the instrument is calculated to be 10^8. This value agrees quite well with the experimentally observed photon flux, giving strong support to the interpretation of our observations. The agreement is better than expected considering the simplifications made in the derivation of Eq. (2) and the estimates made during its application.

We would like to thank D. Kleinman for communication of his paper prior to publication, and H. Guggenheim for growing the $CaF_2:Eu^{2+}$ crystal.

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MAGNETIC BREAKDOWN IN CRYSTALS*

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(Received June 7, 1961)

To establish the various regimes of behavior of electrons in a magnetic field, the cyclotron frequency ω_c is usually compared with τ^{-1}, $\hbar^{-1}kT$, or $\hbar^{-1}E_F$, where τ is a typical relaxation time and E_F the Fermi energy. Comparison of ω_c with $\hbar^{-1}E_g$, where E_g is an energy gap of the pertinent band structure, has not been discussed except to note that the usual derivations of an effective Hamiltonian[1] for the motion of electrons in a magnetic field is no longer valid when $\hbar\omega_c \sim E_g$. No attention has been directed to the question of what actually happens when $\hbar\omega_c$ exceeds E_g; that is the subject of the present Letter.

The splitting of the energy levels due to magnetic fields is of the order of 10^{-8} ev per gauss, whereas gaps in ordinary band structures are rarely as small as 0.1 ev. Our question would be academic were it not for the existence of small gaps of the order 10^{-3} ev or less in some metals because of spin-orbit splitting[2] or of points of accidental degeneracy at or near the Fermi level.[3] These gaps are small compared with the Fermi energy and the remaining typical gaps in the band structure. In such a case the answer to our question is that as H increases, one passes from electron orbits determined semiclassically from the entire band structure to those determined semiclassically by ignoring the small gaps. We call this effect magnetic breakdown. Of course, some additional effects take place, i.e., a spreading of the magnetic levels into narrow bands and additional scattering processes, but these are not significant except in the transition range of magnetic field $\hbar\omega_c \sim E_g$.

Let us consider a simple example of a free-electron metal perturbed by a one-dimensional periodic potential. The one-electron Hamiltonian in the presence of a magnetic field in the z direction is

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} + \frac{|e|}{c}\vec{A}\right)^2 + V_0 \cos\kappa x,$$

where

$$\vec{A} = (0, Hx, 0), \quad k_F \gtrsim \kappa.$$

Since the electron distribution overlaps into the second zone, the Fermi surface consists of an undulating cylinder in the first zone with axis in the x direction and a small pocket of electrons in the second zone [Fig. 1(b)]. When H is small, i.e., $\hbar\omega_c \ll V_0$, the undulating cylinder gives rise to a set of open orbits; as consequences de Haas–van Alphen oscillations arise from the pocket but not from the cylinder and the R_{xx} component of the resistivity tensor increases quadratically with H. When $\hbar\omega_c \gg V_0$,[4] magnetic breakdown essentially restores the free-electron surface [Fig. 1(a)] and only the equatorial section of the "sphere" gives rise to a de Haas–van Alphen period. Moreover, because the open orbits have now disappeared, R_{xx} decreases as H^{-2} to a saturation value which is very approximately the free-electron value. The spreading of the magnetic levels into bands, which have a maximum width of about $2V_0$ for orbits which just intersect the zone faces, gives no appreciable effect, and the additional scattering processes give only second order corrections to the free-electron magnetoresistance[5]

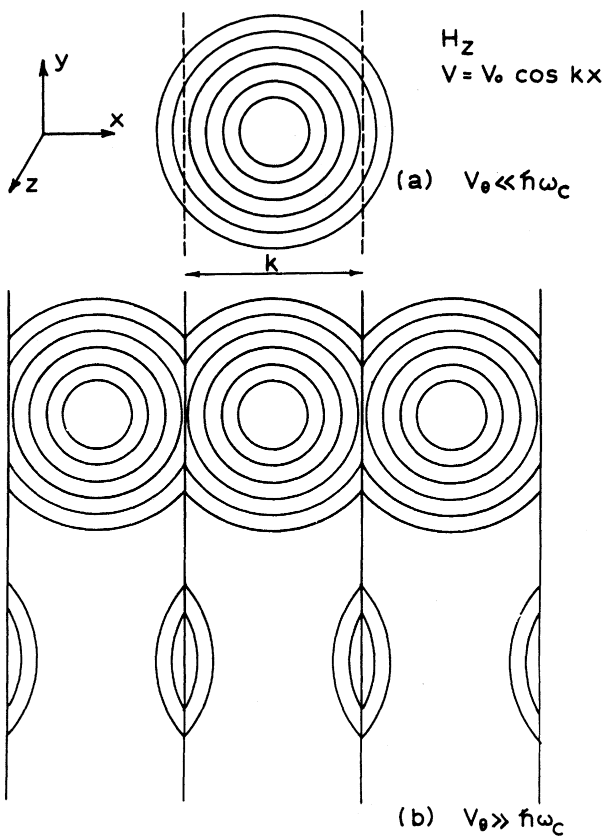

FIG. 1. A simple model to illustrate magnetic breakdown. (a) shows the orbits in k space for the free-electron case which are the same as the orbits when magnetic breakdown occurs; (b) shows the orbits for the low-field case.

that vanish as H^{-2}.

We have seen that in this simple illustration magnetic breakdown changes open orbits into closed orbits and so the H^2 term in the magnetoresistance disappears. More generally, the effect can cause a change of any orbit, i.e., closed, extended, or open, into any other of the same or different category. Consequently, as the field varies it must be possible to observe the disappearance of some periods and the appearance of new ones in experiments such as the de Haas–van Alphen effect, cyclotron resonance, and ultrasonic attenuation.

Possible experimental evidence for magnetic breakdown is the giant orbit observed by Priestley[6] in the de Haas–van Alphen effect in magnesium for fields very near the [000, 1] direction. Such an orbit, with an area larger than the section of the Brillouin zone, cannot occur in any ordinary band structure. However, the existence of an accidental degeneracy near the Fermi energy along the ΓK line (without spin-orbit coupling) and the detailed analysis of the structure of the levels[3] shows that the waist of the Fermi surface in the second zone contacts each "cigar" in the third zone just above and below the basal plane. Spin-orbit coupling should lift the degeneracy, but a sizeable region of k space must remain in which magnetic breakdown occurs for orbits which pass near one set of six points of contact. This would cause transitions between the Fermi surface in the second zone and the "cigars" in the third zone, giving rise to an orbit essentially the same as the equatorial section of the free-electron sphere (Fig. 2); its area gives a value of the period that exactly agrees with the experimental value 1.57×10^{-9} (gauss)$^{-1}$. The narrow angular region (about 4° in radius around the c axis) in which the orbit has been found further supports

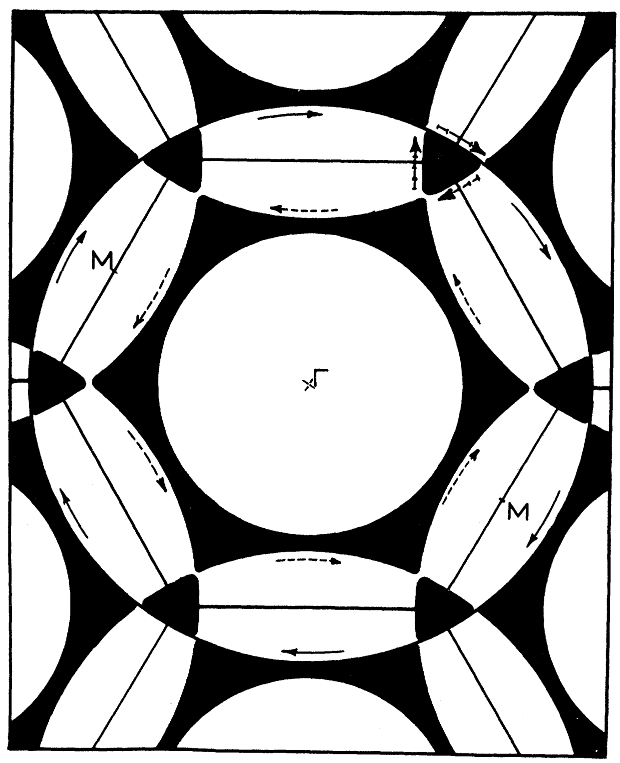

FIG. 2. The giant orbit in magnesium. The figure shows a section of the Fermi surface with a plane parallel to the basal plane and passing through the points of contact removed by spin-orbit splitting. ---→ orbit around the Fermi surface in the second zone at low fields; ⊢⊢⊢→ orbit around "cigars" in the third zone at low fields; ⟶ giant orbit after magnetic breakdown occurs, showing transitions from second zone to third zone to second zone, etc.

this interpretation, because when the field is tipped only slightly off the c axis, the orbit can no longer pass near all six points of contact. The fields used in the experiments (57 to 191 kgauss) should have been enough to overcome the small gaps, but we have not estimated the splitting of the accidental degeneracy.

Another case of magnetic breakdown which may occur at relatively low fields is the magnetoresistance of beryllium for fields perpendicular to the c axis. If the H_1 level[7] of the band structure lies below the Fermi energy, spin-orbit coupling[2] causes the Fermi surface in the third zone to look like two similar cylinders with axes parallel to the c axis; the magnetoresistance must then increase quadratically with H. However, for fields of about 20 kgauss, magnetic breakdown connects these cylinders with the small pockets in the fourth zone, overcoming the small energy gap produced by spin-orbit splitting. The orbits close, and the magnetoresistance must saturate. A similar effect should occur in magnesium at fields very roughly about 200 kgauss from the change in topology of the Fermi surface in the second zone.[2]

In Mg,[3] Zn,[8] and Cd[8] there are two intersecting electron pieces of Fermi surface around L, which are separated by spin-orbit coupling, except at two points of contact on the AL line. For the magnetic field in a plane perpendicular to AL, magnetic breakdown is possible for orbits near the points of contact at any field strength. For arbitrary field orientation $\hbar\omega_c$ must exceed the spin-orbit splitting. In the hexagonal close-packed phases of Li and Na, the piece of Fermi surface around A which is split off from the main body of the Fermi sphere by spin-orbit coupling must show similar behavior because of the vanishing of the splitting along the three lines LAL.

The points of contact on the Fermi surface of graphite[9] are also removed by spin-orbit splitting.[10] Magnetic breakdown phenomena should in principle be observable when the field is nearly normal to the c axis.

Finally, it should be remarked that band gaps introduced by spin-orbit coupling can be magnetic field dependent via the interaction of the magnetic field with the unquenched orbital angular momentum and the electron spin. For the simple example of the degeneracy of p-like states in a reflection plane, the condition for magnetic breakdown is unchanged when the magnetic field is in the plane. On the other hand, magnetic breakdown can occur at somewhat different field strengths for the two different spin orientations when the magnetic field is perpendicular to the plane.

In conclusion we would like to acknowledge several stimulating discussions on this subject with W. A. Harrison.

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*Work supported in part by the Office of Naval Research.

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