Recently Moffat and Bransden have obtained

numerical solutions in terms of the single parameter λ. For $\lambda = -0.20$ they calculate $\nu_R = 3.9$, $\Gamma = 0.6$, and $a_{s0} = 2.0$; J. W. Moffat and B. H. Bransden (private communication).
[10]Y. Nambu, Phys. Rev. 106, 1366 (1957).
[11]R. M. Sternheimer and S. J. Lindenbaum, Phys. Rev. 109, 1723 (1958); J. G. Rushbrooke and D. Radojičić, Phys. Rev. Letters 5, 567 (1960).
[12]M. Stearns and M. B. Stearns, Phys. Rev. 103, 1534 (1956).
[13]G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Phys. Rev. 120, 300 (1960).

ERRATA

EVIDENCE FOR A π-π RESONANCE IN THE $I=1$, $J=1$ STATE. A. R. Erwin, R. March, W. D. Walker, and E. West [Phys. Rev. Letters 6, 628 (1961)].

Footnote 3 should contain an additional reference to G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960). At the end of the text, there should be an additional footnote: [8]It is possible that the effects of large S-wave π-π scattering have been observed by A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters 5, 258 (1960).

PROTON-PROTON INTERACTION. H. Feshbach, E. Lomon, and A. Tubis [Phys. Rev. Letters 6, 635 (1961)].

The last two terms in the square bracket of Eq. (1) for $V_4(r)$ should be

$$-\vec{\sigma}^1 \cdot \vec{\sigma}^2 R_2(\mu r) - S_{12} R_3(\mu r),$$

instead of the same terms with positive signs.

The first two terms in the first square bracket of the expression for $R_1(x)$ just below Eq. (1) should read:

$$\left(\frac{12}{x^2} + \frac{23}{x^4}\right) K_1(2x) \text{ instead of } \frac{12}{x^2} + \frac{23}{x^4} K_1(2x).$$

The first line of p. 636 should have $\xi = 0$ instead of $\xi = 1$, and the second line should have $\xi = 1$ instead of $\xi = 0$.