SUPERCONDUCTING-NORMAL TRANSITION TIME

A. H. Nethercot, Jr.
Thomas J. Watson Research Center, International Business Machines Corporation, Yorktown Heights, New York
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The nonequilibrium properties of superconductors are of particular interest since these properties cannot be calculated directly from the standard Bardeen-Cooper-Schrieffer (BCS) theory. One such problem is the calculation of the speed at which transitions can occur between the normal and superconducting states when the magnetic field is varied. Since the theory does not clearly indicate what fundamental effects might limit this speed, a number of possible explanations have been suggested between which it is difficult to choose (see Appendix). It is the purpose of this Letter to propose a new explanation which proceeds in a relatively direct and straightforward fashion from the BCS theory and which, since it predicts a rather long time, overrides many of the previous suggestions.

In brief, the present explanation is that the switching speed is limited by a spatial rather than by a purely temporal effect: At fast switching speeds and therefore at high frequencies the skin depth becomes so small that this thin layer is prevented from changing its state by the presence of the large amounts of unswitched adjacent material. As will be described below, it appears from present theories[1,2] that skin depths corresponding to switching speeds of 10^{-10} sec are so thin that their properties are severely altered by the presence of the adjacent unswitched material. Hence these skin depths cannot be said to change their state sufficiently to define switching in times on the order of 10^{-10} sec.

The predictions of this theory are in accord with the presently known experimental facts. It is known from experiments with pulses that superconductors do switch in at least several millimicroseconds. However, experiments[3,4] on the nonlinear properties of superconducting tin at 10 000 Mc/sec indicate that switching is only very incomplete or partial in times on the order of 2×10^{-11} sec. Thus the switching time is bracketed between 10^{-9} and 10^{-11} sec. In these 10 000-Mc/sec experiments, the amount of second harmonic power generated when a high-amplitude rf magnetic field is incident against a bulk or thin film superconductor is compared with the harmonic power calculated under the assumption that there is an instantaneous change in the microwave surface impedance from its normal state value to its superconducting value when the field changes from above critical to below critical during a cycle. Distortion of the reflected wave is observed only when the rf field crosses the critical field curve during a cycle. The amount of this distortion is less than one percent of that calculated. This indicates that there is only a partial change in state. In addition, below 3.2°K harmonics are observed only if the bias field, H_0, is greater than H_c, indicating that at low temperatures switching from superconducting to normal is much more difficult than switching from normal to superconducting. Also, the amount of harmonics observed from thin films decreases very rapidly as the films become thinner than 2500 A. Thus, such films do not switch as readily as bulk materials, at least when the switching field is applied to only one side of the film.

The theoretical situation can be discussed from either the point of view of Parmenter[1] or that of Cooper.[2] The former shows that for superconducting-normal-superconducting (S-N-S) or N-S-N sandwiches there exists a critical distance, $2X_c$, for the central portion. For tin $2X_c\approx1500$ A for both the S-N-S and N-S-N sandwiches. For thicknesses, d, smaller than $2X_c$ the central region shows the general properties only of the adjacent material, while for thicknesses, d, greater than $2X_c$ the central region displays its own properties (but if it is superconducting it has a reduced energy gap even if $d>2X_c$). We will discuss first the case in which a bias field, H_0, is applied to a semi-infinite bulk superconductor, where $H_0>H_c$. An additional microwave field, H_{rf}, of large amplitude is also applied to the surface. The effect of this field is to tend to restore the skin depth to the superconducting state during one-half of the cycle. Therefore we wish to consider a thin layer one skin depth thick of superconducting metal in contact with a bulk normal metal (if 0 denotes empty space, an N-S-0 sandwich). Although the N-S-0 sandwich is not treated by Parmenter, it would be expected that the central region could develop a gap only if the thickness is greater than a critical length, $2X_c'$. This distance, $2X_c'$, would be expected to be somewhat less than $2X_c$ since the presence of N material on only one side rather than on both would be less effective in preventing formation of the gap. A rough estimate leads to a value for $2X_c'$ of $0.8(2X_c)$ or 1200 A for the N-S-0 sandwich. If the thickness is only slightly greater than $2X_c'$, the average gap in the central region is ≈1/30 bulk gap.[1] Any gap, no matter how small, suffices to define the superconducting state for the dc currents. However, at high frequencies and $T > 0$, the gap, ΔE, must be greater than a certain amount for some property such as the microwave surface resistance to decrease by a factor of 2 from its normal state value. From inspection of data on the microwave surface resistance versus temperature, this decrease of a factor 2 is seen to occur when $kT \approx \Delta E$. If we consider the temperature range from 3.7 to 2.5°K, it follows that the gap must be less than the bulk gap only by a factor five (rather than by a factor thirty) for this given decrease in surface resistance to occur. This large a gap will occur only at distances rather greater than $2X_c'$, estimated to be 3 to $4X_c'$. Thus the normal skin depth (δ_N) must be greater than 2000 A if this depth is to be able to change its state from N to S. The skin depth becomes approximately 2000 A at $\nu \approx 3\times10^9$ cps, and hence it is predicted that switching cannot occur when the field is changed from greater than critical to less than critical in times shorter than $\tau = 1/2\pi\nu \approx 0.5\times10^{-10}$ sec.

It is predicted that switching from the S to N state is even slower than the above. When $H_0 < H_c$, the penetration of the field is governed by the superconducting skin depth, δ_S. Hence, at the lower temperatures where $\delta_S \approx \lambda$, switching will be extremely difficult since λ (the penetration depth) is only 500 A. This is in agreement with experiment[4] since no nonlinear effects are seen in tin at 10 000 Mc/sec if $H_0 < H_c$ and $T < 3.2°$. It might be objected that this approach leads at low temperatures to a frequency-independent limitation on switching. However, at sufficiently low frequencies new effects can occur such as motion of S-N boundaries and nucleation and growth of new sites. These effects will then allow switching to take place.

In the above, the S-N-0 case has been differentiated from the N-S-0 case with the important parameters being δ_S and δ_N, respectively. If switching is complete, the spatial variation in both N and S material should be included in some complex manner. However, as a first approximation, a high enough frequency can be chosen that switching is only partial and hence only one skin depth is important.

The above treatment can also be carried through on the basis of Cooper's theory.[2] This theory does not give a critical distance below which the gap completely disappears in a superconducting layer, but instead the gap decreases in size indefinitely as the layer becomes thinner. Adopting again the criterion that the gap cannot be less than one-fifth of its bulk value (to establish a change of a factor two in the microwave surface resistance) and discussing the case $H_0 > H_c$ (the N-S-0 sandwich), we find from Eqs. (1) and (3) of reference 2 that

$$\exp[-N(0)Vt_N/t_S] = \tfrac{1}{5},$$

where t_S is the skin depth and t_N is the effective thickness of the normal material. Since $N(0)V \approx 0.30$ for tin, $t_N/t_S = \frac{1}{2}$. If a limit to $(t_N + t_S)$ is set by the bulk correlation distance ($\xi_0 = 2500$ A), we find that $t_S \geq 1700$ A and that the switching time, τ, is longer than 0.3×10^{-10} sec. This value is in reasonable agreement with the 0.5×10^{-10} sec from Parmenter's theory. However, it appears that this value may be too short since, from the uncertainty principle, the correlation distance for reasonably pure bulk materials must be inversely proportional to the energy gap. If the modified gap is $\frac{1}{5}$ its bulk value, the correlation distance is 12 000 A and this leads to a switching speed of 3×10^{-9} sec, a very slow speed. Cooper's theory needs more development to clarify which correlation distance is involved.

The theory of Cooper is particularly suited for the case of thin films. If the switching field is applied to only one side of the film and if the film is thinner than the skin depth (either δ_S or δ_N), the microwave field varies approximately linearly across the film and falls to a very small value on the other side. Therefore, in general, the microwave field falls off more rapidly than it does in bulk material and at a given frequency the interaction potential $N(0)V$ is more diluted and the gap is less. Thus the degree of switching would be less in the thin film. These results for thin films are in accord with experiment[4] since nonlinear effects at $\nu = 10\,000$ Mc/sec rapidly decrease in amplitude for thicknesses less than ≈2500 A.

There is no experimental evidence on the case when the microwave field is applied to both sides of the thin film. If $H_0 < H_c'$, where H_c' is the thin-film critical field, the situation is very similar to the above except that the field varies even more rapidly in space. The switching should be very slow. The other case, $H_0 > H_c'$, is more difficult to treat since the fields do not vary greatly in space. However, the Gor'kov theory indicates that the transition becomes second order in the magnetic field for the thinner films[5] and this causes a limitation on the switching. For first order transitions an infinitesimal change of H_0 from $(H_c' + \epsilon)$ to $(H_c' - \epsilon)$ causes a well-developed gap to appear and a large change in the microwave surface resistance to occur. In a second order transition a much larger change in field is necessary to produce the requisite gap. It is clearly much more difficult to produce a large change in field in a short time ($\approx 10^{-10}$ sec) than it is an infinitesimal change. The switching field should be at least several percent of the critical field to produce a gap $\frac{1}{5}$ of the bulk gap.[5] Since the critical field for thin films is very much greater than that for bulk materials, the requisite field may thus be quite appreciable. A clearer understanding of the relation between the Gor'kov and the spatial dependence theories would lead to a better understanding of these temporal effects as well as of the high-field superconductivity effects associated with filamentary behavior.

We have had informative and interesting conversations with a number of people. Discussions with R. J. von Gutfeld and R. H. Parmenter have been particularly helpful.

Appendix. Since no discussion of the various possible explanations for a limitation of the switching time has been given previously, a brief listing of some of these will be given here. The usual limitation, that of the growth of normal domains limited by eddy currents, has no relevance when skin depths small compared to the correlation distance are involved.

(I) The uncertainty relation $\Delta E \Delta t \approx \hbar$ leads to $\Delta t \approx 10^{-12}$ sec if ΔE is the energy gap. This time is too short.

(II) The correlation distance divided by the phonon velocity (ξ_0/v_s) leads to a time of about 10^{-10} sec and could be interpreted as a signaling time between correlated electrons by means of the interaction responsible for superconductivity. However, it is difficult to guess from the BCS theory whether such an effect should in principle be present and in addition the electrons are physically moving much faster than the phonons.

(III) A relaxation time such as l/v_f (where l is the electron mean free path and v_f the Fermi velocity) should come into the theory since an electron trajectory cannot be greatly altered until it is terminated. However, in the treatment of the anomalous skin effect by the ineffectiveness concept it is not always clear whether l/v_f or δ/v_f should be used.[6] The former would lead to switching times of from 10^{-9} sec to much shorter values and the latter to about 10^{-13} sec. The experimental fact that similar switching behavior is observed in situations where l should be drastically different indicates that l/v_f may not come in directly.

(IV) It has also been speculated[7] that the velocity of second sound ($v_{\rm II}$) in the electron ensemble could possibly limit the rate of conversion of superconducting electrons to normal. This would lead to a time $\tau \approx \lambda/v_{\rm II} \approx 10^{-9}$ sec if[8] $v_{\rm II} \approx [H_c^2(1-t^4)/2\pi\rho]^{1/2} \approx 10^4$ cm/sec. However, it is not clear that relaxation effects would allow this type of second sound to be propagated (especially in impure metals or in evaporated films) or even whether second sound of a different variety[9] and with a velocity of $\sim 10^8$ cm/sec would or would not be preferred.

[1]R. H. Parmenter, Phys. Rev. 118, 1173 (1960).

[2]L. N. Cooper, Phys. Rev. Letters 6, 689 (1961).

[3]A. H. Nethercot, Jr., Proceedings of the Seventh International Conference on Low-Temperature Physics (University of Toronto Press, Toronto, 1960), p. 231.

[4]A. H. Nethercot, Jr., and R. J. von Gutfeld (to be published).

[5]D. H. Douglass, Jr., Phys. Rev. Letters 7, 14 (1961), and references therein.

[6]A. B. Pippard, Advances in Electronics 6, 1 (1954), especially p. 33.

[7]J. Bardeen (private communication).

[8]See J. Bardeen and J. R. Schrieffer, in Progress in Low-Temperature Physics (North-Holland Publishing Company, Amsterdam, 1961), Vol. 3, p. 266.

[9]D. J. Thouless and D. R. Tilley, Proc. Phys. Soc. (London) 77, 1175 (1961).