Inside a thick ring the kinetic angular momenta are given by $\hbar(\nu - e\phi/hc)$ if $\hbar\nu$ is the canonical angular momentum. Therefore with $\phi = hcm/e$ $[\phi = hc(m+\frac{1}{2})/e]$ the pairing with zero current occurs for $(\nu, -\nu)$ $[\nu+\frac{1}{2}, -\nu-\frac{1}{2})]$ in the case of even (odd) n. The description in terms of wave functions (11) or (15) has the advantage of not only showing the equivalence of two apparently different pictures but also of being valid in the case of thin rings where the current does not vanish as assumed by Byers and Yang.

We wish to thank H. P. Duerr, W. Heisenberg, H. Koppe, M. Näbauer, A. de-Shalit, and V. F. Weisskopf for stimulating discussions.

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OSCILLATORY GALVANOMAGNETIC EFFECT IN METALLIC SODIUM*

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We wish to report the observation at 4°K of a novel galvanomagnetic effect in sodium of very high conductivity ($\rho_{300°\mathrm{K}}/\rho_{4°\mathrm{K}} \approx 7500$; $\omega_c\tau \approx 40$ in 10 000 gauss). In this effect, the Lorentz force produces an electrical oscillation of low frequency (ranging from 5 to 35 cycles/sec in our experiment). Our results provide direct evidence of a particular macroscopic electron gas excitation described by Aigrain[1] and given by him the name "helicon" excitation. There is also a connection between our observation and certain effects observed in bismuth at microwave frequencies.[2]

The experimental arrangement (Fig. 1) is based on a configuration for measuring resistivity due to Bean, DeBlois, and Nesbitt.[3] A cylindrical specimen of sodium (height 30 mm and diameter 4 mm) is placed inside primary and secondary coils. On closing or opening the primary circuit, eddy currents are induced in the specimen which can be detected by the secondary coil. From the time constant of the observed voltage in the secondary, one can estimate the resistivity of the specimen. For a sufficiently long time after the primary pulse, the decay is a simple exponential. Our experiment differs from that in reference 3 in that a constant magnetic field (H_0) is applied perpendicular to the axis of the specimen and coils. The constant field H_0 is much larger than the time-dependent magnetic field ($h \approx$ few hundred gauss) produced by the primary coil. The voltage of the secondary coil is observed on closing or opening the primary circuit. Figure 2 shows oscilloscope traces of the secondary voltage as a function of time for various values of the bias field H_0, using sodium for which $\rho_{300°\mathrm{K}}/\rho_{4°\mathrm{K}} \approx 7500$ (mean free path at 4°K $\approx \frac{1}{4}$ mm). In zero magnetic field the normal eddy current decay is observed. The other three traces show the effect of applying H_0. One sees damped oscillations whose frequency is proportional to magnetic field. The frequency at 10 000 gauss is 32 cycles/sec.

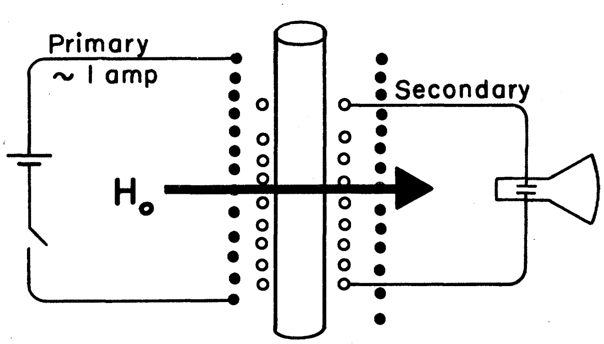

FIG. 1. Schematic diagram of experiment. Primary coil ≈300 turns of No. 28 copper wire; secondary coil ≈1200 turns of No. 43 copper wire. Specimen and coils immersed in liquid helium at 4.2°K.

We believe the oscillations to originate as follows. The abrupt change in the primary field (h) excites stable modes of the electron gas in which the electric vector is circularly polarized about the large magnetic field.[1] The oscillation observed is the component of this circular wave resolved in

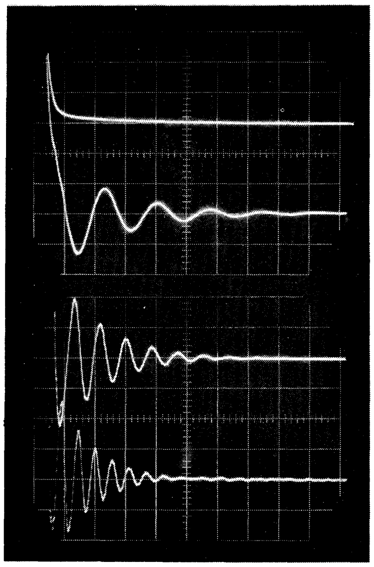

FIG. 2. Voltage in secondary coil as a function of time on opening primary circuit for different values of H_0. Starting from the top, the four curves correspond to $H_0 = 0$, 3600, 7200, and 10 800 gauss, respectively. Ordinate is 5 millivolts per large division; abscissa is 50 milliseconds per large division. Specimen properties: $\rho_{300°\mathrm{K}}/\rho_{4°\mathrm{K}} \approx 7500$; $\omega_c\tau \approx 40$ in 10 000 gauss.

the direction of the axis of the secondary coil. A precise theory of this effect requires the solution of the field equations with the complicated boundary conditions used in our experiment. However, a useful physical picture of the origin of the circularly polarized waves can be derived from the following simplified theory for the infinite solid of very high conductivity.

In a strong magnetic field ($\omega_c\tau \gg 1$), the current vector and the electric field vector are nearly perpendicular, as the component of the electric field along the current direction becomes very small compared to the transverse (Hall) field. We therefore write[4]

$$\vec{\mathrm{E}} + \mu_0 \vec{\mathrm{v}}_D \times \vec{\mathrm{H}} = 0,$$

$$\vec{\mathrm{i}} = ne\vec{\mathrm{v}}_D, \tag{1}$$

using rationalized mks units and conventional notation. We use Eq. (1) for fields and currents varying in space and time, assuming that the mean free path and the relaxation times are short compared to the corresponding periods in the solutions. From Eq. (1) and Maxwell's equations (neglecting displacement current) it follows that

$$\partial\vec{\mathrm{E}}/\partial t = (1/ne)[\nabla\times(\nabla\times\vec{\mathrm{E}})]\times\vec{\mathrm{H}}_0, \tag{2}$$

assuming that $\vec{\mathrm{H}} = \vec{\mathrm{H}}_0 + \vec{\mathrm{h}}(\vec{\mathrm{r}}, t)$, $|\vec{\mathrm{H}}_0| \gg |\vec{\mathrm{h}}|$, and $\vec{\mathrm{H}}_0$ is along the z axis. We solve by substituting $\vec{\mathrm{E}} = \vec{\mathrm{E}}_0 \exp[i(\omega\tau - \vec{\mathrm{k}}\cdot\vec{\mathrm{r}})]$. Equation (2) reduces to a set of three linear homogeneous equations for the components of E_0. The secular equation yields two nonzero frequencies for any given wave vector $\vec{\mathrm{k}}$:

$$\omega = \pm kk_z H_0/ne. \tag{3}$$

The corresponding solutions, omitting a common multiplicative constant, are:

$$\vec{\mathrm{E}}_0 = (k_x^2 + k_z^2, k_x k_y \pm ikk_z, 0),$$

$$\vec{\mathrm{h}}_0 = (k_z/\omega\mu_0)(-k_x k_y \mp ikk_z, k_x^2 + k_z^2, -k_y k_z \pm ikk_x),$$

$$\vec{\mathrm{i}}_0 = \mp k\vec{\mathrm{h}}_0.$$

For propagation along the z direction, these reduce to $k^2(1, \pm i, 0)$; $(k^3/\omega\mu_0)(\mp i, 1, 0)$; $\mp k\vec{\mathrm{h}}_0$, respectively. All the above solutions describe circularly polarized plane waves, the two signs corresponding to right-hand and left-hand screw spatial configurations. The vectors rotate about the magnetic field lines always in the direction corresponding to the cyclotron rotation of the carriers. The $\vec{\mathrm{E}}$ field in general has a component along the propagation direction; the $\vec{\mathrm{h}}$ and $\vec{\mathrm{i}}$ fields do not.

At 10 000 gauss the observed frequency was 32 cycles/sec; with these values Eq. (3) yields a half-wavelength of 3.4 mm for a wave that propagates along the field lines. This is comparable with the corresponding specimen dimension which is 4 mm. The observed linear dependence of the frequency on the magnetic field is also in agreement with Eq. (3).

We have not been able to find any other explanation for the observed oscillations. Mechanical oscillations of the coil in the magnetic field are expected to produce an oscillation whose frequency is determined by the mechanical spring constants and is independent of H_0; the amplitude of the resulting oscillations would depend on H_0 in a manner quite different from that shown in Fig. 2. Moreover, we have taken care to reduce mechanical effects to the minimum. The oscillations observed occur at frequencies which are too low by orders of magnitude to be associated with Landau levels.

The simple theory predicts that the frequency should be independent of conductivity in the high-conductivity limit. This was verified on investigating the oscillation in four sodium specimens whose residual resistance ratios were 7500, 4500, 3200, and 1200, respectively. It was found that oscillations were more damped as the conductivity decreased but the frequency did not change appreciably. At a ratio of 1200, no oscillations were found because of excessive damping, indicating that the conductivity must be very high in order to see the oscillations. Sodium is very favorable in this respect since it has a very small magnetoresistance.

In a variation of the experiment described above, the oscillation was excited under resonant conditions. Helmholtz coils were mounted (outside the cryostat) so that their axis was perpendicular both to the large field H_0 and to the axis of the secondary coil. The latter orientation reduces direct mutual inductance coupling. In Fig. 3 is shown the response of the secondary to a sinusoidal excitation of the Helmholtz coils; the specimen was the same as that used for Fig. 2. The amplitude of excitation current was maintained constant and the frequency varied. A resonant response was obtained. From studies of different specimens, it was found that the Q for the resonance increases with increasing conductivity but the resonant frequency is nearly independent of the conductivity in the range investigated. As yet, no identifiable harmonics have been found. Although the dependence of the resonance on specimen size has not been investigated in detail, it is observed that the resonant frequency increases as the length along H_0 is decreased.

We expect to carry out experiments in which the geometrical arrangement is more amenable to detailed theoretical analysis than the present arrangement; this requires modification of our magnet-cryostat system. This work provides further evidence that one can study in a solid some of the macroscopic electron gas excitations more frequently investigated in gaseous plasmas.[2]

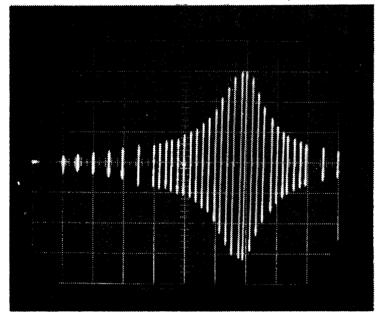

FIG. 3. Voltage in secondary as a function of frequency of exciting coil for fixed $H_0 = 10\,800$ gauss. Ordinate is 1 millivolt per large division; abscissa is 5 cycles/sec per large division. Specimen was the same as that used for Fig. 2.

We are indebted to J. Krumhansl and M. T. Taylor for helpful discussions, and to S. Tallman and E. Johnson for contributions to the experiment. The high-purity sodium was prepared by C. E. Taylor of the Lawrence Radiation Laboratory, Livermore, California.

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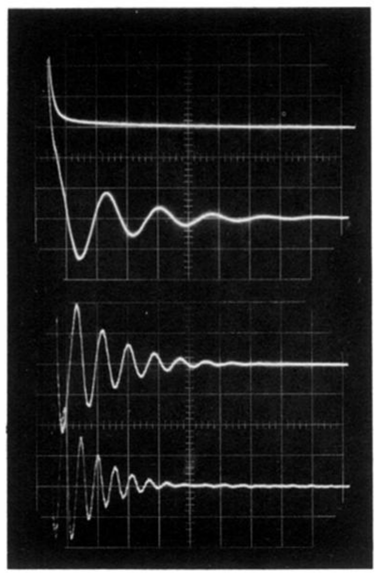

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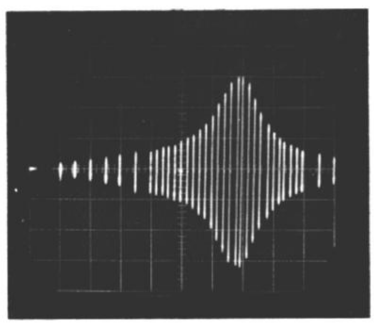

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