FIELD-SWEPT MASER OSCILLATION

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Recently Singer and Wang[1] (SW) have put forth an analysis which purportedly describes the behavior of a very wide variety of maser oscillators. An experimental verification was offered, based on a pulsed maser oscillation waveform from the present writer's work.[2]

The purposes of the present note are as follows: (1) To show that our waveforms, on one of which SW based their Fig. 1, show large qualitative and quantitative discrepancies from their predicted radiation envelope structure. For this purpose we will first discuss their main Eq. (8) and show how its solution (SW Fig. 2) differs from our waveforms. Our reason for the discrepancies is that our oscillator was of the "field-swept" type, in which emission occurs as the Zeeman field value H_0 is swept through resonance, giving a radiation envelope determined largely by the H_0 sweep rate and showing a modulation due, we believe, to a mechanism unrelated to that invoked by SW. (2) To summarize our own analysis[2] of the oscillator behavior, which we feel conforms adequately with our waveforms. In particular our asymptotic form [Eq. (2) below] for relatively fast sweep and weak oscillation will be seen to agree quantitatively within experimental limits, and qualitatively in all details, with an appropriate sample (Fig. 3) from a series of oscillograms.[2] We hope this note will help explain waveforms seen in other paramagnetic maser oscillators involving H_0 sweep.

As for the nature of the SW solution, the nutation-angle substitution[3] $\theta = \int_0^t 2\hbar\mu H(t')dt'$ in their Eq. (8), with $pE \to \mu H$ and N_e constant, the relevant case, gives $\ddot{\theta} + \tau^{-1}\dot{\theta} + (\tau\tau_\gamma)^{-1}\sin\theta = 0$, the simple damped-pendulum equation, where $\tau = 2Q/\omega$ = cavity ringing time and τ_γ = radiation damping time.[4] This writer noted previously[3] (see also Yariv[5]) that even with $\theta(0) \sim \pi$ (initial inverted state), in the pertinent case the radiation amplitude $H = \gamma^{-1}\dot{\theta}$ is given adequately by $\exp(-t/\tau) \times \sin[t/(\tau\tau_\gamma)^{1/2}]$ after the first "cycle" or two. The resulting emission would have mean duration of order τ and would show roughly uniform 100% modulation with period of order $(\tau\tau_\gamma)^{1/2}/2\pi$. In our experiments[2] $\tau \cong 0.6\,\mu$sec. Our observed durations depended on the sweep rate, and ranged[2] from 7 μsec (fastest sweep) to 500 μsec (slowest sweep), with representative values 500, 15, and 90 μsec, respectively, for Figs. 1, 2,

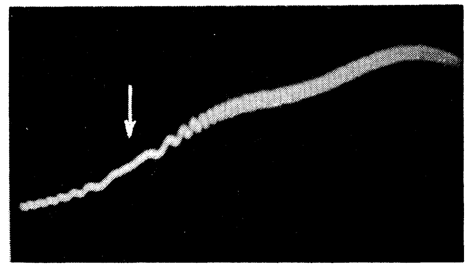

FIG. 1. Field-swept maser oscillation, displaying emission envelope $|H(t)|$. Slow sweep, $dH_0/dt = 800$ gauss/sec. $\beta \cong 0.001$; $\tau_\gamma \cong 0.1\,\mu$sec. Trace length 200 μsec. Fractional Zeeman energy release ~70%. Shown here is the rising portion of the envelope, which decays to the right. The thickness of the trace in the upper part is not noise but unresolved rapid modulation. The arrow is believed in the vicinity of passage through the line-center resonance ($t = 0$), though this point was not localized experimentally. Equation (2) of text is not applicable.

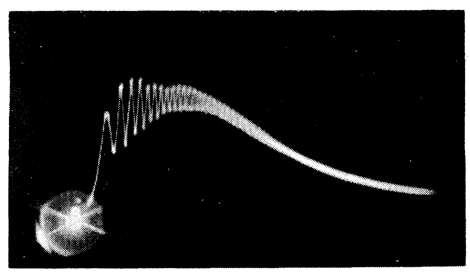

FIG. 2. Field-swept emission. $dH_0/dt = 2\times10^4$ gauss/sec. $\beta \cong 0.03$; $\tau_\gamma \cong 0.1\,\mu$sec. Trace length 30 μsec. Owing to the fast sweep, in spite of about 50% Zeeman energy release the modulation pattern is surprisingly like that of Eq. (2) except for average shallowness. Period of fifth modulation cycle $T_5 = 1.2\,\mu$sec (theor) = 0.8 μsec (exp).

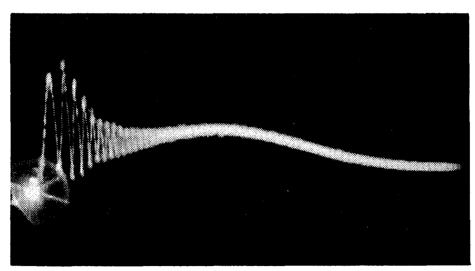

FIG. 3. Field-swept emission. $dH_0/dt = 4\times10^3$ gauss/sec. $\beta \cong 0.03$; $\tau_\gamma \cong 0.6\,\mu$sec. Trace length 200 μsec. Energy release ~30%. $T_5 = 2.8\,\mu$sec (theor) = 4.0 μsec (exp). Equation (2) well describes this plus seven other oscillograms in reference 2, with moderate to rapid sweep and not too strong oscillation.

and 3 shown in this note. Our waveforms show, rather than uniform 100% modulation, a modulation with frequency increasing indefinitely with time, after sweep through resonance, with a percent depth that depends strikingly on the sweep rate.

The possibility of observing two-level paramagnetic maser oscillation showing the pendulum-like behavior[6] expected from a static-H_0 analysis[1,3] was investigated by the writer.[2] We concluded that, when one induces oscillation by the common procedure of sweeping the field H_0 into or through resonance, at least in our apparatus it was impossible to establish oscillation conditions "suddenly" enough to allow emission to occur with essentially static parameters (the field H_0). Emission with H_0 varying in time linearly through resonance was then studied systematically, and was the subject of our work,[2] which we now summarize.

Our theoretical treatment[2] of the field-swept case starts with the coupling equation between a complex radiation field amplitude H and a complex transverse magnetization M. For sweep through resonance rapid enough to allow only a small fractional Zeeman energy release, such that $M_z \cong -M_0$ holds, the spin-dynamic equations are soluble explicitly for arbitrary $H(t)$ and $\Delta H_0(t) = H_0(t) - \omega/\gamma$. We choose $\Delta H_0 = \gamma^{-1}\alpha t$, where α is a constant sweep rate. We assume a Lorentz line of width $(T_2^*)^{-1}$ and, for convenience,[7] that $T_2^* \cong \tau$. One gets

$$\frac{d^2H}{dt^2}+\left(\frac{2}{\tau}-i\alpha t\right)\frac{dH}{dt}+\frac{1}{\tau}\left(\frac{1}{\tau}-\frac{1}{\tau_\gamma}-i\alpha t\right)H=0, \qquad (1)$$

which is readily transformable to Weber's equation.[8] The pertinent solution, in terms of a Weber's function of imaginary order, is

$$H = K\exp[(i\alpha t^2/4 - t/\tau)D_{i/\beta}[-(1+i)(\alpha/2)^{1/2}t],$$

with $K=$ const. From asymptotic forms[2,8] we find, for moderate to rapid sweep,

$$|H(t)| \cong A(t)\left[1+\frac{1}{\beta^{1/2}\alpha^{1/2}t}\sin(\tfrac{1}{2}\alpha t^2+\phi)\right], \qquad (2)$$

where ϕ varies slowly and $\beta = \alpha\tau\tau_\gamma$ is a dimensionless sweep rate. This form described the radiation envelope for $\beta \gtrsim 1/8\pi \cong 0.04$ and $t \gtrsim (2\pi/\alpha)^{1/2}$. $|H(t)|$ rises monotonically during the interval about passage through resonance ($t=0$). $A(t)$, giving the gross shape, rises to simple maximum at roughly $t_1 = \beta^{-1}(2\tau_\gamma/\tau)^{1/3}\tau$, and then decays, asymptotically like $\exp(t/\tau - 1/\beta^2\alpha t^2)$. The sine term gives an amplitude modulation, with fractional depth about 100% (for $\beta \sim 1/8\pi$) initially, but decreasing in percentage as $1/t$, and increasing indefinitely in frequency with t.

The modulation mechanism described by (1) and (2) inherently involves the time-varying H_0, which continually tends to throw the magnetization into the wrong phase, with respect to the radiation field, for oscillation buildup. Modulation due to this mechanism should vanish for vanishingly slow sweep.[9]

Figures 1-3 are from the writer's experimental field-swept oscillator,[2] and display $|H(t)|$. Relevant measured parameters were: $\tau \cong 0.6\ \mu$sec; $T_2^* \cong 0.8\ \mu$sec. τ_γ and $dH_0/dt = \gamma^{-1}\alpha$ were adjustable, respectively, over the ranges 0.1-0.8 μsec and 0.8-40 kilogauss/sec, giving a β range 0.001-0.06. Equation (2) should apply best to Fig. 3. The linearization $M_z \cong -M_0$ of Eq. (1) holds reasonably for Fig. 3 but marginally for Fig. 2. In spite of small β, and nonlinearity, even Fig. 1 has some features in common with Eq. (2). Modulation appears most marked just when Eq. (2) applies. The predicted period $T_n = (\pi/n\alpha)^{1/2}$ from (2), of the nth modulation period to appear, is correct within 50% for T_5 in Figs. 2 and 3. For the expected emission duration T, a useful rough estimate[2] is $2(\tau\tau_\gamma)^{1/2}\beta$, yielding reasonable values 500, 20, and 50 μsec for Figs. 1, 2, and 3, respectively.

We have shown that the maser oscillator emission envelope had in our case a structure dependent almost entirely on the field sweep. Though a simple pendulum-like energy exchange between cavity and emitting sample, described by the "general analysis" of SW, is a basic process,[2] it had nothing to do with our case, with either fast or slow H_0 sweep.

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[1]J. R. Singer and S. Wang, Phys. Rev. Letters 6, 351 (1961).

[2]J. C. Kemp, thesis, University of California, Berkeley, 1960 (unpublished). The relevant portion is available as a University of California Report AFOSR-TN-60-509, issue 275, E.R.L., Berkeley, 1960 (unpublished).

[3]J. C. Kemp, J. Appl. Phys. 30, 1451 (1959). Though this paper covered the rather fundamental "pendulum nutation" mechanism (discussed earlier, without the nonlinearity, in reference 4), the possible role of H_0 sweep in actual experiments was not then appreciated. Adding line broadening to the static-parameter (static H_0) emission problem was discussed in this reference, in reference 5, and further in reference 2 (thesis only), and gives a possible lengthening of emission duration and a modulation frequency which can be smaller on the aver-

age but cannot increase with t over the envelope.

[4]N. Bloembergen and R. V. Pound, Phys. Rev. 95, 8 (1954). $\tau_r^{-1} = \gamma M_0 Q/\mu_0 V$ (mks units[2]), where γ = magnetogyric ratio, M_0/V = available magnetization per unit cavity volume. In SW notation, $\tau_r^{-1} = 4\pi N_e \gamma p Q$ or $4\pi N_e \gamma \mu Q$.

[5]See A. Yariv, J. Appl. Phys. 31, 740 (1960), who omitted a minor approximation (reference 2 Report, Appendix) leading to the pendulum equation, but gave a computer solution justifying our statement that the solution is quite like an exponentially decaying sinusoid.

[6]An unpublished waveform of P. F. Chester, P. E. Wagner, and J. G. Castle, Jr., Scientific Paper 6-94439-8-P4, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania, 1958 (unpublished), p. 10, taken without H_0 sweep, shows the characteristic 100% almost uniform modulation, in my knowledge the only observed such case.

[7]It happened that $T_2^*/\tau \cong 1$ in our experiments. The mathematics can easily accommodate the more general case.

[8]E. T. Whittaker and G. N. Watson, Modern Analysis (Cambridge University Press, New York, 1935), 4th ed., pp. 347-351.

[9]Sweep-rate dependence of modulation depth was also noted by S. Foner, L. R. Momo, and A. Mayer, Phys. Rev. Letters 3, 36 (1959).

HELICITY OF μ^- MESONS; MOTT SCATTERING OF POLARIZED MUONS*

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Current speculations[1] on weak interactions raise the possibility that the neutrino associated with the μ meson may be different from the neutrino of beta decay. Thus the helicity of ν_e may be different from that of ν_μ. This experiment was designed to observe the helicity of the μ^- meson (and hence that of the associated $\bar{\nu}_\mu$) from the reaction:

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu. \qquad (1)$$

The method used was to analyze the polarization of the muons by Coulomb scattering of a transversely polarized μ^- beam (Mott scattering) which predicts a left-right asymmetry due to spin-orbit coupling:

$$\sigma(\theta, \phi) = f(\theta)[1 + g(\theta) P_y \cos\phi]. \qquad (2)$$

Here P_y is the transverse muon polarization, along the y axis, and the z axis is in the direction of the incoming muon momentum. Explicit calculations of the energy-dependent terms, $f(\theta)$ and $g(\theta)$, were performed by Rawitscher,[2] and by Franklin and Margolis.[3] Integration of the equation for the experimental arrangement used leads to a predicted asymmetry,

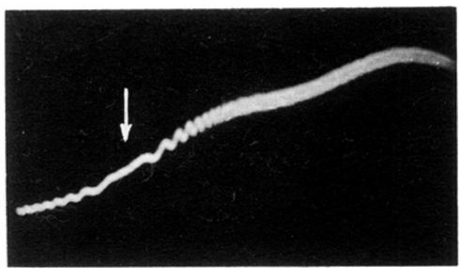
$$(L - R)/(L + R) = \pm 0.09 \text{ for } P_y = \mp 1, \qquad (3)$$

where L and R represent the number of scattering events into the left and right counters with respect to the plane defined by the μ momentum and spin.

Our results, corrected for accidentals and based on ~100 hours of counting (~3×10^8 muons), were $L = 515$ and $R = 618$. This leads to an asymmetry of -0.090 ± 0.031 for our $P_y = 0.9$, from which it is concluded that the helicity of the μ^- meson in reaction (1) is positive (right-handed) in agreement with the V-A theory. Thus the antineutrino in (1) is also right-handed, and hence no evidence is deduced from this result for $\nu_e \neq \nu_\mu$. This conclusion is in agreement with those obtained by measuring the knock-on electrons produced by high-energy longitudinally polarized μ mesons in magnetized iron.[4,5]

The agreement of predicted and observed asymmetry is interesting in that, to our knowledge, there are no pre-existing data on polarized electron or muon scattering in this momentum transfer region ($\langle \Delta q \rangle_{\text{av}} \sim 100$ Mev/c). More explicit discussion of this and of $\sigma(\theta)$ will be published later. We only note here that the inelastic scattering contribution is strongly limited by the low energy of the outgoing muon.

The Mott scattering experiment involved the extraction of a low-energy π^- beam, and the production from this beam of transversely polarized μ mesons by decay in flight. A magnetic channel was designed to this end, and its front inserted into a recessed window in the cyclotron vacuum chamber. The resulting beam of 43-Mev π mesons was further moderated by 3 in. of lithium, placed at the focus of a collimating magnetic system so as to produce a parallel (±2°) beam of (28 ± 2.5)-Mev π mesons in the decay region. The resulting muons selected by decay angle had an average transverse polarization of 90% ($P_y = 0.9$) in the plane of decay, and were uniformly dis-



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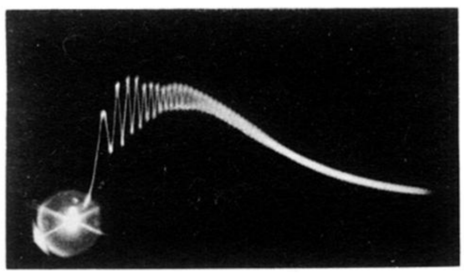

FIG. 2. Field-swept emission. $dH_0/dt = 2\times10^4$ gauss/sec. $\beta \cong 0.03$; $\tau_\gamma \cong 0.1\,\mu$sec. Trace length $30\,\mu$sec. Owing to the fast sweep, in spite of about 50% Zeeman energy release the modulation pattern is surprisingly like that of Eq. (2) except for average shallowness. Period of fifth modulation cycle $T_5 = 1.2\,\mu$sec (theor) $= 0.8\,\mu$sec (exp).

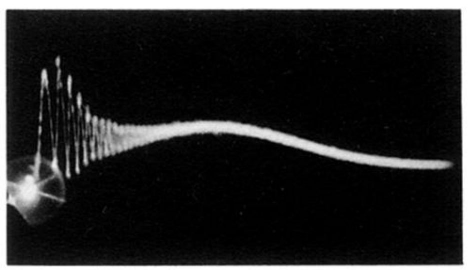

FIG. 3. Field-swept emission. $dH_0/dt = 4 \times 10^3$ gauss/sec. $\beta \cong 0.03$; $\tau_\gamma \cong 0.6\,\mu$sec. Trace length $200\,\mu$sec. Energy release $\sim 30\%$. $T_5 = 2.8\,\mu$sec (theor) $= 4.0\,\mu$sec (exp). Equation (2) well describes this plus seven other oscillograms in reference 2, with moderate to rapid sweep and not too strong oscillation.