REMARK CONCERNING QUANTIZED MAGNETIC FLUX IN SUPERCONDUCTORS

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In a recent experiment,[1] the magnetic flux trapped within a doubly connected superconductor has been measured by cooling a thin ring embracing an external flux below the transition temperature. The experimental result is plotted in Fig. 1, which gives the total flux ϕ through the ring (which can never be changed in the superconducting state) as a function of the external flux ϕ_e. The quantity E also plotted in Fig. 1 will be discussed later on.

The unit $\phi_0 = hc/e$ of Fig. 1 is the unit of quantization briefly mentioned by London[2] and by Onsager[3] which stimulated the experiment. In order to find an indication for the quantization in units of $\phi_0/2$, let us first discuss the argument of London. After this we will give a simple argument that the experiment gives an indication of the existence of bound pairs in the superconductor.

If $\psi_0(\vec{r}_1 \cdots \vec{r}_N)$ is the wave function of the ground state of the superconductor, the wave function

$$\psi = \exp[i\sum_i S(\vec{r}_i)/\hbar]\psi_0 \qquad (1)$$

corresponds to a superfluid flow with a velocity field $\vec{v}(\vec{r})$ given by

$$\vec{p}(\vec{r}) = \operatorname{grad} S(\vec{r}) = m\vec{v}(\vec{r}) + (e/c)A(\vec{r}), \qquad (2)$$

thus obeying London's equation,

$$\operatorname{curl}[m\vec{v} + (e/c)\vec{A}] = 0. \qquad (3)$$

For a closed curve C located entirely within the superconductor and not embracing any hole, the integral

$$\oint_C \vec{p}\cdot d\vec{s} = \oint_C [m\vec{v} + (e/c)\vec{A}]\cdot d\vec{s} = (e/c)\phi \qquad (4)$$

therefore must be zero. This, however, need not be the case if C embraces a hole. As a consequence of (3) the integral then has the same value for all curves around the hole. If the ring is thick enough, the curve can be placed in its interior so that the current density $\sim\vec{v}(\vec{r})$ can be neglected. In this case one has $\phi = \int H df$.

The quantization of the flux ϕ then is supposed to follow from the fact that the wave function is single valued. London states, without proof, that this requires $\oint \vec{p}\cdot d\vec{s} = nh$. This sounds plausible since ψ_0 is single valued so that $\exp[i\sum S(\vec{r}_i)/\hbar]$ also has to be single valued in order to guarantee the same for ψ.

We now want to show that this is not necessary if ψ_0 contains strong correlations of the kind proposed by Bardeen, Cooper, and Schrieffer.[4]

According to BCS the ground-state wave function can be written as

$$\psi_0 = A\prod_{i<j} w(\vec{r}_i - \vec{r}_j), \qquad (5)$$

where A indicates the antisymmetrization procedure and $w(\vec{r})$ is the Fourier transform of $v_k/u_k = w_k$ (u_k and v_k being defined as in BCS).

Let us now introduce cylinder coordinates with the axis along the axis of the ring. If we do not indicate the r and z dependence explicitly, (5) reduces to

$$\psi_0 = A\prod w(\varphi_i - \varphi_j). \qquad (6)$$

As a first approximation one may expect $w(\varphi_2 - \varphi_1)$ to be similar to the wave function in an infinitely extended homogeneous superconductor, i.e., only different from zero in the neighborhood of $\varphi_1 = \varphi_2$, which would be single valued in $(\varphi_1 + \varphi_2)/2$ but not in φ_1 and φ_2 individually. The latter would require

$$w(\varphi) = w(\varphi + 2\pi). \qquad (7)$$

This condition, however, can be fulfilled in replacing w by

$$w_g = \sum_\nu w(\varphi + 2\pi\nu). \qquad (8)$$

One also expects excited states corresponding

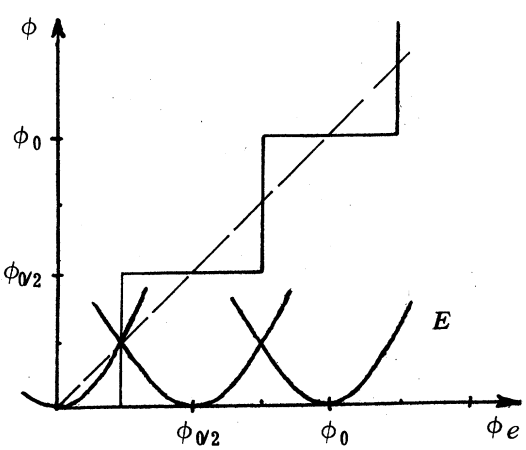

FIG. 1. The experimental values of ϕ (step-like curve, schematically) and the kinetic energy E of the superconductor as function of ϕ_e.

to pairs running around the axis with a total angular momentum n of the type

$$e^{in(\varphi_1+\varphi_2)/2} w_g(\varphi_2 - \varphi_1). \tag{9}$$

Here n has to be an integer in order to make (9) single valued in $(\varphi_1+\varphi_2)/2$. The single valuedness in φ_1 and φ_2 individually can be saved if w_g is replaced by

$$w_u = \sum_\nu w(\varphi + 2\pi\nu)(-1)^\nu, \tag{10}$$

if n is odd.

The wave functions (9) and (10) are not exact eigensolutions of the Schrödinger equation, but they are good approximations deviating from the solutions of the homogeneous medium in only a very small part of the configuration space $0 \leq \varphi_1, \varphi_2 < 2\pi$ if the extension of the pairs is small compared to the dimensions of the ring. The wave functions of the whole system are given by

$$e^{in\sum_i \varphi_i/2} A\prod w_g(\varphi_i - \varphi_j) \text{ for } n \text{ even},$$

$$e^{in\sum_i \varphi_i/2} A\prod w_u(\varphi_i - \varphi_j) \text{ for } n \text{ odd}. \tag{11}$$

We do not consider excitations corresponding to the breakup of pairs and the acceleration of a pair relative to the rest of the pairs since these are only excited by fields above the critical field. The application of (4) then gives the required half-integral values of $e\phi/hc$.

The discussions above only give the wave functions which are compatible with the requirement of single valuedness but do not tell us which of them actually is obtained for a given external flux. For this purpose let us consider the energy of the electrons as a function of the external flux ϕ_e. Since there is no first order change of the wave function of a superconductor in an external field, the energy of the excited states above the ground state is just the kinetic energy of the superfluid flow which can be estimated by assuming $v(r)$ to be constant inside the penetration depth and zero outside. This gives as the excitation energy per particle

$$E/N = \tfrac{1}{2}mv^2(\lambda/\Delta r), \tag{12}$$

where Δr is the thickness of the ring and λ the penetration depth.

The velocity v then can be calculated from Maxwell's equation: $|\mathrm{curl}\,\vec{H}| = 4\pi n_s ev/c$ (n_s is the number of superconducting electrons). If one replaces $\pi r^2 H$ by the flux $(\phi - \phi_e)$ produced by the superconducting electrons, one obtains finally

$$n_s ev = (\phi - \phi_e)c/4\pi r^2\lambda; \quad E = \mathrm{const}(\phi - \phi_e)^2. \tag{13}$$

The result is plotted in Fig. 1 for different values of the total flux ϕ.

Whereas in the superconducting state the transitions between different parabolas, even if energetically possible, are forbidden because of $d\phi/dt = 0$, one expects that the state obtained by cooling the ring down from the normal state with an external flux will be the state of lowest possible energy, i.e., that state which can be obtained by the smallest possible acceleration of the electrons. This would explain completely the experimental result of Fig. 1, in particular the fact that the first flux quantum is trapped already if the external flux has only reached the value of $\phi_0/4$.

The model considered above is of course very poor. We have only treated the case of zero temperature and we have left out all the complicated events which happen during the transition from the normal to the superconducting state. Nevertheless we hope that the indications are strong enough to believe that the experiment of Doll and Näbauer is a (macroscopic) consequence of angular momentum quantization (similar to the quantized vortex lines in He II) and is directly related to the existence of bound pairs of electrons in the superconductor.

Note added in proof. Meanwhile there has appeared a Letter by Onsager[5] explaining the occurrence of odd multiples of $hc/2e$ in essentially the same way as above. We can use our wave function to exhibit the equivalence of this idea to the pairing condition as used by Byers and Yang.[6] For this purpose one only has to notice that introducing

$$w_\nu = \frac{1}{2\pi}\int_{-\infty}^{\infty} w(\varphi)e^{-i\nu\varphi}d\varphi,$$

w_g and w_u can be written as

$$w_g = \sum_\nu w_\nu e^{i\nu\varphi},$$

$$w_u = e^{-i\varphi/2}\sum_\nu w_{\nu+1/2}e^{i\nu\varphi}. \tag{14}$$

The wave functions (11) then can easily be transformed to the usual notation in second quantization:

$$\prod(u_\nu + v_\nu a_{\uparrow\nu+m}{}^* a_{\downarrow-\nu+m}{}^*)|0\rangle; \quad (n \text{ even})$$

$$\prod(u_{\nu+1/2} + v_{\nu+1/2} a_{\uparrow\nu+m+1}{}^* a_{\downarrow-\nu+m}{}^*)|0\rangle; \quad (n \text{ odd}) \tag{15}$$

with $n = 2m$ $(n = 2m+1)$ for even (odd) values of n.

Inside a thick ring the kinetic angular momenta are given by $\hbar(\nu - e\phi/hc)$ if $\hbar\nu$ is the canonical angular momentum. Therefore with $\phi = hcm/e$ $[\phi = hc(m+\frac{1}{2})/e]$ the pairing with zero current occurs for $(\nu, -\nu)$ $[\nu+\frac{1}{2}, -\nu-\frac{1}{2})]$ in the case of even (odd) n. The description in terms of wave functions (11) or (15) has the advantage of not only showing the equivalence of two apparently different pictures but also of being valid in the case of thin rings where the current does not vanish as assumed by Byers and Yang.

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[6]N. Byers and C. N. Yang, Phys. Rev. Letters 7, 46 (1961).

OSCILLATORY GALVANOMAGNETIC EFFECT IN METALLIC SODIUM*

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We wish to report the observation at 4°K of a novel galvanomagnetic effect in sodium of very high conductivity ($\rho_{300°K}/\rho_{4°K} \approx 7500$; $\omega_c\tau \approx 40$ in 10 000 gauss). In this effect, the Lorentz force produces an electrical oscillation of low frequency (ranging from 5 to 35 cycles/sec in our experiment). Our results provide direct evidence of a particular macroscopic electron gas excitation described by Aigrain[1] and given by him the name "helicon" excitation. There is also a connection between our observation and certain effects observed in bismuth at microwave frequencies.[2]

The experimental arrangement (Fig. 1) is based on a configuration for measuring resistivity due to Bean, DeBlois, and Nesbitt.[3] A cylindrical specimen of sodium (height 30 mm and diameter 4 mm) is placed inside primary and secondary coils. On closing or opening the primary circuit, eddy currents are induced in the specimen which can be detected by the secondary coil. From the time constant of the observed voltage in the secondary, one can estimate the resistivity of the specimen. For a sufficiently long time after the primary pulse, the decay is a simple exponential. Our experiment differs from that in reference 3 in that a constant magnetic field (H_0) is applied perpendicular to the axis of the specimen and coils. The constant field H_0 is much larger than the time-dependent magnetic field ($h \approx$ few hundred gauss) produced by the primary coil. The voltage of the secondary coil is observed on closing or opening the primary circuit. Figure 2 shows oscilloscope traces of the secondary voltage as a function of time for various values of the bias field H_0, using sodium for which $\rho_{300°K}/\rho_{4°K} \approx 7500$ (mean free path at 4°K $\approx \frac{1}{4}$ mm). In zero magnetic field the normal eddy current decay is observed. The other three traces show the effect of applying H_0. One sees damped oscillations whose frequency is proportional to magnetic field. The frequency at 10 000 gauss is 32 cycles/sec.

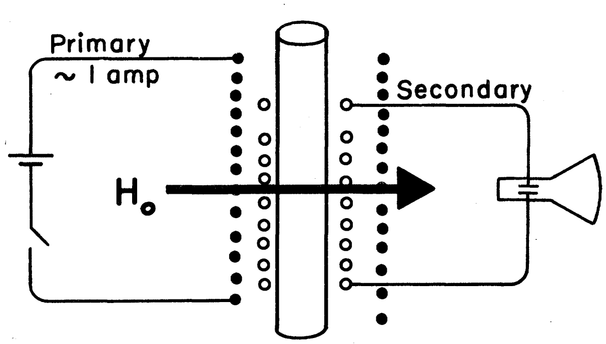

FIG. 1. Schematic diagram of experiment. Primary coil ≈300 turns of No. 28 copper wire; secondary coil ≈1200 turns of No. 43 copper wire. Specimen and coils immersed in liquid helium at 4.2°K.

We believe the oscillations to originate as follows. The abrupt change in the primary field (h) excites stable modes of the electron gas in which the electric vector is circularly polarized about the large magnetic field.[1] The oscillation observed is the component of this circular wave resolved in