SELF-DIFFUSION COEFFICIENT AND NUCLEAR SUSCEPTIBILITY OF LIQUID HELIUM-THREE*

A. C. Anderson, W. Reese, R. J. Sarwinski, and J. C. Wheatley†
Department of Physics, University of Illinois, Urbana, Illinois

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Recent measurements of the specific heat[1,2] and the thermal conductivity[3] of liquid He^3 at low pressures have demonstrated, for temperatures less than approximately 0.04°K, the temperature dependences predicted by the Fermi liquid theory of Landau[4] and Abrikosov and Khalatnikov.[5] However, the magnetization self-diffusion coefficient measured using the spin-echo method by Anderson, Hart, and Wheatley[6] (AHW) obeyed a $T^{-3/2}$ temperature dependence down to 0.03°K in contradiction with the T^{-2} dependence predicted by Hone[7] on the basis of the Fermi liquid theory. As pointed out by AHW, their experiment was susceptible to error, particularly at the lowest temperatures, because of long equilibrium times and difficulties with thermometry. Consequently, the experiment was repeated under much improved conditions and extended to lower temperatures. We now find the diffusion coefficient to be proportional to T^{-2} below 0.04°K - 0.05°K. The nuclear susceptibility is constant below 0.1°K and equal to the value previously measured (AHW) within experimental accuracy. The lowest temperature of these measurements was 0.02°K. The measurements were carried out at a pressure of 12 cm Hg.

The cell containing the He^3 was made from Epibond 100A,[8] and the plug which closed the cell was sealed with Epibond 121.[8] Both resins are nonmagnetic.[9] The space containing He^3 consisted of a cylinder 1.2 cm high and 1.2 cm in diameter with a coaxial cylindrical tail 5.1 cm long and 0.25 cm in diameter. The inside surface of the cell was lined with 400, 0.004-inch diameter, 99.999% pure annealed copper wires about half of which did not extend into the tail. The larger cylinder contained 1.29 g of powdered cerium magnesium nitrate (CMN) which served as a thermometer. The CMN was prevented from entering the tail by a cotton plug. The He^3 was introduced into the cell by a 1/64-in. o.d. by 0.003-in. wall 70-30 cupro-nickel tube which had a negligible magnetic effect.[9] A pi-wound, 3000-turn pickup coil of 0.002-in. diameter manganin wire was placed over 2.0 cm of the tail at the greatest possible distance from the CMN. The above construction gave internal equilibrium times of 15 minutes or less at the lowest temperatures while providing sufficient separation between the He^3 being measured and the CMN to give a negligible temperature-dependent magnetic gradient. The He^3 was cooled by 37.8 g of potassium chrome alum using techniques described elsewhere.[10]

The thermometer was calibrated in the temperature range of 1°K to 0.3°K using a 470-ohm Speer resistor which previously had been calibrated in this temperature range with a CMN magnetic thermometer. The accuracy of the temperature scale is estimated to be 2%.

The He^3 used had a He^4 content of less than 3 parts in 10^5 and a tritium decay rate of less than 900 disintegrations/sec. The apparatus outside the cryostat and the method used to obtain the echoes were similar to those described elsewhere.[11]

The self-diffusion coefficient, D, was measured by the method of spin echoes using an rf field of frequency 85 kc/sec and of amplitude $(2H_1)$ 0.6 gauss measured using a nutation technique. A 90°-180°-180° pulse sequence was used with a fixed time delay between the 90° pulse and the first 180° pulse and a variable time, τ, between the two echoes. The resulting echoes were displayed on an oscilloscope and photographed. The ratio, R, of the amplitudes of the two echoes is given by $R = \exp(-\gamma^2 G^2 D\tau^3/12 - \tau/T_2)$, where G is the magnetic field gradient, $\gamma = 2.038 \times 10^4$ $(\text{gauss-sec})^{-1}$, and T_2 is the transverse relaxation time. The self-diffusion coefficient was calculated neglecting the effect of T_2, which had been found to be greater than 0.74 sec near $T = 0.03$°K.[6] Measurements of D were made with G alternately parallel and antiparallel to the steady field so as to eliminate to first order the effects of residual gradients.

The susceptibility was measured with the same apparatus but with no applied gradient and with only one 180° pulse applied between 2.5 and 5 msec after the 90° pulse. No systematic effect of the time of the echo on the echo height was observed, even at the lowest temperatures. In order to increase the echo height, the He^3 was magnetized in a field of about 100 gauss which was turned off just before the pulse train was initiated. This field remained the same throughout the experiment. The gain of the amplifiers was measured at each point and the echo heights were corrected to constant gain. With the above conditions, the echo height, corrected for gain,

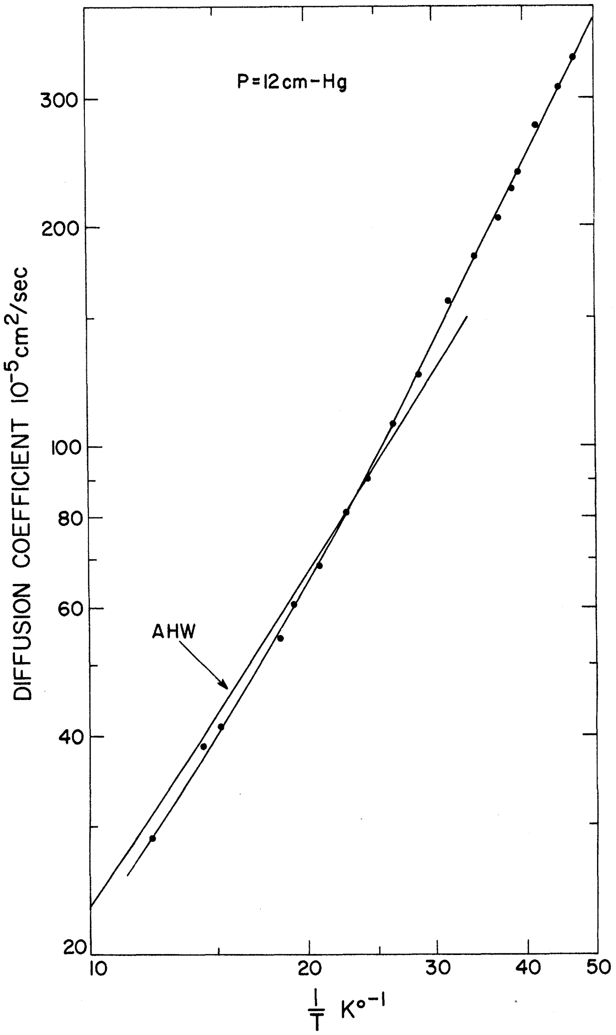

FIG. 1. Self-diffusion coefficient, D, vs $1/T$. The points represent averaged data. The curve has a slope of 2.00 ±0.02 for T less than 0.04°K. The curve labeled AHW is the smoothed curve from reference 6.

should be proportional to the nuclear susceptibility.

The results of the diffusion measurements are shown in Fig. 1. For T less than 0.04°K to 0.05°K we find that $D = 1.54\times10^{-6}T^{-2}$ cm^2 °K^2/sec. The exponent of T in this temperature range is 2 with an estimated accuracy of 1%. The coefficient of D has an estimated accuracy of 7% on the basis of uncertainties in the temperature, time scale, and gradient. At higher temperatures the results are almost parallel to those of AHW but displaced about 5% toward lower T. Their low-temperature points lie below ours, giving a slower temperature dependence than that observed by us. The temperatures used by AHW were made uncertain, especially at the low-temperature end, by the presence of spurious magnetic materials. Moreover, their equilibrium times were several hours at the lowest temperatures, indicating that very small spurious heat inputs could adversely affect the results.

In Fig. 2 we have plotted $(\chi T)_{1.153°\mathrm{K}}/\chi$ vs $1/T$, where the numerator is the product of the echo height at the bath temperature, the bath temperature (1.153°K), and the ratio of the density of the liquid at 0°K to that at 1.153°K as estimated from the data of Lee, Fairbank, and Walker[12]; and where the denominator is the echo height at the value of $1/T$ given on the abscissa. If it is assumed that He^3 obeys Curie's law at the bath temperature, then the ratio $(\chi T)_{1.153°\mathrm{K}}/\chi$ is the effective magnetic temperature, T^*, of the He^3 at low temperatures. Below 0.1°K we find T^* to be constant within experimental accuracy. The average of the data for run 4, which is the best run for this purpose since the check at the bath temperature was made at its conclusion, gives $T^* = 0.347 \pm 0.010$°K for T less than 0.1°K, exactly

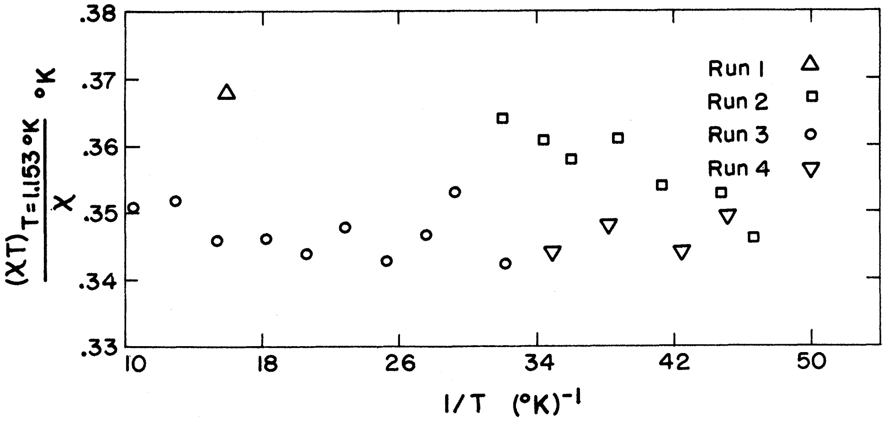

FIG. 2. Effective magnetic temperature $T^* = (\chi T)_{1.153°\mathrm{K}}/\chi$ vs $1/T$. $(\chi T)_{1.153°\mathrm{K}}$ is the product of the susceptibility and temperature at 1.153°K. χ is the susceptibility at temperature T. The points within each temperature interval represent averages for each run.

the result given by AHW. Within each run we found χ to be constant to within about 2%, so it is likely that χ is more nearly constant than is indicated by all the data collected in Fig. 2, for which there is about 5% scatter. The slight drop in χ below 0.05°K observed by AHW and attributed to experimental effects was not observed in these measurements.

The susceptibility can be written [Eq. (5.8), reference 5]

$$\chi = \frac{\beta^2}{4} \frac{(\partial\tau/\partial\epsilon)_\mu}{[1+\frac{1}{4}\bar{\zeta}(\partial\tau/\partial\epsilon)_\mu]} = \frac{\beta^2}{4}\frac{N}{VkT^*}, \tag{1}$$

an equation which defines T^*. In this equation N/V is the number of He^3 atoms per unit volume, $\beta = \gamma\hbar$ is the product of the gyromagnetic ratio and Planck's constant divided by 2π, k is Boltzmann's constant, $(\partial\tau/\partial\epsilon)_\mu$ is the density of states at the Fermi surface, and $\bar{\zeta}$ is the average of the effective spin interaction function over the Fermi surface. Using the most recent empirical values for the molar volume[12,13] and effective mass,[2] the value of T^* given above leads to $[1+\frac{1}{4}\bar{\zeta}(\partial\tau/\partial\epsilon)_\mu] = 0.293$.

Hone[7] writes the diffusion coefficient in the form

$$D = \tfrac{1}{3}V_0{}^2\tau_D[1+\tfrac{1}{4}\bar{\zeta}(\partial\tau/\partial\epsilon)_\mu], \tag{2}$$

where $V_0 = P_0/m^*$ is the quasi-particle velocity at the Fermi surface, τ_D is a relaxation time for diffusion, and the quantity in square brackets is given above. The resulting empirical value of τ_D is $4.6\times10^{-13}T^{-2}$ sec °K^2. If one uses in addition to the above information the most recent value for the speed of sound[14] to estimate[7] the value of τ_D from the Fermi liquid theory, one obtains $\tau_D = 1.0\times10^{-13}\,{}^{\circ}T^{-2}$ sec °K^2. A relaxation time can also be computed from the measured values of thermal conductivity[3] in the Fermi liquid region giving $\tau_K = 6.4\times10^{-13}\,T^{-2}$ sec °K^2. If there were no dependence of the quasi-particle scattering probability on the azimuthal angle of scattering and in addition no spin effects, then by theory[5,7] one has $\tau_D/\tau_K = \frac{4}{3}$; experimentally we observe $\tau_D/\tau_K = 0.70$.

Using the above values of the relaxation times, one finds in both cases that the ratio of the uncertainty in quasi-particle energy to kT is less than 1 at 0.04°K: $\hbar/kT\tau = 0.5$ and 0.7 using τ_K and τ_D, respectively. Moreover, since the specific heat, thermal conductivity, and self-diffusion coefficient all have the predicted temperature dependences below 0.04°K, we conclude that the concept of He^3 as a Fermi liquid at temperatures less than this and at low pressures is valid.

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