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DRAG FORCES IN ROTATING He II[†]

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Despite many measurements on the bulk properties of rotating He II, the first local probing of the velocity field occurred only in 1960 when Pellam[1] studied the torque on a Rayleigh disk immersed in a rotating cylinder filled with He II as a function of disk position and of temperature. The most surprising result of his study occurred in the torque versus temperature measurements, which seemed to indicate that only the superfluid component of He II exerted torque upon the system, so that just below the lambda transition the torque vanished entirely. This result appears to be in conflict with all theories of He II, and provided the motivation for the present study in which the velocity field was studied in a rotating system using a different type of detector, which measures the drag forces exerted on disks placed normal to the stream lines of the rotating helium.

The detectors consisted of pairs of glass disks mounted on quartz crossbeams and suspended inside a rotating cylinder (inside diameter 5.3 cm) by means of quartz fibers fixed at their upper ends in the laboratory coordinate system. The rotating cylinder was contained in a partially silvered glass Dewar. The bottom of the rotating cylinder was sealed by a brass plate, the only connection to the main helium bath being through two 0.005-in. filling holes drilled in the plate about 0.5 mm off axis. The temperature was determined from the helium vapor pressure in the cylinder (scale $T55_E$), thereby assuring that no errors were introduced from flowing He gas. Drag forces on the disks resulted in deflection of the detectors through angles which could be measured by means of a light beam reflected from a mirror mounted on the detector, and thence onto a circular translucent scale 28 cm from the axis of rotation. A range of reproducible angular rotational velocities from 0.21 to 6 rpm was produced by a synchronous motor coupled to speed reduction gear boxes and to a lathe gear box. A leather belt decoupled the drive from the experimental Dewar, thereby reducing vibration to a negligible level.

Two detector assemblies were used. In assembly I the detectors were 1.2 cm in diameter and 0.2 mm thick. They were mounted on a diameter at 2.5 cm center-to-center separation, and suspended from a quartz fiber of torsion constant 8×10^{-2} dyne cm/rad. In assembly II the detectors were 0.6 cm in diameter and 0.2 mm thick, mounted at a center-to-center separation of 2.3 cm. The torsion constant was 4.7×10^{-3} dyne cm/rad. The fiber constants were determined from the observed deflections in air at 300°K at low Reynolds numbers where Stokes' law obtains, so that the drag forces were linear in velocity.[2] The constants obtained in this way were in reasonable agreement with those found by timing the fibers as torsion pendula using a known moment of inertia. An additional advantage of the air calibration procedure is that errors arising from the finite size of the disks tend to cancel when deflections in air are compared to those in helium. System I was additionally calibrated in CS_2 and in liquid N_2. System II was calibrated in He gas at 75°K and 580 mm Hg and at 2°K and 23.7 mm Hg as well as in air. The

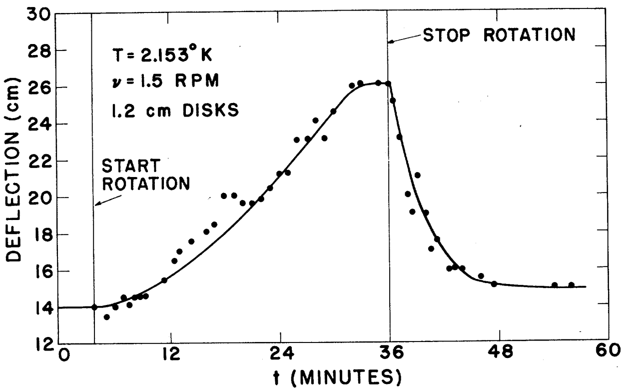

FIG. 1. Development of drag force on a disk in rotating He II. Rotation is begun at 4 minutes and stopped at 36 minutes.

calibrations were self-consistent to within about ±20%.

Three types of measurement were made with these detectors:

(1) For both systems the buildup of the drag forces with time after rotation was begun followed the same type of curve and had the same time scale as was found in the Rayleigh disk experiments.[1] That is, when rotation was started from rest a time delay was observed before any drag was seen, and the equilibrium drag was attained only after about $\frac{1}{2}$ hr. When rotation was stopped, the drag force immediately decreased, and the zero position was regained in a time much shorter than the buildup time. A typical drag versus time curve is shown in Fig. 1, for system I at a temperature of 2.153°K and 1.5 rpm. Rotation was begun at $t = 4$ minutes and stopped at $t = 36$ minutes. Thus in this experiment as well as in the Rayleigh disk experiments, there is evidence for a velocity front propagating from the walls upon initiation of rotation, and there is rigidity associated with the liquid upon cessation of rotation.

(2) The temperature dependence of the equilibrium drag force was studied in both systems at fixed rotational velocities. The results for system I are shown in Fig. 2 for a speed of 4.8 rpm. The drag force is seen to be almost independent of temperature from 1.3°K to the lambda temperature, T_λ, with a slight minimum near 1.9°K. The drag appears to be continuous across the lambda transition. However, above T_λ fluctuations in the detector became quite large, so that it was very difficult to take measurements and the points are subject to large error. The constancy of the drag force with temperature is in striking contrast with the torque measurements obtained with the Rayleigh disk.[1] The temperature variation of drag obtained with system II at fixed rotational velocities is similar to that for system I.

(3) At fixed temperatures below T_λ the drag force was studied as a function of the velocity (v) of rotation. For system I the drag force was found to be quadratic in velocity at all temperatures not too near T_λ. This is the velocity dependence expected for a disk immersed in a classical fluid at Reynolds numbers $\mathcal{R}$ greater than about 50.[2] In the present experiments, $\mathcal{R}$ $(=\rho v d/\eta_n)$ always exceeds 100 so that classical drag may be expected. The dimensionless drag coefficient C_D (defined as the ratio of the observed drag force to $\frac{1}{2}\rho v^2$ times the area of the disk) ob-

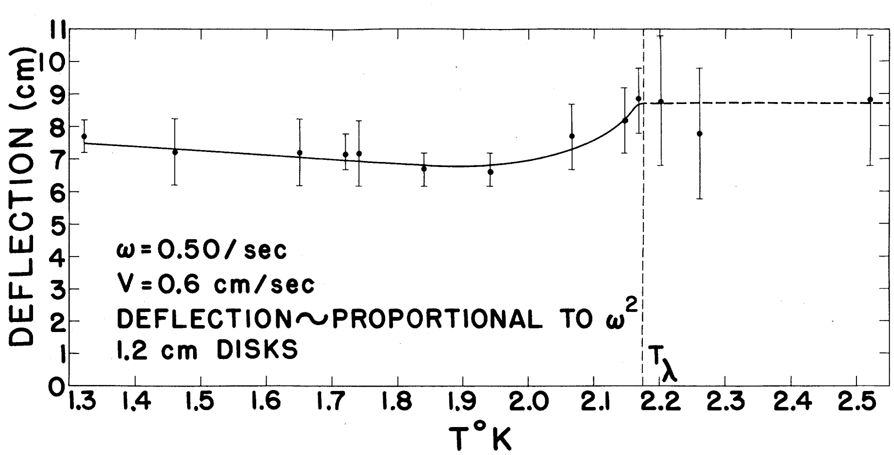

FIG. 2. Temperature dependence of the drag force on a disk in rotating helium.

tained from the data of Fig. 2 is about 0.25 at all temperatures. This drag coefficient is approximately equal to that obtained in the sphere drag measurements of Dowley, Firth, and Hollis Hallett,[3] an agreement which is somewhat surprising since the classical drag coefficient in this range of Reynolds numbers is 0.47 for spheres, while it is 1.17 for disks.[2] The behavior in the smaller system II was much different from system I. Except close to T_λ the drag force at fixed temperature varied as v^n with $1 \leq n < 1.3$. Figure 3 presents some representative drag versus rotational velocity measurements for systems I and II. At temperatures within 0.1°K of T_λ the behavior of both systems becomes more complicated. The exponent n becomes a function of velocity, but always approaches two at high velocities. The high-velocity drag coefficient is always about 0.25.

The significance of the linearity of the drag force with velocity in system II is not yet clear, and may well be an instrumental effect. This behavior and the behavior close to T_λ are still under study. However, because of the present lack of explanation for the Rayleigh disk results, we venture the following suggestion which offers a consistent method for interrelating those results with the linear drag found in the present experiment.

In classical fluids drag forces linear in velocity occur only for flow at low Reynolds numbers. The linearity of the system II measurements and the instantaneous decrease in the drag force upon cessation of rotation suggest that there is rigidity associated with rotating He II. This rigidity could arise from the vortex lines which must exist in rotating He II, and which probably exert drag forces upon bodies placed in rotating He II. Since the vortex lines are excitations of ground-state superfluid, they must be associated with the normal fluid, and would be seen in a drag experiment as an excess normal fluid viscosity. The Reynolds number describing the normal fluid flow would then be quite small, so that Stokes' drag linear in velocity could occur with attendant flow separation. Since the vortex line density is independent of temperature, the drag force is also. The line density is proportional to the rotational velocity, but the excess viscosity is not necessarily proportional to the line density. Such an excess normal fluid viscosity should influence the Rayleigh disk experiment. The equation for the torque on a Rayleigh disk is derived under the assumption of high Reynolds number flow with no flow separation. In rotating He II the total torque on a disk

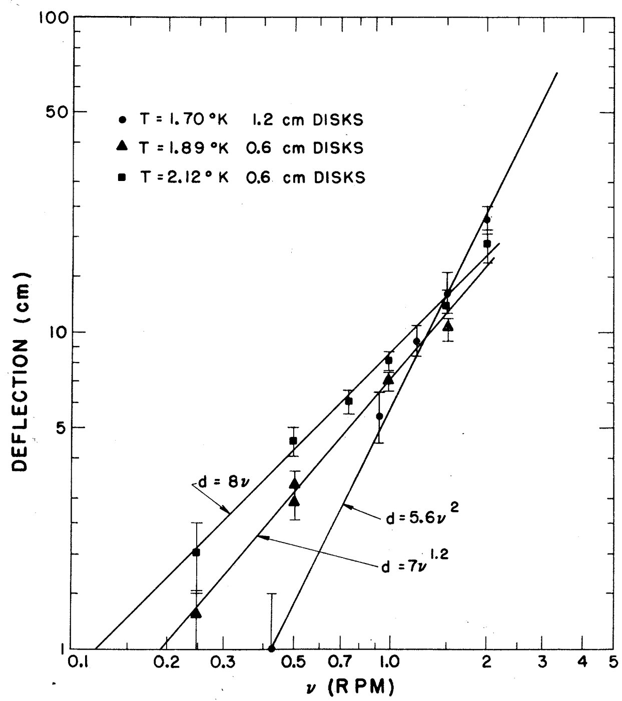

FIG. 3. Variation of the drag force on disks in He II with rotational velocity. Two distinct types of behavior are found.

with this assumption should be proportional to $\rho_s v_s^2 + \rho_n v_n^2$, and presumably $v_n = v_s$ since rotating He II is known to possess classical angular momentum.[4] If flow separation of the normal fluid occurs, the second term vanishes, and the total torque is proportional to $\rho_s v_s^2$, in agreement with Pellam's observations.

Vinen[5] has argued that vortex line turbulence may arise from heat currents in He II. If the above speculations are valid, it should be possible to increase the excess viscosity by heat currents. Preliminary experiments tend to support this idea, but large fluctuations of the detector due to unstable flow prevent definite conclusions at this time.

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EVIDENCE FOR SPIN-ORBIT SPLITTING IN THE BAND STRUCTURE OF ZINC AND CADMIUM*

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Cohen and Falicov[1] have discussed the effect of spin-orbit coupling on the topology of the Fermi surfaces of the hexagonal close-packed metals. They conclude that spin-orbit splitting lifts the degeneracy between the first and second bands at the corners of the Brillouin zone and separates the small pockets of holes (arm caps) in the first band from the region of holes (arms) in the second. This gives rise to a closed-hole surface (cap surface) in the first band and a second-band surface that is infinitely extended along the hexagonal axis [Fig. 2(b) of reference 1] due to the intersections of arms. Both of these surfaces have extremal cross sections at the hexagonal face of the zone which one would expect to be observable in de Haas–van Alphen, cyclotron resonance, and ultrasonic attenuation experiments.

During the course of detailed de Haas–van Alphen studies of zinc and cadmium, we have observed oscillations in the magnetic susceptibility whose periods correspond to estimated values for the aforementioned extremal cross sections and which we have been unable to account for with Fermi surface models that neglect spin-orbit coupling. The de Haas–van Alphen periods were obtained by a torsion method in magnetic fields up to 23 kilogauss, using a null deflection technique and automatic recording. The angular dependences of the periods essential to this discussion are shown in Fig. 1. θ is the angle between the magnetic field direction and the hexagonal axis.

In Cd, for small θ the values of the Δ and x periods can be determined to within approximately 2% because all of the oscillations present are of comparable amplitude. As θ is increased the amplitude of the main arm oscillation increases rapidly, while effects due to arms in the other symmetry directions [periods indicated by dashed lines in Fig. 1(b)] are hardly observable. The amplitudes of the x and Δ oscillations drop rapidly, with Δ undetectable at $\theta \cong 1°$ and x at $\theta \cong 10°$.

In Zn, analysis is more difficult because x and Δ occur only as a weak modulation on a strong carrier—the arm periods. It was possible to obtain an unambiguous value for the Δ points out to a θ of 6°. However, there is no way of distinguishing from our data whether x lies below the arm period as shown in Fig. 1(a) or an equivalent distance above. The lower value was selected to be consistent with the relative sizes of the spin-orbit splitting in Zn and Cd (reference 1) and with the effect of this splitting on the difference in cross section of arm junction and cap surface as explained below. For θ greater than 7°, beating among the three arm periods makes detection of the Δ oscillation difficult due to its decreasing amplitude and to the presence of a comparable arm period. Therefore, the angular extent of the Δ period is not sharply determined. Near $\theta = 15°$, where the Δ period might beat distinctly with the arm periods, a careful search yielded no sign of its presence up to 23 kilogauss nor did any unexplained periods appear at larger θ.

The arm data in Zn agree with the few periods observed by Verkin and Dmitrenko,[2] while those in Cd fit very well with the work of Berlincourt,[3] but no mention of the x and Δ periods appears in either paper. We feel that the identification of Δ's as cap surface periods and x's as arm junction periods is correct because the other periods observed in Zn and Cd agree with the general form of portions of the Fermi surface predicted by the single orthogonalized plane wave construction,[4] and because this construction requires arm junctions and cap surfaces in the single zone scheme.

The presence of the spin-orbit energy gap decreases the cross section of the cap surface and increases that of the arm junction from the common value that these cross sections would have in