# HELICITY OF THE PROTON FROM Λ DECAY*

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The purpose of this note is to report a result for the helicity of the proton from Λ decay. We find that our data strongly favor positive helicity.

This result is obtained from a study of about 2000 Λ's produced by $K^-$ absorption in He using the Duke helium chamber,[1] and a low-energy $K^-$ beam at Berkeley.[2] The study, based on 105 proton scattering events from Λ decays, was made as part of a continuing experiment, a portion of which is described in detail elsewhere.[3]

The helicity of the proton in Λ decay is defined as the sign of the proton polarization measured in the decay rest frame. As shown by several authors,[4] this polarization is given by

$$\vec{\sigma}^* = -\alpha \vec{k}^*/|\vec{k}^*|, \qquad (1)$$

where $\alpha$ is the asymmetry parameter of the decay $\Lambda \to \pi^- + p$, or in terms of the $s$ and $p$ amplitudes, $\alpha = 2\mathrm{Re}(s^*p)/(s^2+p^2)$ and $\vec{k}^*$ is the c.m. momentum of the proton. The proton polarization, longitudinal in the Λ rest frame, becomes partially transverse when transformed into the laboratory frame since (nonrelativistically) the spin remains rigid while the momentum $\vec{k}^*$ transforms into $\vec{k}_i$(lab), making an angle Γ with respect to $\vec{k}^*$. The transverse component, $\vec{\sigma}^*\sin\Gamma$, can then be measured by means of subsequent elastic scattering.

In principle both the magnitude and the sign of $\alpha$ can be obtained by measuring the (azimuthal) angular distribution of the scattered protons. In practice, however, since only a small portion of the available data has as yet been analyzed, only the sign of $\alpha$ is determined with good accuracy in this report.

Severe limits on the magnitude of $\alpha$ have already been set from the results of the up-down asymmetry experiments at Brookhaven and Berkeley.[5] The latest unpublished Berkeley results[6] indicate that $|\alpha| \geq 0.85$. We employ this result as a constraint on the determination of the sign of $\alpha$ in the present experiment. Details of the analysis are given below.

If an incoming proton beam of (lab) momentum $\vec{k}_i$ and arbitrary polarization $\vec{\sigma}$ is scattered from a spin 0 nucleus, the angular distribution[7] of the elastically scattered protons which emerge with polarization $\vec{P}_{an}(\theta, \epsilon)$ is

$$f(\theta, \epsilon) \sim 1 + \vec{\sigma}\cdot\vec{P}_{an}(\theta, \epsilon), \qquad (2)$$

where $\theta$ = (proton) center-of-mass scattering angle, $\epsilon$ = lab proton energy at the point of scattering, $\vec{k}_f$ = outgoing proton lab momentum, and $\vec{P}_{an}(\theta, \epsilon) = P_{an}(\theta, \epsilon)\vec{k}_i \times \vec{k}_f / |\vec{k}_i \times \vec{k}_f|$ = analyzing polarization.[8] This expression can be written in a form more useful for analysis by expressing $\vec{\sigma}\cdot\vec{P}_{an}(\theta, \epsilon)$ in terms of the angle Γ defined above, and Ψ, an azimuthal scattering angle which is conveniently chosen as the angle between the Λ decay plane and the $p$-He scattering plane. The distribution then becomes

$$f(\Psi)d\Omega = [1 + \alpha \sin\Gamma P_{an}(\theta, \epsilon)\sin\Psi]d\Omega. \qquad (3)$$

The quantities Γ and Ψ are directly obtainable[9] from a kinematic analysis of each event, and since the analyzing polarization is known[10] (see Fig. 1), $\alpha$ may be determined by an appropriate best-fit procedure using (3) as the theoretical distribution.

The important characteristics of the various subsamples chosen for analysis are given in Table I. Of the total of ≈150 Λ-decay proton scattering events found by the scanners, only 105 survived the selection criteria for sample $A$ which were imposed in order to remove the Coulomb scatter-

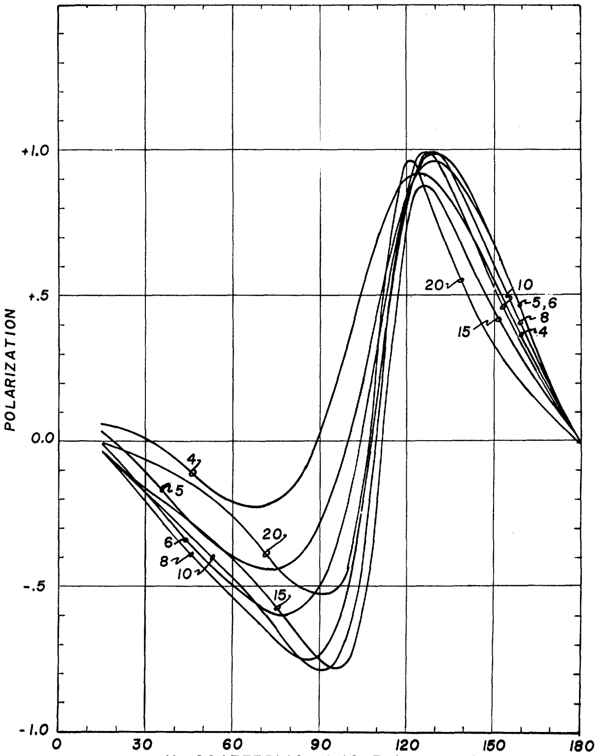


FIG. 1. Analyzing polarization for proton helium scattering as a function of c.m. angle for various proton (lab) kinetic energies. Data at and above 10 Mev are taken from J. L. Gammel and R. M. Thaler, Phys. Rev. 109, 2041 (1951). Data below 10 Mev are calculated from phase shifts given by A. Juveland and W. Jentschke, Z. Physik 144, 521 (1956).

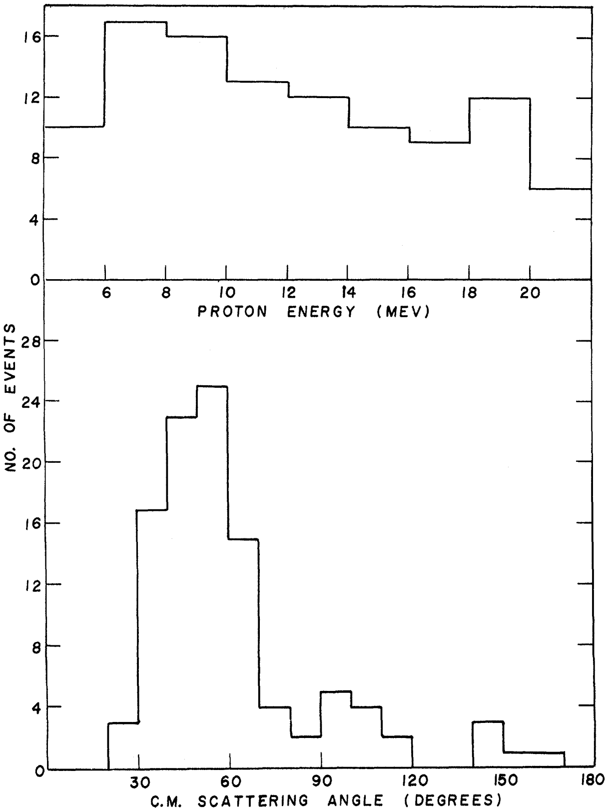


FIG. 2. (a) Proton energy spectrum of Sample $A$ ($\theta \geq 26°$, 4 Mev $\leq \epsilon \leq 22.5$ Mev). (b) Center-of-mass angular distribution of Sample $A$.

ings and possible inelastic contamination. These selection criteria are: (1) $\theta \geq 26°$, (2) 4 Mev $\leq \epsilon \leq 22.5$ Mev. Of the total of 105 events in $A$, 100 stop in the chamber. The energy spectrum and angular distribution of all events is shown in Fig. 2.

Each event in the sample was subjected to a full kinematic analysis which yielded best-fit values of the dip and azimuth of each track. This information, when coupled with the accurate energy determination provided by the range of the scattered proton, permitted the calculation of $\sin\Psi$ and $P_{an}$ to an average accuracy of 15% and 10%, respectively.[11]

The determination of the sign of $\alpha$ was carried out using three techniques, (a) the method of maximum likelihood, (b) comparison of the measured

Table I. Characteristics of the proton scattering sample. $N$ = No. of events in sample; $\epsilon$ = average proton energy at scattering point; $\theta$ = average c.m. scattering angle; $\bar{P}_{an}$ = average analyzing polarization; $\langle P_{an}\sin\Gamma\rangle_{av}$ = average effective polarization of sample; $\eta_{exp}$ = experimental value of right-left asymmetry; $\langle\eta\rangle$ = predicted value of right-left asymmetry using the measured $\langle P_{an}\sin\Gamma\rangle_{av}$.

| Sample definition | $N$ | $\bar{\epsilon}$ | $\bar{P}_{an}$ | $\langle\sin\Gamma\rangle_{av}$ | $\langle P_{an}\sin\Gamma\rangle_{av}$ | $\eta_{exp}$ | $\langle\eta\rangle$ | $\bar{\theta}$ |
|---|---|---|---|---|---|---|---|---|
| $A$. $\theta \geq 26°$, 4 Mev $\leq \epsilon \leq 22.5$ Mev | 105 | 12.0 | 0.36 | 0.77 | 0.28 | 0.20 | $-0.18\alpha$ | 60.5° |
| $B$. $\theta \geq 26°$, 4 Mev $\leq \epsilon \leq 22.5$ Mev, $P_{an}\sin\Gamma > 0.25$ | 57 | 10.6 | 0.47 | 0.86 | 0.40 | 0.15 | $-0.26\alpha$ | 69.0° |
| $C$. $\theta \geq 40°$, 4 Mev $\leq \epsilon \leq 22.5$ Mev | 85 | 11.8 | 0.41 | 0.76 | 0.31 | 0.20 | $-0.20\alpha$ | 66.5° |

right-left asymmetry with its expected value, and (c) a $\chi^2$ fit of (7) to the $\Psi$ distribution.

The likelihood function, $L(\alpha)$, representing the total probability for observing our 105 events in sample $A$, is a product of the distributions (3) evaluated for each event:

$$L(\alpha)=\prod_{i=1}^{20}[1+\alpha P_{an}(\theta_i,\epsilon_i)\sin\Gamma_i\sin\Psi_i]. \qquad (4)$$

A plot of $L(\alpha)$ vs $\alpha$ for sample $A$ is shown in Fig. 3. Although the best-fit magnitude of $\alpha$, as determined from the position of the maximum of $L(\alpha)$, is quite consistent with the limits set by the up-down asymmetry experiments ($0.85\leq|\alpha|\leq1$), it is clear from the width of $L(\alpha)$ that the present data cannot determine $|\alpha|$ with comparable precision. We therefore accept the previous determination of $|\alpha|\approx1$, and ask the question: "Given the magnitude of $\alpha$, what is the likelihood of the hypothesis $\alpha=-1$, relative to the hypothesis $\alpha=+1$"? One such relative likelihood measure is given by the ratio $L(\alpha=-1)/L(\alpha=+1)\approx2\times10^3$ which strongly favors[12] $\alpha=-1$.

Another such relative measure can be obtained by comparison of the measured azimuthal angle asymmetry $\eta$, defined[13] by

$$\eta_{exp}=[N_R(\sin\Psi\geq0)-N_L(\sin\Psi<0)]/(N_R+N_L), \qquad (5)$$

with its expected value $\langle\eta\rangle$ given by

$$\langle\eta\rangle=(2/\pi)\alpha\langle\sin\Gamma\, P_{an}(\theta,\epsilon)\rangle_{av}. \qquad (6)$$

For our sample $A$, $\langle\eta\rangle=-0.18\alpha$, and the measured asymmetry is $\eta_{exp}=0.20\pm0.1$. Thus the data are consistent with the requirement $|\alpha|\approx1$, and are in good agreement with the hypothesis $\alpha=-1$, but are in disagreement with the hypothesis $\alpha=+1$ by about 3.8 standard deviations.[12]

An alternate approach is based on a $\chi^2$ analysis of the observed distribution in $\Psi$ (see Fig. 4). The expected distributions $g(\Psi)d\Psi$ for $\alpha=\mp1$, respectively, follow from (3), integrated over solid angle:

$$g(\Psi)d\Psi=[1+\alpha\langle\sin\Gamma\, P_{an}(\theta,\epsilon)\rangle_{av}\sin\Psi]d\Psi, \qquad (7a)$$

$$g(\Psi)d\Psi=[1-\alpha\langle\sin\Gamma\, P_{an}(\theta,\epsilon)\rangle_{av}\sin\Psi]d\Psi, \qquad (7b)$$

where $\langle\sin\Gamma\, P_{an}(\theta,\epsilon)\rangle_{av}$ is the experimental average given in Table I.[14] The most probable value of $\chi^2$ for this fit is 5.0. The distribution (7a) is in excellent agreement with the data of Fig. 4, yielding a $\chi^2$ of 3.9, corresponding to a probability of 56% for the observation of the data, if $\alpha=-1$. On the other hand, the distribution (7b) has a $\chi^2$ of 25.7, corresponding to a probability of less

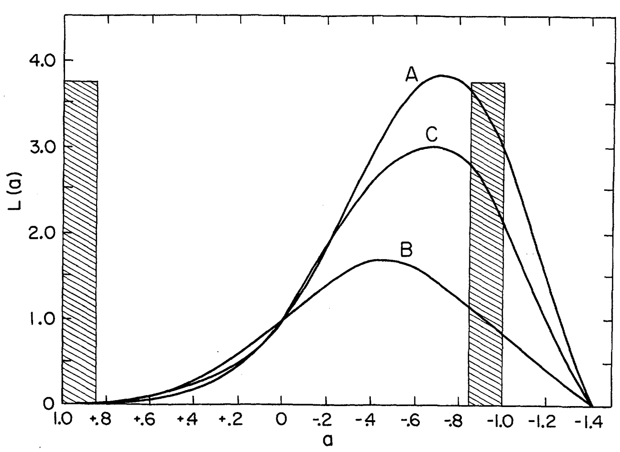


FIG. 3. Likelihood function $L(\alpha)$ vs $\alpha$ for Samples $A$, $B$, and $C$. The shaded areas represent regions allowed by up-down asymmetry experiment. Sample $B$ is defined by $P_{an}\sin\Gamma\geq0.25$, 4 Mev $\leq\epsilon\leq$ 22.5 Mev; sample $C$ is defined by $\theta\geq40°$, 4 Mev $\leq\epsilon\leq$ 22.5 Mev.

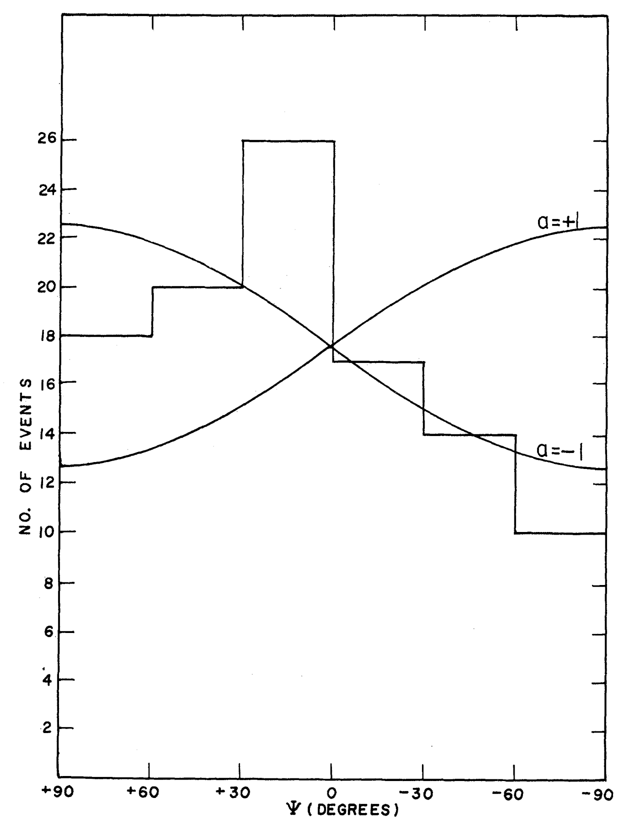


FIG. 4. Distribution of angle $\Psi$ between $\Lambda$ decay and proton scattering planes. The expected distributions for $\alpha=\pm1$ are represented by smooth curves.

than 1 % for the observation of the data, if $\alpha = +1$.

Aside from the purely statistical errors given above, we have considered several possible sources of systematic error—inelastic contamination, Coulomb contamination, scanning bias against events in certain regions of energy and scattering angle (see Fig. 2), and finally polarization of the $\Lambda$ sample. We believe all of these effects to be either small or nonexistent for reasons discussed below.

Firstly, each of the scattered protons had an energy below threshold for $p + He^4 \rightarrow d + He^3$, which constitutes 90 % of the inelastic cross section at low energy. Secondly, the $\theta \geq 26\%$ criterion was designed to eliminate ~90 % of the Coulomb events. In any event a small contamination of Coulomb scatters will not affect the maximum-likelihood estimate of $\alpha$ because these events, at small $\theta$, would be assigned a small $P_{an}$ (see Fig. 1).

In principle, since the angle $\Psi$ between the production and decay planes is uncorrelated with scattering energy and the angle, the bias against low $\epsilon$ and small $\theta$ events should not affect the $\Psi$ distribution. To test this point experimentally, we selected two subsamples $B$ and $C$ according to the criteria given in Table I. $B$ is a high effective polarization sample rich in low-energy events; $C$ is a sample composed of large scattering angle events. Both the likelihood functions (see Fig. 3) and the asymmetries (see Table I) for $B$ and $C$ are in good agreement with their expected values, thus confirming the internal consistency of the data.

Lastly, we consider the possibility of the existence of a polarization of the $\Lambda$ sample, which would complicate the analysis to some extent.[4] The majority of the $\Lambda$'s in the sample were produced via a two-step $\Sigma$-conversion process,[15] i.e.,

$$K^- + N \rightarrow \Sigma + \pi, \qquad (8a)$$

$$\Sigma + N' \rightarrow \Lambda + N'. \qquad (8b)$$

The outgoing $\Lambda$'s, if polarized at all, must be polarized perpendicular to the production plane of (8b). However, since <u>all</u> such production planes are accepted, the outgoing $\Lambda$'s must be polarized.

Owing to all the above considerations, we include no systematic effects in our estimation of the errors in this experiment.

In order to make a direct comparison of our results with those of other experiments of this type, we must disregard the constraint placed on the magnitude of $\alpha$ by the up-down asymmetry experiments. Then, from the likelihood function of Fig. 3, the most likely value of $\alpha$ is

$$\alpha = -0.75^{-0.15}_{+0.50}, \qquad (9)$$

where the error on the positive side corresponds to the half-width of $L(\alpha)$. The result (9) can be compared with those of Boldt et al.[16] and Birge et al.,[17] who find $\alpha = +0.85 \pm 0.20$ and $-0.45 \pm 0.5$, respectively. Our result is clearly in disagreement with the former; it is in agreement with that of Birge et al., but is even more indicative of a negative $\alpha$.

Furthermore, we should like to emphasize that by taking the known limits on $|\alpha|$ properly into account, as described above, one obtains considerably stronger results. We prefer to summarize these results as follows: For values of $|\alpha|$ consistent with the known limits, our data favor the hypothesis that $\alpha$ is negative over the hypothesis that $\alpha$ is positive by at least 3 standard deviations.

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[1]M. M. Block, H. A. Fairbank, E. M. Harth, T. Kikuchi, C. M. Meltzer, and J. Leitner, Proceedings of the International Conference on High-Energy Accelerators and Instruments, CERN (European Organization for Nuclear Research, Geneva, 1959), p. 461.

[2]N. Horwitz, J. Murray, M. Ross, and R. Tripp, University of California Radiation Laboratory Report UCRL 8269, 1958 (unpublished). J. Murray and P. Schlein (private communication).

[3]M. M. Block et al., Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 419 ff.

[4]T. Lee and C. Yang, Phys. Rev. 108, 1353 (1957), R. Gatto, University of California Radiation Laboratory Report UCRL 3795, 1957 (unpublished); J. Leitner, Nuovo cimento 8, 68 (1958).

[5]F. Eisler et al., Phys. Rev. 108, 1353 (1957). F. S. Crawford, M. Cresti, M. L. Good, K. Gottstein, E. M. Lyman, F. T. Solmitz, M. L. Stevenson, and H. K. Ticho, Phys. Rev. 108, 1102 (1957).

[6]Private communication from F. Crawford. A similar estimate (~0.8) is given by M. Schwartz (private communication).

[7]L. Wolfenstein, Ann. Rev. Nuc. Sci. 6, 43 (1956).

[8]This is the definition of polarization adopted by L. Wolfenstein, Phys. Rev. 75, 1664 (1949).

[9]The angle $\Gamma$ is directly obtainable from $\Lambda$-decay kinematics at the decay vertex. It must then be corrected because of the difference in rotation frequencies of the

proton spin and momentum vectors during the flight path from the decay vertex to the scattering point. This frequency difference is proportional to the anomalous moment of the proton. See D. F. Nelson, A. A. Schupp, R. W. Pidd, and H. R. Crane, Phys. Rev. Letters 2, 492 (1959). The average correction in our experiment is quite small (~2°).

[10]J. L. Gammel and R. M. Thaler, Phys. Rev. 109, 2041 (1958). A. Juveland and W. Jentschke, Z. Physik 144, 521 (1956). The phase shifts of the latter paper and the formula given in reference 8 were used to calculate the values of $P_{an}$ for energies below 10 Mev.

[11]This is true only for events in which the proton stops. Events in which the proton leaves the chamber have more severe errors in general.

[12]The value $|\alpha| > 0.85$ given in reference 6 may be somewhat uncertain since it is based on the assumption that only $s$ and $p$ waves are present in $\Lambda + K^0$ production by 1-Bev/$c$ $\pi^-$ on protons. However, even if we take $|\alpha| = 0.7$, our measured asymmetry $\eta_{exp}$ would still differ by about 3 standard deviations from its expected value for $\alpha = +0.7$, and the ratio $L(-0.7)/L(+0.7)$ is $\approx 10^{+2}$.

[13]This definition holds only for events in which $P_{an}$ is of one sign—viz, $P_{an} < 0$. For events in which $P_{an} > 0$, "right" and "left" must be interchanged.

[14]This is not an algebraic average, but rather is the average magnitude of ($\sin\Gamma P_{an}$). We associate a negative sign with it since most of the scattering events have $P_{an} < 0$. For events with $P_{an} > 0$, we reverse the sign of $\sin\Psi$ in Fig. 4.

[15]See reference 3, page 423.

[16]E. Boldt, H. S. Bridge, D. O. Caldwell, and Y. Pal, Phys. Rev. Letters 1, 256 (1958).

[17]R. W. Birge and W. B. Fowler, Phys. Rev. Letters 5, 254 (1960).

## ERRATUM

SELF-CONSISTENT CALCULATION OF THE MASS AND WIDTH OF THE $J = 1$, $T = 1$, $\pi\pi$ RESONANCE. Fredrik Zachariasen [Phys. Rev. Letters 7, 112 (1961)].

Due to an error in the numerical computations the mass value $m_\rho \sim 950$ Mev is wrong and should be changed to $m_\rho \sim 350$ Mev. While the agreement with the experimental value is slightly less good, it is perhaps encouraging to have too large a coupling constant going with too small a mass. An improved calculation might then give a smaller coupling constant, corresponding to a weaker attraction which could be consistent with a larger mass, while it is hard to see how a smaller coupling constant could go with a smaller mass.

I am indebted to J. Mathews for pointing out that the mass was incorrect, and to C. Zemach for aid in computing the correct value.